Inherent Steering Characteristics Coming from the Configuration of a Vessel

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要　　約

船舶의 運動特性은 船型과 密接한 關係를 갖는다. 여러가지 運動特性中에서 船舶의 操縱과 特別히 關係가 깊은 것은 針路安定性이다. 이 特性은 船舶設計에 있어서 大端히 重要하지만 船舶操縱에서는 한 船舶을 操船하는데 事前에 必히 알아두어야 할 事項이다.

本 論文의 目的은 操船者들에게 船型에 따라서 생기는 針路安定性을 明白히 알림으로써 船舶의 安定運航을 도모하는데 있다.

最近 美國, 日本, 유럽등의 나라에서 많은 學者들이 이러한 運動特性을 많이 硏究하여오고 있으나 그들의 硏究는 操船을 위한 것이 아니고 造船을 위한 경우가 大部分이다.

筆者는 척형船型의 代表的인 model과 비대형船型의 代表的인 model을 골라서 이들의 動的인 針路安定性을 計算하였다. 그 結果 비대형船舶은 船型에서 由來하는 針路不安定性이 있음을 明白히 알 수 있었다.

Nomenclature

- H : draft of a ship
- I_z : moment of inertia of a ship with respect to z-axis
- I'_z : dimensionless moment of inertia of a ship with resped to z-axis $\left[=I_z/\left(\frac{1}{2}\rho L^4 H\right)\right]$
- L : length of ship
- m : mass of ship
- m' : dimensionless mass of a ship $\left[=m/\left(\frac{1}{2}\rho L^2 H\right)\right]$
- m'_y : dimension less virtual mass of a ship in lateral direction $[=(m'-Y'_{\dot{v}})]$
- N : total yaw moment
- N' : dimensionless total yaw moment $\left[=N/\left(\frac{1}{2}\rho L^2 H u^2\right)\right]$
- N'_r : $\partial N'/\partial r'$
- N'_v : $\partial N'/\partial v'(=-N_\beta')$
- N_r' : $\partial N'/\partial r'$
- n'_z : dimensionless virtual moment of inertia of a ship with respect to z-axis $[=(I'_z-N_{\dot{r}}')]$
- N'_β : $\partial N'/\partial\beta$
- N'_δ : $\partial N'/\partial\delta$, dimensionless yaw moment derivative with respect to rudder angle
- R : radius of rotation
- R'_i : initial turning rate
- r : turning rate$(=d\psi/dt)$
- r' : dimensionless turning rate$(=rL/u=L/R)$
- t : time
- t' : dimensionless time $(=ut/L)$
- u : ship speed in x-axis direction
- U : ship speed $(u^2+v^2)^{\frac{1}{2}}$
- v : ship speed in y-axis direction
- v' : v/U
- X : total x directional force
- Y : total lateral force
- Y' : dimensionless total lateral force $\left[=Y/\left(\frac{1}{2}\rho L H U^2\right)\right]$

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Y_r' : $\partial Y'/\partial r'$
Y_v' : $\partial Y'/\partial v'(=-Y_\beta')$
$Y_{\dot{v}}'$: $\partial Y'/\partial \dot{v}'$
Y_β' : $\partial Y'/\partial \beta$
Y_δ' : $\partial Y'/\partial \delta$, dimensionless lateral force derivative with respect to rudder angle.

β : drift angle $\left(=-\sin^{-1}\frac{v}{U}\right)$
δ : rudder angle
ρ : density of water
σ : stability index
ψ : yaw angle

I. Introduction

Handling performance of a vessel is greatly related with her steering characteristics which, the author thinks, are consisted of two kinds of motion characteristics; namely course stability and maneuverability.

The correct prediction of these qualities, espcially the steering characteristic is as much important in ship handling as in ship design.

It is the purpose of this paper to provide ships handlers better understanding of steering characteristics and then to help them in safe controlling and mareuvering of vessels hy presenting distinct inherent steering characteristic difference that lies between a fine-form vessel and a full-form vessel.

