

기류가 있는 관에서 가열에 의한 열음향력의 발생

(Thermoacoustic Power Generation by a Heater in a Tube with Air Current)

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Abstract

Thermoacoustic oscillation induced by a heater in a tube with air current is studied theoretically. Linearized perturbation equations are derived in dimensionless form under the assumption that the system is one dimensional. The equation to predict the acoustic power generation from a heating surface is derived and calculated by solving differential equations numerically. The effect of the mean velocity of the air current is illustrated. The energy conversion mechanism is shown by pressure-volume diagram like a heat engine.

要 約

기류가 있는 관에서 발생하는 열음향振動에 관하여 이론적으로 연구하였다. 加熱히터가 平面熱源이란 假定하에 線形擾動方程式으로 지배방정식을 세우고 열음향력을 예측하기 위한 式을 유도하였다. 加熱面으로부터 空氣로 전달되는 熱傳達의 振動成分에 의하여 열음향력을 구하기 위한 式이 表示되었다. 지배방정식을 수치적으로 풀어서 진동 열전달을 구하였으며 이의 열음향發生에 기여하는 成分을 가열온도에 대하여 기류속도의 함수로 나타내었으며 기류의 속도가 열확산속도 $\sqrt{\omega \alpha}$ 보다 약간 큰 경우 最大로 되는 것이 발견되었다. 열에너지로부터 음향에너지로의 에너지 변환을 열기관에서와 같이 압력 체적 선도로 표시하였다.

I. INTRODUCTION

Thermally induced oscillation due to heat addition in a tube by combustion or heaters

are well known phenomena. The oscillation occurring when heat is added to an internal grid located in the lower half of a vertical pipe with ends open and with air current

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is a typical thermoacoustic oscillation known as Rijke phenomenon.

The criterion for any type of thermoacoustic oscillations was first given by Rayleigh⁽¹⁾ as follows: "If heat be given to the air at the moment of greatest condensation or be taken from it at the moment of greatest rarefaction, the vibration is encouraged." This statement was proved by Putnam and Dennis.⁽²⁾ They found the phase difference between the oscillating heat input and pressure must be less than 90° to encourage oscillation. Chu,⁽³⁾ also, derived a formulation for the stability of systems containing a heat source but the derivation is not complete because adiabatic compression was assumed throughout the heating region.

Recently, Madarame⁽⁴⁾ investigated the Rijke oscillation both theoretically and experimentally. The experimental results showed a good agreement with the analysis qualitatively. However, the analysis is not enough for the quantitative prediction of the acoustic power generation. In his analysis, only the energy equation was solved under the assumption that velocities are constant. Volume expansion also was obtained by spatial integration of temperature fluctuations.

The objective of this study is to present complete mathematical formulations to predict the acoustic power generation and make clear the energy conversion mechanism. The power is obtained by solving numerically differential equations set up in dimensionless form. The effect of the mean velocity of the air current is illustrated and the energy conversion mechanism is explained by an engine analogy using pressure-volume diagram.

2. ANALYSIS

When a plane heater is placed normal to the air current in a tube as shown in Fig. 1 and the mean velocity of the air current is assumed to be uniform, the system can be regarded as one dimensional. The acoustic pressure is almost uniform throughout the heating region, since the thermal boundary near the heater is very small compared to the length of the pipe which is approximately half the wave length. Therefore, the governing equations are the continuity, energy and perfect gas equations

$$\frac{\partial \rho^*}{\partial t^*} + \frac{\partial}{\partial x^*} (\rho^* u^*) = 0, \quad (1)$$

$$\rho^* C_p^* \left(\frac{\partial T^*}{\partial t^*} + u^* \frac{\partial T^*}{\partial x^*} \right) - \frac{\partial P^*}{\partial t^*} = \frac{\partial}{\partial x^*} \left(\lambda^* \frac{\partial T^*}{\partial x^*} \right), \quad (2)$$

$$P^* = \rho^* R^* T^*, \quad (3)$$

where p is the pressure, ρ is the density, T is the temperature, u is the velocity, λ is the thermal conductivity, C_p is the specific heat, R is the gas constant and subscript * denotes dimensional quantities.

