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Common Fixed Point Theorems in Probabilistic Metric Spaces and Extension to Uniform Spaces*

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>***Abstract***<

Let$(X,\mathcal{F})$ be a probabilistic metric space with a t-norm. Common fixed point theorems and convergence theorems generalizing the results of Ćirić, Fisher, Sehgal, Istrăţescu-Săcuiu and others are proved for three mappings P,S,T on X satisfying $F_{Pu,Pv}(qx)\geq \min\{F_{Su,Tv}(x), F_{Pu,Su}(x), F_{Pv,Tv}(x), F_{Pu,Tv}(2x), F_{Pv,Su}(2x)\}$ for every $u,v\in X$, all $x>0$ and some $q\in(0,1)$. One of the main results is extended to uniform spaces.

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1. Introduction.

V.M. Sehgal [13] initiated the study of contraction mappings on probabilistic metric spaces (PM-spaces) (cf. also [14] and [15]). Ćirić [3] introduced the notion of 'generalized contraction' on a PM-space (see Remark 1 below). On the other hand Jungck [9] generalized the well known Banach Contraction Principle by introducing a contraction condition for a pair of self-mappings on a metric space (see also [16]). Fisher [4] studied a contraction condition for a triplet of self-mappings on a metric space and obtained the essential part of Jungck's result [9] as a corollary. Recently Hadžić [5] has extended Fisher's result [4] to PM-spaces. In the present note we combine the ideas of Ćirić [3] and Fisher [4] (cf. also Singh [17]), and introduce the notion of 'generalized contraction'

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type condition for a triplet of self-mappings on a PM-space (cf. Definition 1). Then we prove two common fixed point theorems (cf. Theorems 1 and 2) for the generalized contraction triplet which include Hadžić's contraction condition [5] for a triplet of mappings on a PM-space. We give application and prove two convergence theorems for three sequences of mappings and the sequence of their common fixed points. Finally we present an extension of the main result to uniform spaces, which includes several fixed point theorems on metric, PM- and uniform spaces.

2. Preliminaries.

A PM-space is an ordered pair $(X, \mathcal{F})$ where X is a nonempty set of elements and $\mathcal{F}$ is a mapping from $X \times X$ to $\mathfrak{L}$, the collection of all distribution functions. The value of $\mathcal{F}$ at $(u, v) \in X \times X$ is represented by $F_{u,v}$. The functions $F_{u,v}$ are assumed to satisfy the following conditions

(a) $F_{u,v}(x)=1$ for all $x>0$ iff $u=v$;

(b) $F_{u,v}(0)=0$;

(c) $F_{u,v}=F_{v,u}$;

(d) if $F_{u,v}(x)=1$ and $F_{v,w}(y)=1$ then $F_{u,w}(x+y)=1$.

A mapping t; $[0,1] \times [0,1] \to [0,1]$ is called a t-norm (or T-norm, see [12]) if it satisfies:

(e) $t(a,1)=a$, $t(0,0)=0$;

(f) $t(c,d) \geq t(a,b)$ for $c \geq a$, $d \geq b$;

(g) $t(a,b)=t(b,a)$;

(h) $t(t(a,b),c)=t(a,t(b,c))$;

for all a, b, c, d in $[0,1]$.

A Menger space is a triplet $(X, \mathcal{F}, t)$, where $(X, \mathcal{F})$ is a PM-space and t-norm t is such that the inequality

$$\text{(d}'\text{)} \quad F_{u,w}(x+y) \geq t\{F_{u,v}(x),\ F_{v,w}(y)\}$$

holds for all $u, v, w \in X$ and all $x \geq 0, y \geq 0$.

Note that among a number of possible choices for t, $t(a,b)=\min\{a,b\}$ or simply "$t=$ min" is the strongest possible universal t (cf. [12, page 318]). Moreover, if t satisfies

$t(x,x) \geq x$ for every $x \in [0,1]$ then (f) and (d') imply

$$(d'')\quad F_{u,w}(x+y) \geq \min\{F_{u,v}(x),\ F_{v,w}(y)\}$$

for all $u,\ v,\ w \in X$ and all $x \geq 0,\ y \geq 0$.

Due to the simplicity and universality of "$t=\min$", (d") will be used frequently.

For the details of the topological preliminaries refer to Schweizer and Sklar [12] and Istrătescu [6].

