開水路에 作用하는 不定壓力에 관한 數値模型

Numerical Method for Transient Pressure on Canals

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要 旨

본 研究의 目的은 開水路의 水位의 變動에 따른 自由 地下水面과 開水路에 作用하는 不定壓力의 分布를 計算할 수 있는 數値 模型을 開發하는데 있다. Diagnostic Eq. 은 Point SOR 방법에 의해서, 그리고 Prognostic Eq. 은 Implicit Lax-Wendroff 방법에 의하여 해석하였다. Simulation 조건들에서 地下水 浸透面의 변화를 豫測하기 위하여 透水性 및 不透水性開水路에 대하여 네가지 다른 경우에 대한 結果를 나타내었다.

Abstract

The purpose of this paper is to develop a mathematical model which can be used to compute the position of the free surface due to water level fluctuations in the canal and the transient pressure distributions along the canal lining. The diagnostic equation has been solved by the point successive over-relaxation method, and the linearized prognostic equation has been solved by the implicit Lax-Wendroff scheme. Four different cases in the simulation conditions are presented for both permeable and impermeable canal lining to predict the transient seepage surface development.

1. Introduction

The problem of canal seepage and its slope stability encompasses the fields of groundwater hydrology and geotechnical engineering. Excessive hydrostatic pressure build-up due to dewatering in the canal and soil instability could result in canal failure. Certain hydraulic heads (and pore pressures) exist along the canal lining under either steady-state or unsteady-state groundwater flow. If reasonable estimates can be made of water-table configuration and the distribution of soil type, it should be possible to predict the pore pressure distributions along the canal lining and potential slip surfaces by means of a flow-net or analytical or numerical solutions. Unsaturated flow is not considered here, since the pressure is negative. Ignoring the unsaturated flow may introduce some errors in the analysis. This is primarily because the flow region normally consists of both saturated and unsaturated components. Dupuit-Forchheimer assump-

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tions are applied to the unconfined flow.

The objective of this report is to develop a mathematical model which can be used to compute the position of the free surface due to water level fluctuations in the canal and the transient pressure distributions along the canal lining. The problem of canal lining underdrainage is studied. The results of this study can be used to determine the safe drawdown of the canal subject to the specified allowable pressures on lining.

2. Formulation of the Problem

Canal (or aqueduct) seepage problem can be classified into the four cases under different configurations as represented by Figure 1. Cases I and II represent the conditions of a shallow aquifer (or high water-table), whereas Cases III and IV correspond a deep aquifer (or low water-table). Also, Cases I and III have an impermeable bottom boundary, while Cases II and IV have a drain as the bottom boundary.

Bouwer's analog simulation results (1965) indicate that a distance of $10W_b$(W_b=canal bottom width) is sufficiently long for the establishment of a water-table with essentially constant slope for Case I, and an essentially horizontal water-table for Case II. When the depth of aquifer approaches $5W_b$ for Case II or W_b for Case IV, the dimensionless seepage rates would be relatively close to that of an infinitely deep drainage layer. The seepage from canals with natural or artificial, thin, slowly permeable layers along the wetted per-

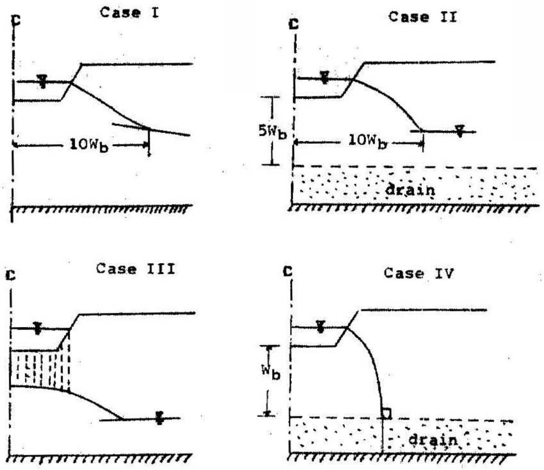

그림 1. Regional Hydraulic Conditions for Seepage Analysis (W_b denotes the canal bottom width)

