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ON ANTI-INVARIANT SUBMANIFOLDS OF COSYMPLECTIC MANIFOLDS

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A normal almost contact metric manifold is said to be cosymplectic if its fundamental 2-form and contact form are both closed. Cosymplectic manifolds and their submanifolds have been studied by D.E. Blair ([1], [2]), G.D. Ludden ([2], [11]), S.I. Goldberg ([8]), S.S. Eum ([4], [5], [17]), K. Yano ([8], [17]) and U-H. Ki ([9], [17]).

In the last decade, the study of anti-invariant submanifolds of Kaehlerian and Sasakian manifolds has provided us with a great deal of new and valuable results ([3], [10], [12], [16], [18], etc.). However, the study of anti-invariant submanifolds of cosymplectic manifolds is not performed yet. The purpose of the present thesis is to study anti-invariant submanifolds of cosymplectic manifolds and obtain some results.

In chapter I, we recall fundamental concepts of cosymplectic manifolds and prepare structure equations for anti-invariant submanifolds of cosymplectic manifolds. Lastly we obtain some propositions.

In chapter II, we study anti-invariant submanifolds, which are tangent to the structure vector field, of cosymplectic manifolds. In §1 we obtain basic formulas and define η-umbilical submanifolds of cosymplectic manifolds. In §2 we prove that an η-umbilical submanifold of a constant curvature space with respect to γ_{ji} is locally symmetric and conformally flat. §3 is devoted to the study of submanifolds with parallel f-structure in the normal bundle. When the ambient space is of constant ϕ-holomorphic sectional curvature, we conclude that the submanifolds are of the form: (a) locally a product of constant curvature space and one dimensional space (b) locally flat (c) conformally flat. In §4, we investigate submanifolds of cosymplectic manifold with vanishing cosymplectic Bochner curvature tensor. We obtain some conditions in order that the submanifold is locally a product of conformally flat Riemannian manifold and one dimenisonal space.

In chapter III, we study anti-invariant submanifolds, which are normal to the structure vector field, of cosymplectic manifolds. In §1 and §2 we obtain basic formulas and investigate the Ricci tensor and scalar curvature of the submanifolds.

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§3 is devoted to the study of submanifolds with parallel f-structure. We also obtain a necessary and sufficient condition for the submanifold to be of constant curvature. In §4 we prove that a totally umbilical submanifold of a cosymplectic manifold with vanishing cosymplectic Bochner curvature tensor is conformally flat.

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STRUCTURES OF A HYPERSURFACE IMMERSED IN A PRODUCT OF TWO SPHERES

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Recently, K. Yano and M. Okumura [20] defined the (f, g, u, v, λ)-structure induced on submanifolds of codimension 2 of an almost Hermitian manifold or real hypersurfaces of an almost contact metric manifold, which is a very useful method in studying Riemannian manifolds admitting that structure. Also, Yano [18] studied the differential geometry of a product of two spheres $S^n \times S^n$ and proved that the (f, g, u, v, λ)-structure is naturally induced on $S^n \times S^n$ as a submanifold of codimension 2 of a $(2n+2)$-dimensional Euclidean space or a real hypersurface of $(2n+1)$-dimensional unit sphere $S^{2n+1}(1)$.

G.D. Ludden and Okumura [13] studied the so-called invariant hypersurface of $S^n \times S^n$, which is derived from the almost product structure defined by its projection operators on $S^n \times S^n$.

On the other hand, it is well-known that the so-called $(f, g, u, v, w, \lambda, \mu, \nu)$-structure is naturally induced on submanifolds of codimension 3 of an almost Hermitian manifold or real hypersurfaces of a manifold with (f, g, u, v, λ)-structure (cf. [8], [9], [22]). Therefore, real hypersurfaces immersed in $S^n \times S^n$ admit the $(f, g, u, , v, w, \lambda, \mu, \nu)$-structure deduced from the (f, g, u, v, λ)-structure defined on $S^n \times S^n$. From this point of view, S.S. Eum, U-H. Ki and Y.H. Kim [5] researched partially real hypersurfaces of $S^n \times S^n$ by using the concept of k-invariance.

The purpose of the present paper is devoted to study some intrinsic characters of real hypersurfaces immersed in $S^n \times S^n$, characterize global properties of them by using some integrable condition and prove that the generic submanifold of $S^n \times S^n$ with the almost contact metric structure is the real hypersurface.

In chapter I, we recall the intrinsic properties of $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$ and have some algebraic relationships and structure equations of hypersurfaces of $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$.

In chapter II, we determine mainly a minimal hypersurface of $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$ satisfying $\lambda^2 + \mu^2 + \nu^2 = 1$.

In chapter III, we find the necessary and sufficient condition for a hypersurface of $S^n \times S^n$ being k-antiholomorphic and prove its global properties.

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In chapter IV, we define an integrable condition for the induced structure on a hypersurface of $S^n\times S^n$ which is called to be normal and look into an intrinsic character of a normal k-antiholomorphic hypersurface of $S^n\times S^n$.

In chapter V, we find the necessary and sufficient condition for a hypersurface of $S^n\times S\times$ for being k-invariant, and prove that it is isometric to $S^{n-1}\times S^n$.

In chapter VI, we have a global form of a complete hypersurface of $S^n\times S^n$ under some algebraic conditions.

In the last chapter VII, we prove that a generic submanifold of $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$ admitting an almost contact metric structure is a real hypersurface.

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