DEFINITION 1. Three mappings $P,\ S,\ T$ on a PM-space $(X, \mathfrak{F})$ will be called a generalized contraction triplet $(P;\ S,\ T)$ iff there exists a constant $q \in (0,1)$ such that for every $u,\ v \in X$,

$$(1)\quad F_{Pu,Pv}(qx) \geq \min\{F_{Su,Tv}(x),\ F_{Pu,Su}(x),\ F_{Pv,Tv}(x),\ F_{Pu,Tv}(2x),\ F_{Pv,Su}(2x)\}$$

for all $x > 0$.

DEFINITION 2. If there exists a point u_0 in X and a sequence $\{u_n\}$ in X such that

$$(2)\quad Su_{2n+1} = Pu_{2n},\ Tu_{2n+2} = Pu_{2n+1},\ n=0,\ 1,\ 2, \cdots\cdots,$$

then the space X will be called $(P;\ S,T)$-orbitally complete with respect to u_0 or simply $(P(u_0);\ S,T)$-orbitally complete iff the Cauchy sequence $\{Pu_n\}$ converges in X.

If S and T are identity mappings on X then the space X will be called $P(u_0)$-orbitally complete. X is said to be P-orbitally complete if it is $P(u_0)$-orbitally complete for every $u_0 \in X$; [2, 8].

DEFINITION 3. S is said to be $(P(u_0);\ S,T)$-orbitally continuous if the restriction of S on the closure of $\{Pu_n\}$ is continuous.

3. Fixed Point Theorems.

We shall need the following result:

LEMMA. Let $\{y_n\}$ be a sequence in a Menger space $(X, \mathfrak{F}, t)$, where t is continuous and satisfies $t(x,x) \geq x$ for every $x \in [0,1]$. If there exists a constant $q \in (0,1)$ such that

$$(3)\quad F_{y_n, y_{n+1}}(qx) \geq F_{y_{n-1}, y_n}(x)$$

for all n, then $\{y_n\}$ is a Cauchy sequence in X.

PROOF. Let ε, λ be positive reals. Then for $m>n$ we have by (d′),

$$F_{y_n, y_m}(\varepsilon) \geq t\{F_{y_n, y_{n+1}}(\varepsilon - q\varepsilon),\ F_{y_{n+1}, y_m}(q\varepsilon)\}$$
$$\geq t\{F_{y_0, y_1}(\varepsilon - q\varepsilon)q^{-n},\ F_{y_{n+1}, y_m}(q\varepsilon)\}, \text{ by (3).}$$

Taking $(\varepsilon - q\varepsilon)q^{-n} = a$, it follows that

$$F_{y_n, y_m}(\varepsilon) \geq t\{F_{y_0, y_1}(a),\ t(F_{y_{n+1}, y_{n+2}}(q\varepsilon - q^2\varepsilon),\ F_{y_{n+2}, y_m}(q^2\varepsilon))\}$$
$$\geq t\{F_{y_0, y_1}(a),\ t(F_{y_0, y_1}(a),\ F_{y_{n+2}, y_m}(q^2\varepsilon))\}.$$

By the associativity of t and $t(x,x) \geq x$,

$$F_{y_n, y_m}(\varepsilon) \geq t\{F_{y_0, y_1}(a),\ F_{y_{n+2}, y_m}(q^2\varepsilon)\}.$$

Repeated use of these arguments gives

$$F_{y_n, y_m}(\varepsilon) \geq t\{F_{y_0, y_1}(a),\ F_{y_{m-1}, y_m}(q^{m-n-1}\varepsilon)\}$$
$$\geq t\{F_{y_0, y_1}(a),\ F_{y_0, y_1}(q^{-n}\varepsilon)\}$$
$$\geq t\{F_{y_0, y_1}(a),\ F_{y_0, y_1}(a)\}$$
$$\geq F_{y_0, y_1}((\varepsilon - q\varepsilon)q^{-n}).$$

Therefore, if N be so chosen that $F_{y_0, y_1}(\varepsilon - q\varepsilon)q^{-N} > 1-\lambda$, it follows that

$$F_{y_n, y_m}(\varepsilon) > 1-\lambda \quad \text{for all } n \geq N.$$

Hence $\{y_n\}$ is a Cauchy sequence in X.