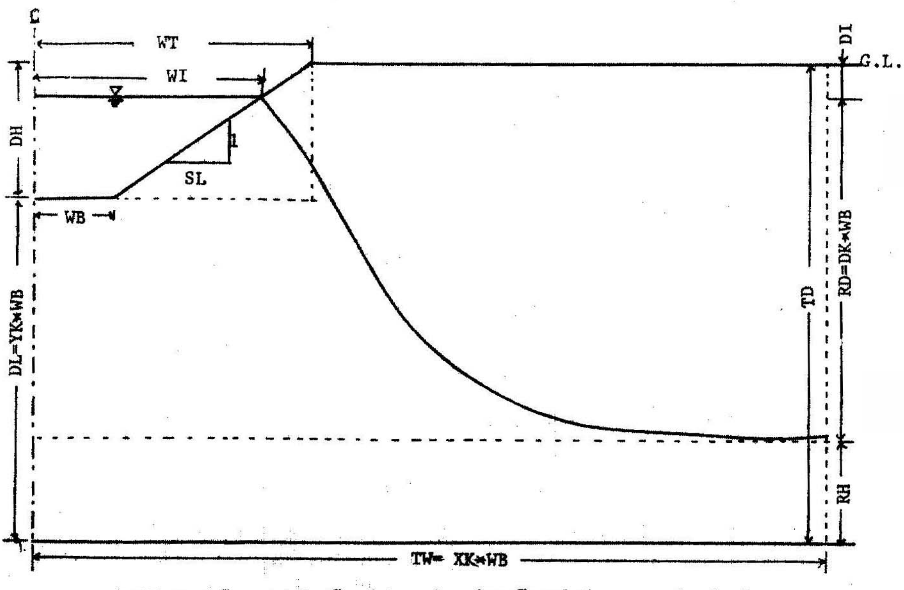

그림 2. Geometric Configuration for Canal Seepage Analysis

imeter (Case III) will be unsaturated. The pressure head of the soil water in the underlying material can be approximated by the critical pressure head when the sigmoid curve relating hydraulic conductivity and negative pressure is replaced by a step function.

Upon examining several possible cases in connection with canal lining underdrainage (Bouwer, 1965), it appears that the case with a high water-table and an impermeable bottom boundary closely represents a typical system after a period of normal operation. Figure 2 shows some details about the geometric configuration to study the problem of seepage surface development along the lining under an unsteady state condition. The slope stability problem, although very important, is not within the scope of this study.

3. Groundwater Hydraulics

The governing equations of transient free surface flow in a saturated porous medium consist of the Laplace equation and the wave equation. The total piezometric potential satisfies an elliptic partial differential equation called a diagnostic equation and the motion of the free surface is described by a nonlinear hyperbolic partial differential equation called a prognostic equation.

The continuity equation (conservation of mass) is:

$$\nabla\cdot(K\nabla h)=0,\quad h=y+p/\gamma \qquad (1)$$

These equations are valid for steady and unsteady flow.

The kinematic boundary condition for a two-dimensional case is:

$$n\frac{\partial H}{\partial t}=K\left(hx\frac{\partial H}{\partial x}-hy\right) \qquad (2)$$

where n is the effective porosity of the medium and $H(x,t)$ is the position of the free surface. Since the free surface is also an isobaric surface, the dynamic boundary condition becomes

$$\frac{\partial H}{\partial x}=hx+hy\frac{\partial H}{\partial x} \qquad (3)$$

Substituting hx from Eq. (3) into Eq. (2),

$$\frac{n}{K}\frac{\partial H}{\partial t}=(1-hy)\left(\frac{\partial H}{\partial x}\right)^2-hy \qquad (3)'$$

Linearizing by the Taylor series expansion about an approximate $\tilde{H}x$,

$$\frac{n}{K}\frac{\partial H}{\partial t}=(1-hy)\left(2\tilde{H}x\frac{\partial H}{\partial x}-\tilde{H}x^2\right)-hy \qquad (4)$$

It is the application of quasi-linearization to the nonlinear differential equation (Cheng and Li, 1973).