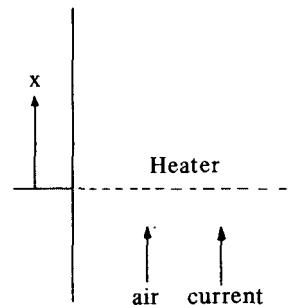


Fig. 1 The Rijke Tube with a Plane Heater

When the sound is a standing wave of angular frequency ω^* , variables can be separated into steady state and oscillating terms in complex form

$$\begin{aligned} P^* &= P_m^* + \hat{P}^* e^{i\omega^* t^*}, \\ \rho^* &= \rho_m^* + \hat{\rho}^* e^{i\omega^* t^*}, \\ T^* &= T_m^* + \hat{T}^* e^{i\omega^* t^*}, \\ u^* &= u_m^* + \hat{u}^* e^{i\omega^* t^*}, \end{aligned} \quad (4)$$

where subscript m denotes steady state and cap $\hat{\Lambda}$ denotes complex amplitude. The variables can be made dimensionless as follows

$$\begin{aligned} x &= x^* \sqrt{\alpha^*/\omega^*} t^* = t^* \omega^* \alpha^*, \quad u = u^*/\sqrt{\omega^* \alpha^*}, \\ \rho &= \rho^*/\rho_{m,o}^*, \quad P = P^*/P_{m,o}^*, \quad T = T^*/T_{m,o}^*, \\ \lambda &= \lambda^*/\lambda_{m,o}^*, \end{aligned} \quad (5)$$

where subscript o means the upstream side of the air before heating and α is thermal diffusivity. Substituting (4) into (1) and using the nondimensional variables of (5), we can obtain steady state and linearized acoustic equations

$$\rho_m u_m = u_{m,o}, \quad (6)$$

$$u_{m,o} (T_m - 1) = \lambda_m \frac{dT_m}{dx}, \quad (7)$$

$$1 = P_m T_m, \quad (8)$$

$$i\hat{\rho} + \frac{d}{dx} (\rho_m \hat{u} + \hat{\rho} u_m) = 0, \quad (9)$$

$$\begin{aligned} & (i\hat{T} + U_m \frac{d\hat{T}}{dx} + \hat{u} \frac{dT_m}{dx} + \hat{u} \frac{dT_m}{dx}) / T_m \\ & + \hat{\rho} / \rho_m U_m \frac{dT_m}{dx} - \frac{\gamma-1}{\gamma} i\hat{p} = \frac{d}{dx} (\lambda_m \frac{d\hat{T}}{dx} + \hat{\lambda} \frac{dT_m}{dx}), \end{aligned} \quad (10)$$

$$\hat{P} = \frac{\hat{\rho}}{\rho_m} + \frac{\hat{T}}{T_m}, \quad (11)$$

where the relation $C_p^* = \frac{\gamma}{\gamma-1} R^*$ was used.

Dimensionless heat conduction from the heating surface is

$$\hat{Q} = \lambda_m \frac{d\hat{T}}{dx} + \hat{\lambda} \frac{dT_m}{dx} \quad (12)$$

and the temperature dependance of the thermal conductivity can be written approximately as

$$\lambda_m = T_m^\beta, \quad (13)$$

$$\hat{\lambda} = \beta \lambda_m \frac{\hat{T}}{T_m}. \quad (14)$$

Near the heater the temperature gradient is very sharp, so that the pressure fluctuation is negligible compared to the temperature or density fluctuation. Neglecting pressure, substituting P_m and ρ from (8) and (11) into (9) and (10) and using (12) we can obtain

$$\frac{d\hat{u}}{dx} = \frac{dT_m}{T_m} \hat{u} + \left(\frac{i}{T_m} - u_{m,o} \frac{dT_m}{dx} \right) \hat{T} + \frac{u_{m,o}}{\lambda_m} \hat{Q}, \quad (15)$$

$$\frac{d\hat{T}}{dx} = -\beta \frac{dT_m}{T_m} \hat{T} + \frac{\hat{Q}}{\lambda_m}, \quad (16)$$

$$\begin{aligned} \frac{d\hat{Q}}{dx} &= \frac{dT_m}{T_m} \hat{u} + \left(\frac{i}{T_m} - u_{m,o} \frac{dT_m}{dx} \right) \hat{T} \\ &+ \frac{u_{m,o}}{\lambda_m} \hat{Q} \end{aligned} \quad (17)$$

When the Mach number of the mean flow is small the acoustic energy convected by it is negligible. Therefore, the acoustic power generated per unit area of the heating surface can be obtained by

$$W = \frac{Re}{2} [\overline{\hat{p}} (\hat{u}(0) - \hat{u}(-\infty))], \quad (18)$$

where the complex notation for time average was used, Re denotes the real part of a complex number, p is the complex conjugate of p and dimensionless acoustic power is $W = W^* / (P_m^*, 0 \sqrt{\alpha_0^* \omega^*})$