THEOREM 1. Let $(X, \mathfrak{F}, t)$ be a Menger space, where t is continuous and satisfies $t(x,x) \geq x$ for every $x \in [0,1]$, and $P, S, T\colon X \longrightarrow X$. Further, let $(P; S, T)$ be a generalized contraction triplet, $PT=TP$ and $PS=SP$. If there exists a point u_0 in X such that X is $(P(u_0); S, T)$-orbitally complete and P, S and T are $(P(u_0); S, T)$-orbitally continuous, then P, S and T have a unique common fixed point and $\{Pu_n\}$ converges to the fixed point.

PROOF. By (1),

$$F_{Pu_{2n}, Pu_{2n-1}}(qx) = F_{Pu_{2n-1}, Pu_{2n}}(qx)$$

$$\geq \min\{F_{Pu_{2n-2},Pu_{2n-1}}(x), F_{Pu_{2n}, Pu_{2n-1}}(x), F_{Pu_{2n}, Pu_{2n-2}}(2x)\}.$$

This gives

$$F_{Pu_{2n}, Pu_{2n-1}}(qx) \geq F_{Pu_{2n-1}, Pu_{2n-2}}(x),$$

since

$$F_{Pu_{2n}, Pu_{2n-2}}(2x) \geq \min\{F_{Pu_{2n}, Pu_{2n-1}}(x), F_{Pu_{2n-1},Pu_{2n-2}}(x)\}.$$

Similarly

$$F_{Pu_{2n+1}, Pu_{2n}}(qx) \geq F_{Pu_{2n}, Pu_{2n-1}}(x).$$

In general

$$F_{Pu_{n+1},Pu_n}(qx) \geq F_{Pu_n,Pu_{n-1}}(x).$$

Therefore, $\{Pu_n\}$ is a Cauchy sequence by the Lemma in X, and converges to some point z in X. By the continuity conditions on P and S, $PSu_{2n+1} \to Pz$ and $SPu_{2n+1} \to Sz$. So $Pz = Sz$, since P and S commute.
Similarly $Tz = Pz$. So, from

$$F_{Pu_{2n+1}, Pz}(qx) \geq \min\{F_{Su_{2n+1}, Tz}(x), F_{Pu_{2n+1}, Su_{2n+1}}(x),$$
$$F_{Pz,Tz}(x), F_{Pu_{2n+1},Tz}(2x), F_{Pz, Su_{2n+1}}(2x)\},$$

it follows that

$$F_{z,Pz}(qx) \geq \min\{F_{z,Pz}(x), 1, 1, F_{z,Pz}(2x), F_{Pz,z}(2x)\} = F_{z,Pz}(x),$$

which yields $Pz = z$. Thus z is the common fixed point of P, S and T.

In order to prove the uniqueness of z, let $y (\neq z)$ be another common fixed point. Then for all $x > 0$,

$$F_{y,z}(qx) = F_{Py, Pz}(qx)$$
$$\geq \min\{F_{Sy,Tz}(x), F_{Py,Sy}(x), F_{Pz,Tz}(x), F_{Py,Tz}(2x), F_{Pz,Sy}(2x)\}$$
$$= \min\{F_{y,z}(x), 1, 1, F_{y,z}(2x), F_{z,y}(2x)\},$$

proving $y = z$.

It is evident from the above proof that if S and T be identity maps on X, then P may be noncontinuous. Hence we have

REMARK 1. If S and T be identity mappings in Definition 1, then P satisfying (1) is a generalized contraction (on a PM-space) introduced by Ćirić [3]. So, if S and T be identity mappings on X then Ćirić's result [3, Theorem 1] is obtained as a corollary to the above theorem.

THEOREM 2. Let $(x,\mathfrak{F},t)$ be a complete Menger space where t is continuous and satisfies $t(x,x)\geq x$ for every $x\in[0,1]$, and P,S,T: $X\to X$. Further let $(P(u_0);\ S,T)$ be a generalized contraction triplet, $PT=TP$, $PS=SP$ and $P(X)\subseteq S(X)\cap T(X)$. If P,S and T be continuous then P,S and T have a unique common fixed point.

We remark that Theorems 1—2 generalize the corresponding results in [4] and [9].

4. An Application

Now we shall apply Theorem 2 to establish the following result.