For given boundary conditions, alternating sequence of the coupled governing equations, equations (1) and (3)/(4) should be solved using the finite-difference or the finite-element method. Under some restricted situations, four types of uncoupled equations have been used to describe the transient free surface (Eagleson, 1970).

Dupuit approximation:

$$n\frac{\partial H}{\partial t}=K\frac{\partial}{\partial x}\left(H\frac{\partial H}{\partial x}\right) \qquad (5)$$

Linearization(I):

$$n\frac{\partial H}{\partial t}=KH\frac{\partial^2 H}{\partial x^2} \qquad (6)$$

Linearization(II):

$$n\frac{\partial H^2}{\partial t}=KH\frac{\partial^2 H^2}{\partial x^2} \qquad (7)$$

Deep-Water wave:

$$n\frac{\partial H}{\partial t}=-K\frac{\partial H}{\partial z} \qquad (8)$$

Although many numerical methods for those governing equations have been applied to the earth dam model and a ditch drainage problem (Todsen, 1971), the canal or aqueduct seepage problem has been confined to the steady-state condition only.

The initial condition of the free-surface can be assumed to be horizontal, linear, parabolic or exponential depending upon the steady state solutions of the various approximation

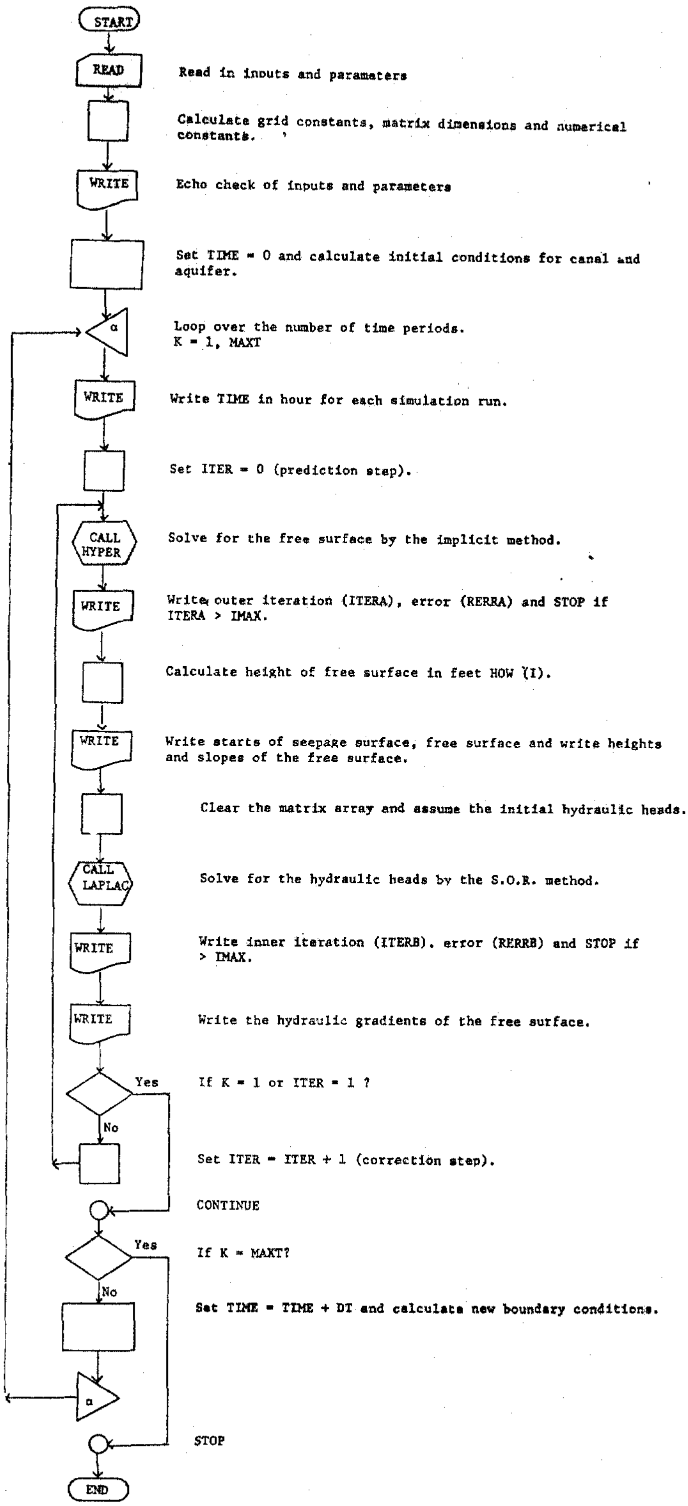

그림 3. Program Flowchart(Main).