Nowadays in advanced maritime countries such as U.S.A, Japon and other countries in Europe, they study and investigate steering characteristics of various vessels enthusiastically, but main interest of their study lies in ship design and not in safe handling and controlling of vessels.

The author calculated dynamic course stabilities of two kinds of ideal models, one of which represents a fine-form ship and the other a full-form ship, based on hydrodynamic data of forces and moments obtained at the rotating-arm facility of the Davidson Laboratory of Stevens Institute of Technolagy in the State of New Jersey in U.S.A.

The result of calculutions indicated that a ship of full-form configuration has inherent course instabilty.

Though significant nonlinearities affect ship motions in maneuvers, application of linear theory is sufficient for prediction of the maneuvering characteristics of vessels on calm-water surface for handling references.

II. Equations of motion

A set of coordinate axis with origin fixed at the center of gravity of the ship, as shown in Fig. 1, is used to describe ship motions in the horizontal plane.

Longitiudinal and transverse horizontal axes of the ship are represented by the x and y-axes. By reference to these coordinate axes, the equations of motion can be written in the form

$$\left.\begin{array}{l} m(\dot{u}-ru)=X \\ m(\dot{v}+ru)=Y \\ I\dot{r} \quad\quad =N \end{array}\right\} \quad \cdots\cdots (1)$$

where X, Y and N represent total hydrodynamic forces and moment generated by ship motions, rudder and etc.

In the calm-water case, hydrodynamic forces are considered to be functions of hull motions and rudder angle.

For example, in the case of hydrodynamic moment,

$$N'=N'(v',\ r',\ \delta,\ \dot{v}',\ \dot{r}',\ \dot{\delta}')$$

and hydrodynamic force to y direction.

$$Y'=Y'(v',\ r',\ \delta,\ \dot{v}',\ \dot{r}',\ \dot{\delta}')$$

where prime means dimensionless.

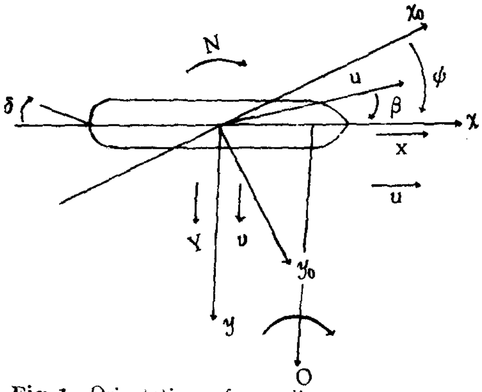

Fig. 1. Orientation of coordinate axes fixed in ship, showing drift angle wth respect to velocity vector and yaw angle with respect to fixed axes.

1. Yaw equation

$$I'\dot{r}'=N_v'v'+N_r'r'+N_\delta'\delta+N_{\dot{v}}'\dot{v}'+N_{\dot{r}}'\dot{r}'+N_{\dot{\delta}}'\dot{\delta}'$$

$$n'_z\dot{r}'-N_r'r'-N_{\dot{v}}'\dot{v}'-N_v'v'=N_{\dot{\delta}}'\dot{\delta}'+N_\delta'\delta \quad\cdots\cdots (2)$$

2. Sway equation

$$m'\dot{v}'+m'r'=Y_v'v'+Y_r'r'+Y_\delta'\delta+Y_{\dot{v}}'\dot{v}'+Y_{\dot{r}}'\dot{r}'+Y_{\dot{\delta}}'\dot{\delta}'$$

$$m_y'\dot{v}'-Y_v'v'-Y_{\dot{r}}'\dot{r}'+(m'-Y_r')r'=Y_{\dot{\delta}}'\dot{\delta}'+Y_\delta'\delta \quad\cdots\cdots (3)$$

II. Dynamic Stability

The dynamic stability of a vessel is directly related to magnitude of yaw and sway deviations caused by small initial disturbances.

The linearized equations of motions can he applied effectively for this treatment.