Since the velocity difference is the volume expansion rate and

$$\hat{u}(0) - \hat{u}(-\infty) = \hat{Q}(0) - \hat{Q}(-\infty)$$

from (15) and (17), the equation (18) can be written as

$$W = \frac{\text{Re}}{2} \left(\overline{\hat{P}} \frac{d\hat{V}}{dt} \right), \quad (19)$$

or

$$W = \frac{\text{Re}}{2} \left(\overline{\hat{P}} \hat{Q}_{in} \right), \quad (20)$$

where V represents the dimensionless oscillating volume and \hat{Q}_{in} is $\hat{Q}(0) - \hat{Q}(-\infty)$.

The equation (20) can be written in dimensional form as

$$W^* = \frac{r-1}{2r\rho_m^*} \text{Re} \left\{ \overline{\hat{P}}^* \hat{Q}_{in}^* \right\}, \quad (21)$$

This equation is a complete mathematical formulation for the acoustic power generation from a heating surface. The equation (19) and (20) clarify that the oscillatory heat input from the heating surface generates acoustic power by pressure-volume work like a heat engine.

3. NUMERICAL SOLUTION AND RESULTS

The steady state temperature T_m can be obtained integrating the equation (7) with

the condition $T_m = T_r$ at $x=0$, and $\beta = 0.85$ for air. The particle velocity, the oscillating temperature and the oscillating heat conduction are to be obtained by solving the equations with suitable boundary conditions.

When the thermal capacity of the heater is large, the temperature fluctuation on the heater surface is negligible. The particle velocity at $x = -\infty$ depends on the location of the heater in the pipe. Since the phase of the particle velocity leads that of the pressure by 90° in the lower half of the pipe with open ends by the standing wave theory, $\hat{u}(-\infty)$ can be assumed to be i . Therefore, the boundary conditions can be written as

$$\begin{aligned} \hat{T}(-\infty) &= T_r & T_m(0) &= T_r, \\ \hat{T}_m(-\infty) &= 1 & T_m(-\infty) &= 1, \\ \hat{u}(-\infty) &= i \end{aligned} \quad (22)$$

Since the equations are linear and homogeneous, solutions are proportionate to the value of $u(-\infty)$. The equations were solved numerically by shooting method to satisfy the three boundary conditions of the equation (22).

The particle velocity is shown in Fig. 2. The increase of the amplitude and the change of the phase depend on the heating temperature and the dimensionless mean velocity.

The fluctuating temperature is shown in Fig. 3. The phase of the temperature is behind that of pressure, which means the phase of condensation leads that. Such a phase change by heating shows that the compression can not be regarded as adiabatic at the heating region and "condensation" should be replaced by "pressure" in the Rayleigh criterion.

The temperature profile indicates the phase of the heat input by conduction from the heater leads that of pressure. The real part of this heat input is shown in Fig. 4. Since the pressure was set to be real and positive at the heating region, the thermoacoustic power generation is proportionate to $\text{Re}(\hat{Q}_{in})$ by the equation (20). We can see the acoustic power generation increases according to the heating temperature and becomes maximum when the dimensionless mean velocity is a little larger than 1.

Since $\frac{d\hat{V}}{dt} = \hat{Q}_{in}$ from (19) and (20), the amplitude of the oscillating volume expansion can be written as

$$\hat{V} = -i \hat{Q}_{in} \quad (23)$$

The pressure-volume diagram is shown in Fig. 5. The cycle area is the acoustic energy converted from thermal energy for one cycle. In the figure, the influence of the mean flow is clearly shown.

The acoustic power generation becomes maximum when the mean velocity of the air current is near the square root of the angular frequency times the thermal diffusivity. If the heater is placed in the upper half of the pipe $u(0) = -i$, then the results are reversed, which means damping occurs instead of driving.

4. SUMMARY

Complete mathematical formulations are presented to predict the acoustic power generation due to oscillating heat release from a heating surface. The numerical calculation of the acoustic power shows that it becomes maximum when the mean velocity of the air current is near the square of the angular frequency times the thermal diffusivity. The energy conversion mechanism is explained by an engine analogy that the oscillating heat input produces the acoustic power by means of pressure-volume work.