methods (Eagleson, 1970). It seems to be that the linear or exponential case approximates the initial form of the mound for the system which is compatible with the given geometric configuration in Fig. 1.

$$H(x)=RH+RD\frac{TW-x}{TW-WI},\quad WI\leq x\leq TW \quad (9)$$

or

$$H(x)=RH+RD\exp\cdot\left[\frac{SL(WI^2-x^2)}{2WI\cdot RD}\right],\quad x\geq WI \quad (10)$$

The boundary condition on the canal lining might be assumed to be either permeable or impermeable. In the permeable case, drawdown variations as a function of time in the canal impose a time varying boundary conditions. At the right hand side regional boundary, the Dupuit-Forchheimer approximations are used; the left hand boundary is the axis of symmetry.

4. Computational Procedures

The diagnostic equation (1), is an elliptic partial differential equation which can be solved either by the finite-difference or the finite-element method. Suitable finite difference schemes include the Gauss-Seidel method, the successive over-relaxation (SOR) and alternating direction implicit method(ADI). For an inhomogeneous and anisotropic aquifer with a complicated configuration, the finite-element method is preferred over other conventional numerical schemes. It does not appear there will be difficulty in solving the diagnostic equation using either one of the above-metioned methods.

The prognostic equation, Equation (3)′, for H is a "time quasilinear" hyperbolic equation which is a special case of the general nonlinear equation.

$$u_{tt}+f(x,t,u,u_x,u_{xx})u_{xt}+g(x,t,u,u_x,u_{xx})u_t = P(x,t,u,u_x,u_{xx}) \quad (11)$$

The nine-point computational molecule of Ames (1968) will generate an implicit "linear" tridiagonal system at each time step which is explicitly solvable by the Thomas algorithm. The Dupuit approximation, Equation (5), is a nonlinear diffusion equation of the following type.

$$f(u)\ \frac{\partial u}{\partial t}=\frac{\partial}{\partial x}\left(g(u)\frac{\partial u}{\partial x}\right) \quad (12)$$

A predictor-corrector modification of the Crank-Nicolson procedure (Douglas and Jones, 1963) is possible so that the resulting algebraic problem is linear. Other linearized hyperbolic differential equations can be solved by the Lax-Friedrich scheme, the Lax-Wendroff scheme or the Richtmyer's leap-frog scheme.

To study the problem of the transient seepage surface development along the canal lining, it is proposed to solve an alternating sequence of two governing equations (1) and (2) subject to the boundary condition (3) on the free surface. For a given initial position of the free surface and the hydraulic gradient on the free surface, the free surface quation (4) will be solved to determine the new position and the corresponding slope. Then the Laplace equation (1) will be solved for the interior region of flow to determine the hydraulic head distribution and the corresponding gradient. These processes (subroutine HYPER and LAPLAC) will repeat twice for each time increment; one for prediction and the other for correction (Fig. 3).

To provide a fine resolution along the canal lining which is compatible with the water level fluctuation in the canal, each subroutine consists of two segments; one using coarse computational grids for the regional groundwater flow and the other using fine grids for the local flow. The height of the free surface and the hydraulic head from the coarse grids provide the interface boundary conditions for the fine grids.