THEOREM 3. Let $(X,\mathfrak{F},t)$ be a complete Menger space, where t is continuous and satisfies $t(x,x)\geq x$ for every $x\in[0,1]$, and P,S and T three mappings from the product space $X\times X$ to X such that

$$P(X\times\{v\})\subseteq S(X\times\{v\})\cap T(X\times\{v\});$$
$$P(S(u,v),v)=S(P(u,v),v)$$

and $P(T(u,v),v)=T(P(u,v),v)$
for all u,v in X. Suppose that

$$F_{P(u,v),P(u',v')}(qx)$$

$$(3.1)\qquad \geq\min\{F_{S(u,v),T(u',v')}(x),\ F_{P(u,v),S(u,v)}(x),\ F_{P(u',v'),T(u',v')}(x),\ F_{P(u,v),T(u',v')}(2x),\ F_{P(u',v'),S(u,v)}(2x),\ F_{v,v'}(x)\}$$

for all u,u', v,v' in X and for all $x>0$ and some constant $q\in(0,1)$. If P,S and T be continuous then there exists exactly one point b in x such that

$$P(b,b)=S(b,b)=T(b,b)=b.$$

PROOF. Let $v=v'$ in (3.1). Then

$$\begin{aligned}F_{P(u,v),P(u',v)}(qx)\geq\min\{&F_{S(u,v),T(u',v)}(x),\\&F_{P(u,v),S(u,v)}(x),\\&F_{P(u',v),T(u',v)}(x),\ F_{P(u,v),T(u',v)}(2x)\\&F_{P(u',v),S(u,v)}(2x)\}.\end{aligned}$$

Therefore for a fixed v in X, Theorem 2 yields that there exists a unique $u(v)$ in X such that

(3.2) $\quad P(u(v),v)=S(u(v),v)=T(u(v),v)=u(v).$

Therefore for any v, v' in X we have by (3.1),

$$\begin{aligned}F_{u(v),u(v')}(qx)&=F_{P(u(v),v),P(u(v'),v')}(qx)\\&\geq\min\{F_{u(v),u(v')}(x),\ F_{u(v),u(v)}(x),\\&\quad F_{u(v'),u(v')}(x), F_{u(v),u(v')}(2x),\\&\quad F_{u(v'),u(v)}(2x),\ F_{v,v'}(x)\}\\&=F_{v,v'}(x).\end{aligned}$$

Since this is true for all $x>0$, $u(\cdot)$ is a contraction mapping on the complete Menger space X. So there exists a unique b in X such that $u(b)=b$. Hence by (3.2),

$$P(b,b)=S(b,b)=T(b,b)=b$$

5. Convergence Theorems.

Let P and $P_n(n=1,2,\cdots)$ be mappings on a PM-space. If $P_n\to P$ uniformly then Istrătescu [6, page 342] (cf. also [3, Cor. 2.1]) and Ćirić [3, Theorem 2] have investigated the conditions under which the sequence of the common fixed points of P_n converges to the fixed point of P. Similar investigations have been made by Istrătescu and Săcuiu [7] in the case of two sequences of mappings on a PM-space. Istrătescu [6, page 342] has also proved a convergence theorem if $P_n\to P$ pointwise. In this section we consider three sequences of mappings, and first prove the following result.

THEOREM 4. Let $(X,\mathfrak{F},t)$ be a Menger space, where t is continuous and satisfies $t(x,x)\geq x$ for every $x\in[0,1]$. Let P_n, S_n and T_n be mappings from X to itself with a common fixed point z_n for each $n=1,2,\cdots$. Let $(P;\ S,T)$ be a generalized contraction triplet on X with z as their common fixed point. If the sequences $\{P_n\}$, $\{S_n\}$ and $\{T_n\}$ converge uniformly to P, S and T respectively on $\{z_n,\ n=1,2\cdots\}$ then $z_n\to z$.

PROOF. We have for any n,

$$
\begin{aligned}
F_{z_n,\,z}(\varepsilon) &= F_{P_nz_n,\ Pz}\left(\frac{1-q}{2}\varepsilon+\frac{1+q}{2}\varepsilon\right)\\
&\geq \min\left\{F_{P_nz_n,Pz_n}\left(\frac{1-q}{2}\varepsilon\right),\ F_{Pz_n,\ Pz}\left(\frac{1+q}{2}\varepsilon\right)\right\}\\
&\geq \min\left(F_{P_nz_n,Pz_n}\left(\frac{1-q}{4}\varepsilon\right),\ F_{Pz_n,Pz}\left(\frac{1+q}{2}\varepsilon\right)\right\}. \qquad (4.1)
\end{aligned}
$$