Assuming the rudder fixed amidship in equation (2) and (3), a set of homogeneous differential equations with constant coefficients is obtained;

$$\left.\begin{array}{l} n_z'\dot{r}'-N_r'r'-N_{\dot{v}}'\dot{v}'-N_v'v'=0 \\ m_y'\dot{v}'-Y_v'v'-Y_{\dot{r}}'\dot{r}'+(m'-Y_r')r'=0 \end{array}\right\} \quad\cdots\cdots (4)$$

The linearized equatians of yaw and sway motions can be written as follows :

$$\left.\begin{array}{l} a_{11}\dot{r}'+a_{12}r'+a_{13}\dot{v}'+a_{14}v'=0 \\ a_{21}\dot{r}'+a_{22}r'+a_{23}\dot{u}'+a_{24}v'=0 \end{array}\right\} \quad\cdots\cdots (5)$$

where

$$a_{11}=n_z'=I_z'-N_{\dot{r}}'\simeq 2I'_z \qquad a_{21}=-Y'_{\dot{r}}\simeq 0$$

$$a_{12}=-N_r' \qquad a_{22}=-(Y_r'-m')$$

$$a_{13}=-N'_{\dot{v}}\simeq 0 \qquad a_{23}=m_y'\simeq 2m'$$

$$a_{14}=-N_v' \qquad a_{24}=-Y_v'$$

Equations (5) are a system of homogeneous linear differential equations with constant coefficients. The most general solution of a system of this sort is a sum of exponential terms. With $\delta=0$ there are two terms in each sum :

$$r'=R_1'e^{\sigma'_1 t'}+R'_2e^{\sigma'_2 t'}=R'e^{\sigma' t'}$$
$$v'=V'_1e^{\sigma'_1 t'}+V'_2e^{\sigma'_2 t'}=V'e^{\sigma' t'}$$

and

$$\left.\begin{array}{l}\dot{r}'=R'\sigma'e^{\sigma' t'}\\ \dot{v}'=v'\sigma'e^{\sigma' t'}\end{array}\right\} \qquad (6)$$

Substituting equations (6) into equations (5) we get the following set of algebraic equations :

$$\left.\begin{array}{l}(a_{11}\sigma'+a_{12})R'+(a_{13}\sigma'+a_{14})V'=0\\ (a_{21}\sigma'+a_{22})R'+(a_{23}\sigma'+a_{24})V'=0\end{array}\right\} \qquad (7)$$

Equations (7) posses non-trivial solutions for R' and V' only if the determinant of their coefficient is zero. The resulting algebraic equations for the exponents σ'_i is quadratic :

$$\triangle=\begin{vmatrix}(a_{11}\sigma'+a_{12}) & (a_{13}\sigma'+a_{14})\\ (a_{21}\sigma'+a_{22}) & (a_{23}\sigma'+a_{24})\end{vmatrix}=A\sigma'^2+B\sigma'+C=0 \qquad (8)$$

where

$$A=a_{11}a_{23}-a_{21}a_{13}=n'_z m'_y$$

$$B=\begin{vmatrix}a_{11} & a_{14}\\ a_{21} & a_{24}\end{vmatrix}+\begin{vmatrix}a_{12} & a_{13}\\ a_{22} & a_{23}\end{vmatrix}=a_{11}a_{24}+a_{12}a_{23}$$
$$=-n'_zY'_v-m'_yN'_r$$

$$C=a_{12}a_{24}-a_{22}a_{14}=N'_rY'_v-N'_v(Y'_r-m')$$

Equations (8) has two roots σ'_1 and σ'_2.

The values of R' and V' of equation (7) depend on the initial conditions of the motion, but the stability is a property of the differential equations (7) alone, and can be inferred from tle exponents σ'_i.

Thus if the real parts of σ'_1 and σ'_2 are both negative, the solutions r' and v' given by equations (7) will approach zero as t' appreaches infinity, regardless of the initial conditions; in this case the motion is stable in the sense that the ship returns to a new straight course following a small arbitrary inital disturbance. On the other hand if either of σ'_1 or σ'_2 has a positive real part, every initial condition will lead to values of r' and v' that increases with increasing time t', so that the motin winds up into a spiral; in this case the motion is unstable in the sense that a straight course can not be gained without using rudder after even a small arbitrary initial disturbance.