5. Numerical Methods

The diagnostic equation (1) has been solved by the point successive over-relaxation method (LAPLAC). Let $\bar{h}_{i,j}^{(k)}$ be the components of the k^{th} Gauss-Seidel iteration. Then, for the regular computational molecule

$$\bar{h}_{i,j}^{(k)}=\frac{\Delta y^2}{2(\Delta x^2+\Delta y^2)}(h_{i-1,j}^{(k)}+h_{i+1,j}^{(k-1)}) + \frac{\Delta x^2}{2(\Delta x^2+\Delta y^2)}(h_{i,j-1}^{(k)}+h_{i,j+1}^{(k-1)}) \quad (13)$$

The SOR technique is defined by means of the relationship

$$h_{i,j}^{(k)}=(1-w)\ h_{i,j}^{(k-1)}+w\bar{h}_{i,j}^{(k)} \quad (14)$$

The relaxation parameter w is determined by the optimum value

$$w^{(k)}=\frac{2}{1+\sqrt{1-\lambda}}=\frac{2}{1+\sqrt{1-\text{EIGEN}}} \quad (15)$$

where the spectral radius of the Gauss-Seidel iteration matrix λ can be approximated by the following asymptotic property

$$\text{EIGEN}=\frac{\sum|h_{ij}^{(k)}-h_{ij}^{(k-1)}|}{\sum|h_{ij}^{(k-1)}-h_{ij}^{(k-2)}|} =\frac{w^{(k)}\sum|\bar{h}_{ij}^{(k)}-h_{ij}^{(k-1)}|}{w^{(k-1)}\sum|\bar{h}_{ij}^{(k-1)}-h_{ij}^{(k-2)}|}=\frac{\text{SUM}}{\text{SUMA}} \quad (16)$$

The convergence criteria is the relative error.

$$\text{RERR}=\frac{\sum|h_{ij}^{(k)}-h_{ij}^{(k-1)}|}{\sum h_{ij}^{(k)}}= \frac{w^{(k)}\sum|\bar{h}_{ij}^{(k)}-h_{ij}^{(k-1)}|}{\sum h_{ij}^{(k)}}=\frac{\text{SUM}}{\text{SUMB}} \quad (17)$$

For the irregular boundaries near the free surface,

$$h_{xx}=\frac{2}{\Delta x^2}\left[\frac{h_{i-1,j}-h_{ij}}{\theta_1(\theta_1+\theta_3)}+\frac{h_{i+1,j}-h_{ij}}{\theta_3(\theta_1+\theta_3)}\right]$$

and $$h_{yy}=\frac{2}{\Delta y^2}\left[\frac{h_{i,j+1}-h_{ij}}{\theta_2(\theta_2+\theta_4)}+\frac{h_{i,j-1}-h_{ij}}{\theta_4(\theta_2+\theta_4)}\right]$$

where $\theta_1, \theta_2, \theta_3$ and θ_4 are the fraction of the standard spacing Δx or Δy from $h_{i,j}$ to $h_{i-1,j}$, $h_{i,j+1}$, $h_{i+1,j}$ and $h_{i,j-1}$, respectively.

The linearized prognostic equation (4) has been solved by the implicit Lax-Wendroff scheme (HYPER). Let $ST=n/K$, then

$$\frac{\partial H}{\partial t}=2\frac{\tilde{H}_x}{ST}(1-h_y)\frac{\partial H}{\partial x}-\frac{h_y}{ST}-\frac{\tilde{H}_x^2}{ST}(1-h_y)$$

$$(\alpha_i-2\alpha_i^2)H_{i-1,k}+(1+4\alpha_i^2)H_{i,k}-(\alpha_i+2\alpha_i^2)H_{i+1,k}=\beta_i$$

where $\alpha_i=\frac{\Delta t\ \tilde{H}_x}{\Delta x ST}(1-h_y)|_i$,

$$\beta_i=H_{i,k-1}-\frac{\Delta t}{ST}(h_y+(1-h_y)\tilde{H}_x^2)|_i \quad (18)$$

This is a linear tridiagonal system which is explicitly solvable by the Thomas algorithm. For additional smoothing, $h_x|_i$ can be replaced by the average of $h_x|_{i-1}$ and $h_x|_{i+1}$.

$$h_x|_i=0.5(h_x|_{i-1}+h_x|_{i+1}) \quad (19)$$

After each successive iteration, the variable coefficients are cyclically adjusted to satisfy the constraint equation (3).