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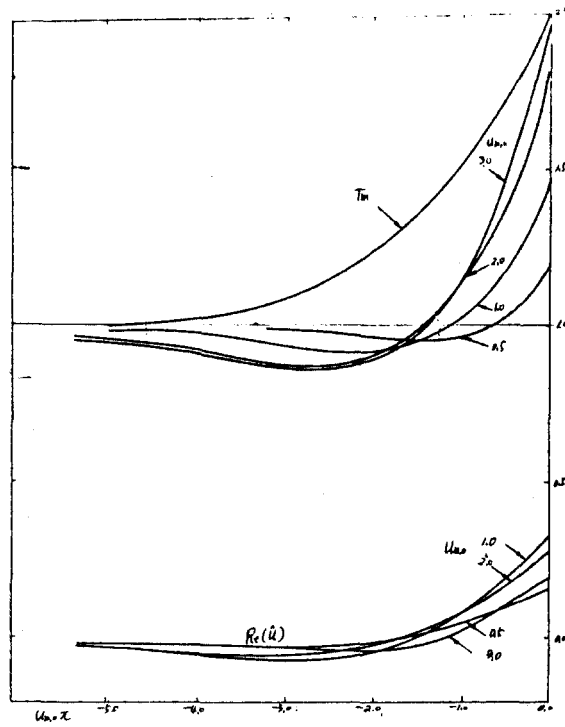


Fig. 2. U , the amplitude of the particle velocity when $Tr=2.0$

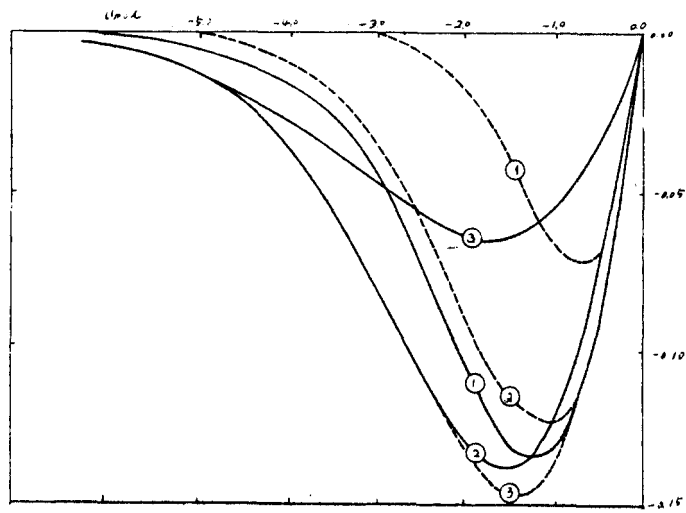


Fig. 3. T , the amplitude of acoustic temperature when $Tr=2.0$

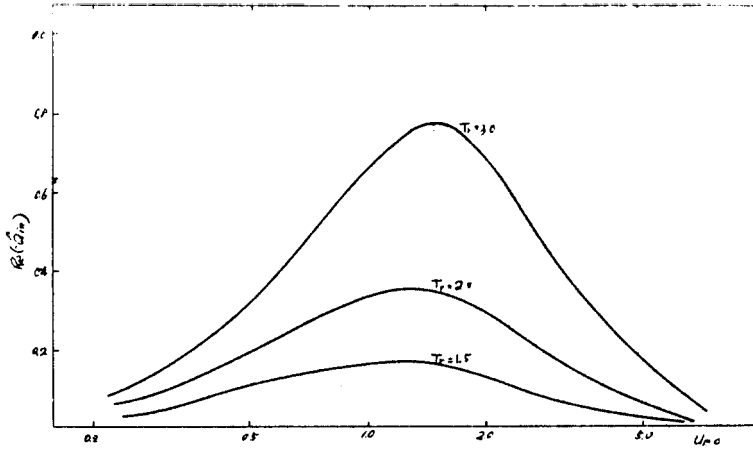


Fig. 4. Oscillating heat input by the heater in phase with the acoustic pressure when $U(\infty)=i$

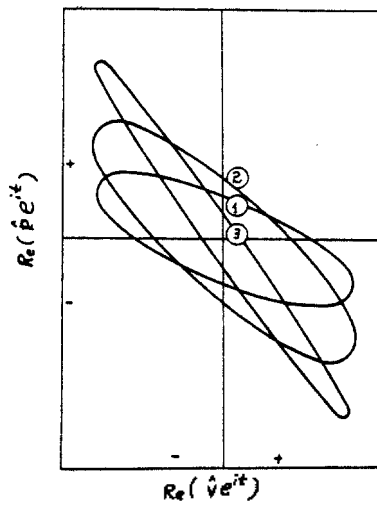


Fig. 5. Pressure-Volume diagram