The solutions of equation (8) are

$$\sigma'_{1\cdot 2}=\frac{-B\pm(B^2-4AC)^{\frac{1}{2}}}{2A} \qquad (9)$$

It follows from equation (9) that the necessary and sufficient conditions that the real parts of $\sigma'_{1\cdot 2}$ be negative and the motion therefore be stable, are

$$\left.\begin{array}{l}A>0\\ B>0\\ C>0\end{array}\right\} \qquad (10)$$

The values of A and B are always positive for all actual ships, so far as is known.

Thus $C>0$ alone is criterion for stability an straight course :

$$N_r'Y_v'-N_v'(Y_r'-m')>0 \quad \cdots\cdots (11)$$

We now examine the different types of motion that will prevail depending on the nature of $\sigma'_{1,2}$.

1. when the values of $\sigma'_{1,2}$ are real

(1) $\sigma_{1,2}<0$ or $C>0$

This is the case of a dynamically stable ship, for which an initial yaw angle or angular velocity damps out exponentially.

(2) $\sigma'_{1,2}>0$ cr $C<0$

This is the case of a dynamically unstable ship, for which an initial yaw angle or angular velocity increases exponentially with time passing.

From above results, an obvious test presents itself for determining whether or not a ship is dynamically stable. The ship is started on straight course with rudder held amidships and the subsquent motion observed.

Since no water is completely still, the motion will be disturbed slightly. If the ship continues an straight course or one close to straight line, it is dynamically stable, while, if it goes into a circle in either direction, it is dynamically unstable.

2. When the values of $\sigma_{1,2}$ are of complex

This occurs when there is greater static stability than is necessary for dynamic stability. The motion following an initial yaw angle becomes oscillatory. No known ship meets this condition.

Ⅳ. Calculation and comparision of the stability indices of a fine-form ship and full-form ship

Now we caculate the values of stability indices of the ideal model of a fine-form ship; Series 60 and the model of a full-form VLCC, T80 and calculate their yaw responses, to sudden yaw disturbance of 1 degree with zero angular velocity. The following tables are their hydrodynamic coefficients;

table 1 (Series 60)

$m'=0.170$	$N'_\delta=-0.024$
$m'_y=0.341$	$Y'_v=-0.305$
$n'_z=0.022$	$Y'_r-m'=+0.0896$
$N'_r=-0.0954$	$-0.171=-0.0814$
$N'_v=-0.0701$	$Y'_\delta=0.05$

table 2 (T 80)

$m'=0.01370$	$N'_\delta=-0.00140$
$m'_y=0.02460$	$Y'_v=-0.01650$
$n'_z=0.00152$	$Y'_r-m'=-0.00975$
$N'_v=-0.00600$	$Y'_\delta=0.00305$
$N'_r=-0.00310$	

1. Calculations for the fine-form ship(series 60)

(1) Stability indices

$$\left.\begin{array}{l}0.0220\dot{r}'+0.0701r'+0.0954v'=0\\0.0814r'+0.3410\dot{v}'+0.3050v'=0\end{array}\right\} \quad\cdots\cdots(12)$$

From equations (12) we obtain stability indices as follows :

$$\sigma'_{1,2}=\frac{-B\pm(B^2-4AC)^{\frac{1}{2}}}{2A}$$

$$\sigma'_1=-0.508 \qquad \text{whece} \quad A=0.007502$$
$$\sigma'_2=-3.573 \qquad B=0.030614$$
$$C=0.013615$$

as C>0, this ship has dynamic course stability. It is clear then that $|1/\sigma'_1|$ has the physical significance of being approximately the number of ship lengths required for an initial disturbance to diminish to 1/e of its initial value. For the above ship $|\sigma_1'|=0.508$, so $1/0.508$ is about 2, which means that when the ship advance 2 times her length with rudder amidships, initial disturbarce will almost disapear.