$$h_y|_i=1-\frac{hx}{Hx}|_i \quad (20)$$

The convergence criteria is the maximum change of the slope Hx.

$$\text{RERR}=\max\left|\frac{\partial H}{\partial x}-\tilde{H}x\right| \quad (21)$$

The exact starting position of the free surface on the canal lining can be computed by

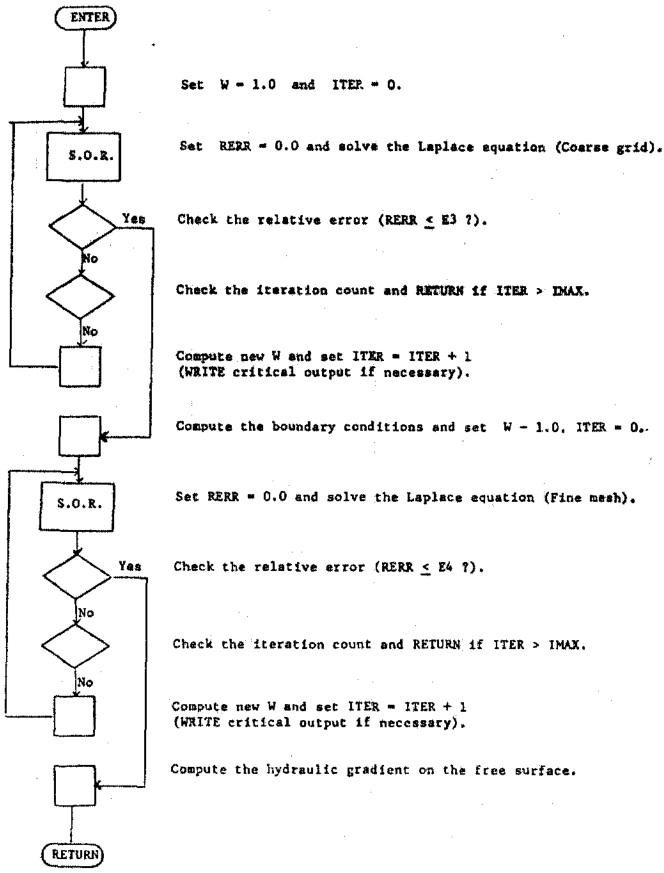

그림 4(a). Subroutine LAPLAC

backward extrapolation using the equation (3).

$$H_{i-1,k}=H_{i+1,k}-\frac{2\Delta xhx}{1-hy}|_i$$
$$=H_{i+1,k}-2\Delta xHx|_i \quad (22)$$

The constraint equation (3) describes both the seepage surface of the permeable case and the free surface of the aquifer. Figure 4 shows the flowchart of these subroutines.

6. Program inputs and outputs

The canal physical dimensions (WB, WT, DH, SL) and the stopping criteria (max. number of iteration IMAX: error bounds E1 and E2 for the sub. HYPER, error bounds E3 and E4 for the sub. LAPLAC) are specified by the DATA statement. The regional hydro-

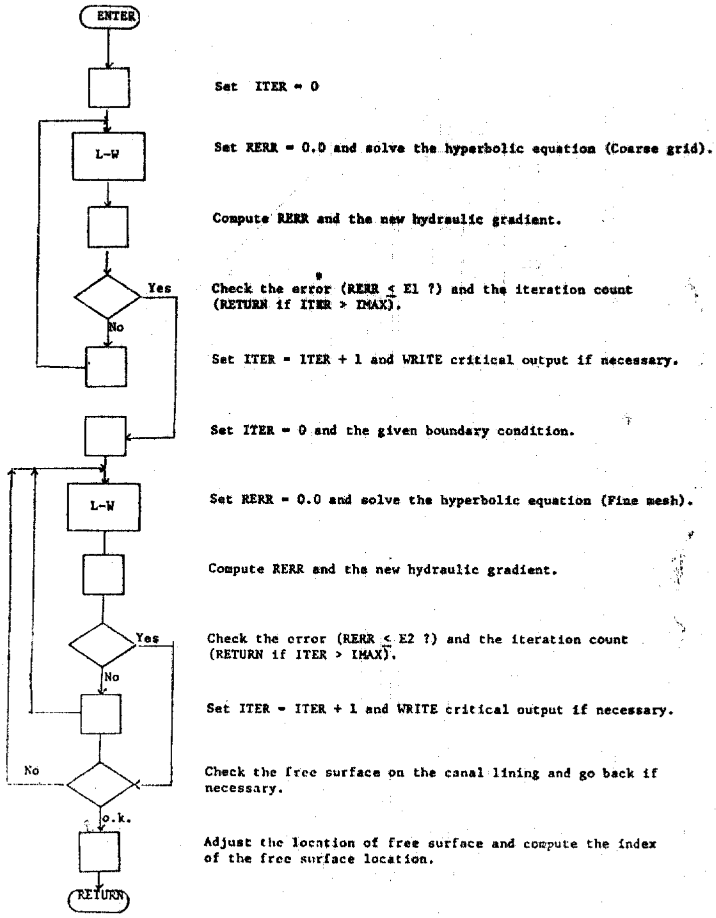

그림 4(b). Subroutine HYPER.

logic conditions (XK, YK, DK, ST) and the drawdown variations in the canal (DI, VEL, DT, TMAX) are given by the READ statement. Note that the computational grids (Δx, Δy) are automatically adjusted to the canal slope (SL) for a given drawdown velocity (VEL), time increment (DT) and simulation period (TMAX).

Output option (FLAG) has been provided; 0 for the permeable case of the canal lining, 1 for the impermeable case. These two extreme cases indicate the envelope of the realistic case where the semi-impermeable layer restricts the seepage through the canal lining. Only the outputs for the free surface in the fine grid region is printed out. For a full output, the corresponding comment statements in the main program should be removed. Two WRITE statements of the fine and coarse grid computations are included for each subroutine to check critical outputs during the program implementation.

Typical values of the "effective" porosity n are 0.10~0.30 for sands and 0.03~0.05 for clays (Eagleson, 1970). Average values of the hydraulic conductivity K are $1\text{-}10^2$cm/sec for gravel. 10^{-3}-1cm/sec for clean sands and $10^{-6}\text{-}10^{-3}$cm/sec for clayey sands, fine sands (De-Wiest, 1965). Because samples with higher n generally also have higher K, the probable range of ST($=n/K$) is 0.01 (coarse sands)—100 (fine sands) hr/ft. For small values of ST for the coarse sands, the seepage from the canal lining will be unsaturated and the saturated-unsaturated formulation will be needed.

Maximum drawdown velocity (VEL) has been suggested to be 2 FT/DAY from the transient analysis of the flow by pumping and gate motion. For the sensitivity analysis of different drawdown velocities, the matrix dimensions should be adjusted to provide an ade- quate simulation by checking corresponding statements in the main program. Even though the given numerical method is unconditionally stable, the time increment (DT) should be restricted for the accuracy consideration. For comparable results with an explicit method, the step size of the implicit method can be "several" times longer than that of the Courant condition$\left(DT \leq \frac{ST}{|hx|} \cdot \Delta x\right)$.