(2) Yaw responses

As the values of stability indices were calculated, the solution of the equations of yaw and sway motions will be

$$\left.\begin{array}{l}r'(t')=R'_1e^{-0.508t'}+R'_2e^{-3.573t'}\\v'(t')=V'_1e^{-0.508t'}+V'_2e^{-3.573t'}\end{array}\right\} \quad\cdots\cdots(13)$$

Now we can calculate the values of R' and V' from the given initial conditions which describe at $t'=0$, $r'(0)$ is zero and $v'(0)=\pi/180$, the value of which is ⊖0.01745 because ⊖v' means ⊕B'.

$$\left.\begin{array}{l}R'_1+R'_2=0\\V'_1+V'_2=-0.01745\end{array}\right\} \quad\cdots\cdots(14)$$

Applying initial conditions to equations (4) we get followings :

$$\dot{r}'(0)=\frac{N'_v v'(0)}{n'_z}=\frac{0.0954\times0.01745}{0.022}=0.075670$$

$$\dot{v}'(0)=\frac{Y'_v v'(0)}{m'_y}=\frac{0.3050\times0.01745}{0.341}=0.015608$$

From above values we get following :

$$\left.\begin{array}{l}-0.508R'_1-3.573R'_2=0.075670\\-0.508V'_1-3.573V'_2=0.015608\end{array}\right\} \quad\cdots\cdots(15)$$

From equations (14) and (15) we get followings :

$$R'_1=0.02469 \qquad V'_1=-0.01525$$
$$R'_2=-0.02469 \qquad V'_2=-0.00220$$

Now we get final equations of yaw and sway motions :

$$\left.\begin{array}{l}r'(t')=0.02469e^{-0.508t'}-0.02469e^{-3.573t'}\\v'(t')=-0.01525e^{-0.508t'}-0.00220e^{-3.573t'}\end{array}\right\} \quad\cdots\cdots(16)$$

Yaw angle $\phi(t')$ can be obtained by integrating $r'(t')$:

$$\phi(t')=\int r'(t')dt'$$

$$=-\frac{0.02469}{0.508}e^{-0.508t'}+\frac{0.02469}{3.573}e^{-3.573t'}+C_o$$

where C_o is to be calculated from initial conditions, $\Psi(0)=0.01745(=1 \text{ degree})$.

$$\therefore \phi(t')=-0.04860e^{-0.508t'}+0.00691e^{-3.573t'}+0.05914 \quad \cdots\cdots(17)$$

The calculated values of yaw responses and its figure of time history are as table 3 and figure 2.

table 3 (Calculated values of yaw responses)

t'(time)	0	1	2	4	5	10	15	∞
$e^{-0.508t'}$	1	0.60	0.36	0.13	0.05	0.01	0.00	0.00
$e^{-3.573t'}$	1	0.03	0.00	0.00	0.00	0.00	0.00	0.00
$r'(t')\times 10^2$	0	1.42	0.89	0.32	0.12	0.02	0.00	0.00
$-v'(t)\times 10^2$	1.75	0.93	0.55	0.20	0.07	0.01	0.00	0.00
$\phi(t')\times 10$	0.20	0.40	0.50	0.30	0.60	0.60	0.60	0.60

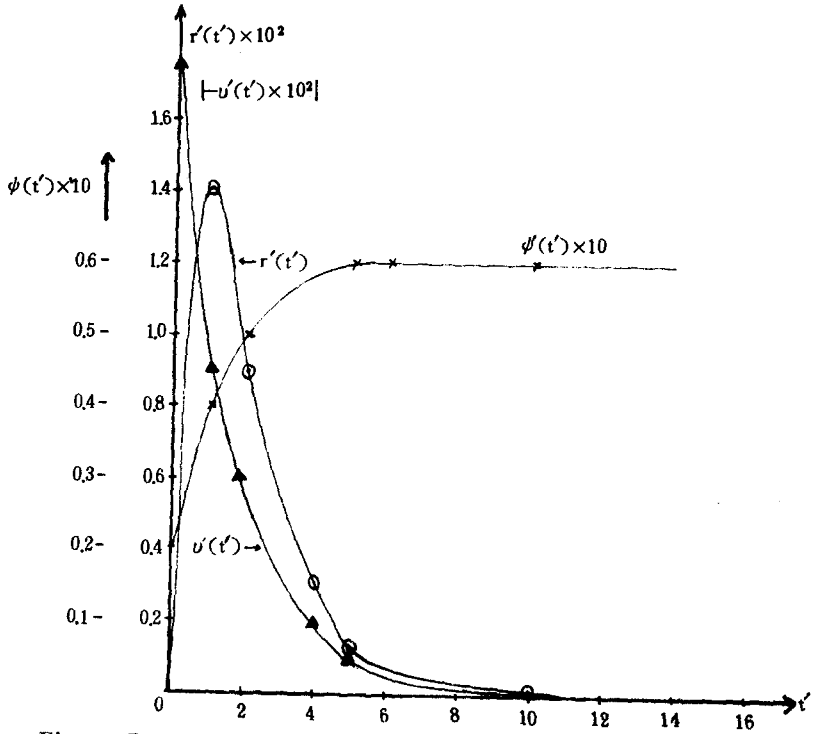

Fig. 2 Responses of the fine-form ship to initial disturbance of yaw angle 1 degree accompanying no argular velocity with rudder fixed at zero

2. Calculations for the full-form ship(T 80)

(1) stability indices

$$\left.\begin{array}{l} 0.00152\dot{r}'+0.00310r,+0.00600v'=0 \\ 0.00975r'+0.02460\dot{v}'+0.01650v'=0 \end{array}\right\} \cdots\cdots (18)$$

From equations (18) we obtain stability indices :

$$\sigma'_{1,2}=\frac{-B\pm(B^2-4AC)^{\frac{1}{2}}}{2A} \qquad \text{where}$$

$\sigma_1'=0.184$ $\qquad$ $A=0.0000374=3.74\times10^{-5}$

$\sigma_2'=-2.893$ $\qquad$ $B=0.0001013=1.013\times10^{-4}$

$C=-0.00000735=-7.35\times10^{-6}$

As $C<0$, this ship is dynamically unstable ship. It means that this ship will not follow straight course and runs turning along a circle line without using control surface.

(2) Yaw responses

From initial conditions we get followings :

$$\left.\begin{array}{l} r'(0)=R'_1+R'_2=0 \\ v'(0)=V'_1+V'_2=-0.01745 \end{array}\right\} \cdots\cdots (19)$$

$$\dot{r}'(0)=0.0689$$

$$\dot{v}'(0)=0.0117$$

$$\left.\begin{array}{l} 0.184R'_1-2.893R'_2=0.0689 \\ 0.184V'_1-2.893V'_2=0.0117 \end{array}\right\} \cdots\cdots (20)$$

From equations (19) and (20) we get :

$R'_1=0.0224$ $\qquad$ $V'_1=-0.01267$

$R'_2=-0.0224$ $\qquad$ $V'_2=-0.00485$

$$\left.\begin{array}{l} r'(t')=0.0224e^{0.184t'}-0.0224e^{-2.893t'} \\ v'(t')=-0.01267e^{0.184t'}-0.00485e^{-2.893t'} \end{array}\right\} \cdots\cdots (21)$$

$$\phi(t')=0.12174e^{0.184t'}+0.00774e^{-3.893t'}+C_0$$

$$\therefore\ \phi(t')=0.12174e^{0.184t'}+0.00774e^{-2.893t'}+0.11203 \cdots\cdots (22)$$

The calculated values of yaw responses and its figure of time history are as table 4 and figure 3.

table 4 (Calculated values of yaw respanses)

t'(time)	0	1	2	4	5	10	15	∞
$e^{0.184t'}$	1	1.20	1.45	2.09	2.51	6.30	15.8	∞
$e^{-2.893t'}$	1	0.06	0.00	0.00	0.00	0.00	0.00	0
$r'(t')\times10$	0	0.26	0.33	0.47	0.56	1.41	3.54	∞
$-v'(t')\times10$	0.08	0.15	0.18	0.27	0.32	0.80	2.00	∞
$\phi(t')\times10$	1.18	0.40	0.65	1.42	1.94	6.55	18.1	∞

Comparing figure 2 and 3 we know that the motions of series 60 are converging whereus those of T 80 wind up into bigger evaporation step by step.