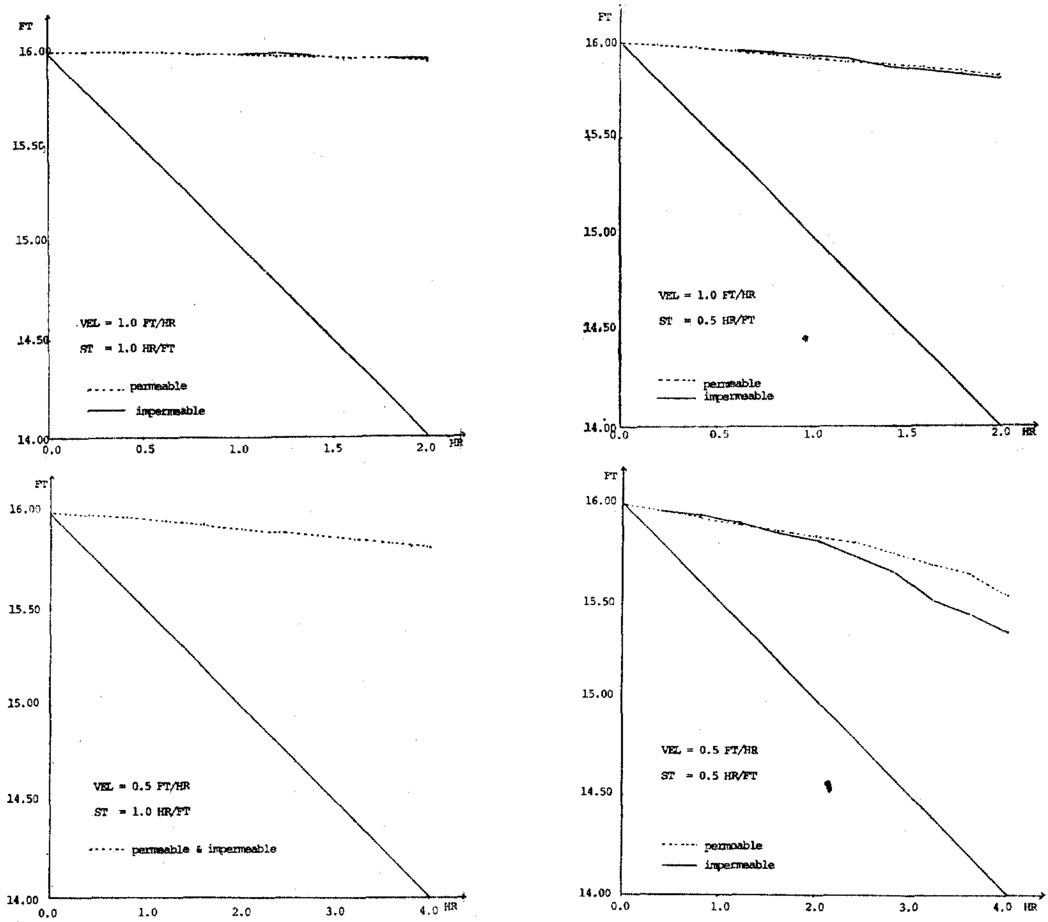

그림 5. Transient free surface height on the lining from the canal bottom(Lower triangle represents the drawdown in the canal)

7. Simulation Results

Four different cases in the simulation conditions (model input VEL and model parameter ST) are presented for both permeable and impermeable canal lining to predict the transient seepage surface developments (Fig. 5). First, the following intuitive reasonings are confirmed (where $XK=20.0$, $YK=10.0$, $DK=6.0$ and $DI=4.0$ FT).

Obs. 1) For a given drawdown velocity, fine medium (large ST) develops longer seepage surface than coarse medium (small ST).

Obs. 2) For a given porous medium, fast drawdown velocity develops longer seepage surface than slow drawdown. More important observations useful for a further simulation study are

Obs. 3) If both VEL and ST are relatively small, both permeable and impermeable cases should be investigated to incorporate the model uncertainty of the canal lining (otherwise, more refined model should be developed to incorporate this).

Obs. 4) Furthermore, the impermeability approximation of the canal lining does not necessarily give the critical condition for a slope stability analysis.

Typical storage requirement and computation time on the MV/8000 for a given simulation run (both permeable and impermeable cases) are less than 300 K and 100 second, respectively.

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Notation

DH; total depth of channel

DI; vertical distance between water surface in channel and ground level

DL; vertical distance between impermeable layer and channel bottom

H; position of free surface

$\hat{H}$; approximate value of *H*

h; piezometric head

K; hydraulic conductivity

n; effective porosity of the medium

p; hydrostatic pressure

RD; vertical distance between water surface and groundwater table

RH; thickness of aquifer

SL; channel wall slope

TD; total depth of impermeable layer from the ground level

TW; total length between the center of channel and the point of constant groundwater table

WB; width of channel bottom

WI; width of water surface in channel

WT; total width of channel

w; relaxation parameter

x, y, z; three dimensional axis or distance; and

γ; specific gravity of fluid

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