We should give great attention to the latter case because most of all very large vessels are of full-form, which means most of all VLCC have inherent course instability from their configuration.

There are great many characteristic differences between a course stable vessel and a unstable one when we maneuver vessels on various seas, especially in restricted water area.

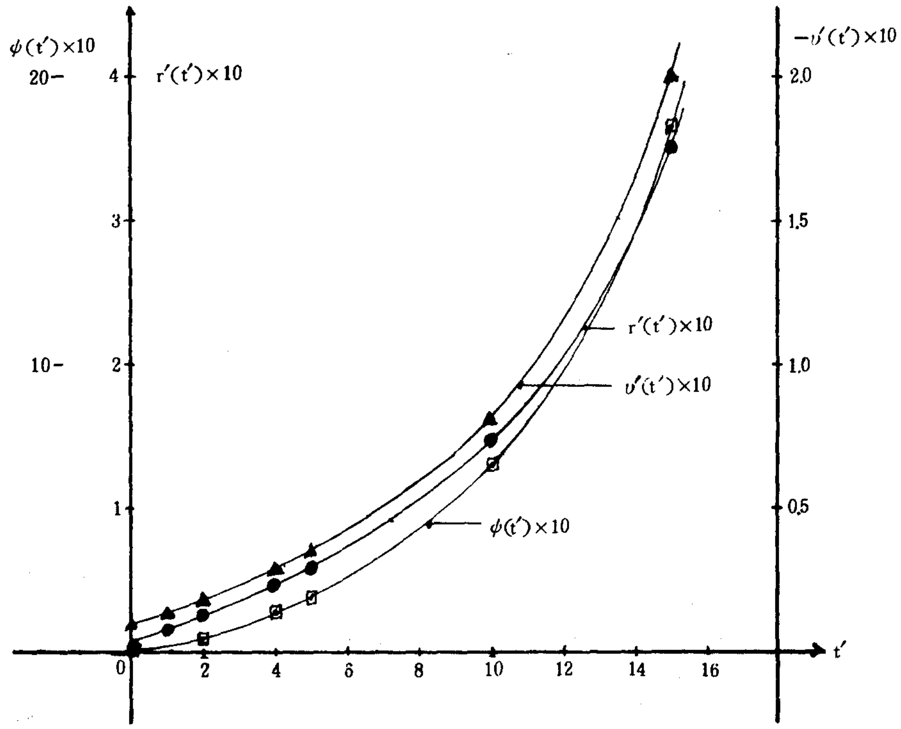

Fig. 3 Responses of the full-form vessel to initial disturbanc of yaw angle 1 degree accompanying no angular velocity with rudder amidships

V. Conclusions

It is the purpose of this paper to provide the handlers of various vessels better informations and understanding of the inherent steering characteristics coming from the ship configuration.

Results of general interest, especially from the view point of practical ship handling, are summarized as follows :

1. Fine-form ship configuration provides the vessel inherent course stability, whereas that of full-form inherent course instability.

2. Quickness of the induced motion vanishment of a fine-form vessel and that of the motion windng of a full-form vessel are to be determined by the value of $1/\sigma'_1$.

In this paper the value of $1/\sigma_1'$ of series 60 is about ⊖2, that means when this ship travels ahout 2 times its length it will almost settle down on a straight course, whereas the value of $1/\sigma_1'$ of T80 is about ⊕5.5, which meant that after this ship runs about 5.5 times her length, the motions will get violent winding from its original status(see figure 2 and 3).

Additional :

1. Generally all of large tankers (VLCC) have inherent course instability with rudder fixed amidships.
2. The inherent course instability combining with shallow water and bank effect makes safe handling of a VLCC much more difficult than that of a ordinary cargo vessel on coastal sea areas.

So we must prepare without fail sufficient assisting tugs in advance when a VLCC passes through a narrow channel or enters in congested sea area.

The cost of tugs is too little to be compared with expenses to be incurred in case of a accident without arranging them for cost saving.

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