By (1),

$$
\begin{aligned}
F_{Pz_n,Pz}\left(\frac{1+q}{2}\varepsilon\right) &= F_{Pz_n,Pz}\left(q\frac{1+q}{2q}\varepsilon\right)\\
&\geq \min\left\{F_{Sz_n,Tz}\left(\frac{1+q}{2q}\varepsilon\right),\ F_{Pz_n,Sz_n}\left(\frac{1+q}{2q}\varepsilon\right),\right.\\
&\qquad \left. F_{Pz,Tz}\left(\frac{1+q}{2q}\varepsilon\right),\ F_{Pz_n,Tz}\left(\frac{1+q}{q}\varepsilon\right),\ F_{Pz,Sz_n}\left(\frac{1+q}{q}\varepsilon\right)\right\}\\
&= \min\left\{F_{Sz_n,z}\left(\frac{1+q}{2q}\varepsilon\right),\ F_{Pz_n,Sz_n}\left(\frac{1+q}{2q}\varepsilon\right)\right\},\\
&\qquad \text{since } Pz=Tz=z,\\
&\geq \min\left\{F_{Sz_n,\ S_nz_n}\left(\frac{1-q}{2q}\varepsilon\right),\ F_{z_n,\ z}\left(\frac{1+2q}{2q}\varepsilon\right),\right.\\
&\qquad \left. F_{Pz_n,P_nz_n}\left(\frac{1+q}{4q}\varepsilon\right),\ F_{P_nz_n,\ Sz_n}\left(\frac{1+q}{4q}\varepsilon\right)\right\}.\\
&\geq \min\left\{F_{Sz_n,\ S_nz_n}\left(\frac{1-q}{4}\varepsilon\right),\ F_{z_n,\ z}\left(\frac{1+2q}{2q}\varepsilon\right),\right. \qquad (4.2)\\
&\qquad \left. F_{Pz_n,\ P_nz_n}\left(\frac{1-q}{4}\varepsilon\right)\right\},\\
&\qquad \text{since } P_nz_n=S_nz_n=z_n.
\end{aligned}
$$

Since $\{P_n\}$ and $\{S_n\}$ converge uniformly to P and S, there exist $\varepsilon,\lambda>0$ such that

$$
F_{P_nz_n,\,Pz_n}\left(\frac{1-q}{4}\varepsilon\right)>1-\lambda \text{ and } F_{S_nz_n,\ Sz_n}\left(\frac{1-q}{4}\varepsilon\right)>1-\lambda
$$

for all $n\geq N=N(\varepsilon,\lambda)$. So from (4.1) and (4.2) we have for all $n\geq N$,

$$F_{z_n,z}(\varepsilon)>1-\lambda, \text{ since } F_{z_n,z}(\varepsilon)<F_{z_n,z}(\frac{1+2q}{2q}\varepsilon).$$

Thus $z_n\to z$.

THEOREM 5. Let $(X,\mathfrak{F},t)$ be a Menger space, where t is continuous and satisfies $t(x,x)\geq x$ for every $x\in[0,1]$. Let a triplet $(P_n;\ S_n,T_n)$ of self-mappings on X be a generalized contraction with (the same) generalized contraction constant q and z_n as their common fixed point for each $n=1,2,\cdots$. If $\{P_n\}$, $\{S_n\}$ and $\{T_n\}$ converge respectively pointwise to selfmappings P,S and T of X with z as their common fixed point, then $z_n\to z$.

PROOF. For any n,

$$F_{z_n,z}(\varepsilon)=F_{P_nz_n,Pz}(\frac{1+q}{2}\varepsilon+\frac{1-q}{2}\varepsilon)$$

$$\geq\min\{F_{P_nz_n,P_nz}(\frac{1+q}{2}\varepsilon),\ F_{P_nz,Pz}(\frac{1-q}{2}\varepsilon)\}$$

(5.1) $$\geq\min\{F_{P_nz_n,P_nz}(\frac{1+q}{2}\varepsilon),\ F_{P_nz,Pz}(\frac{1-q}{4}\varepsilon)\}.$$

Since $(P_n;\ S_n,T_n)$ is a generalized contraction triplet, we have

$$F_{P_nz_n,P_nz}(\frac{1+q}{2}\varepsilon)=F_{P_nz_n,P_nz}(q\frac{1+q}{2q}\varepsilon)$$

$$\geq\min\{F_{S_nz_n,T_nz}(\frac{1+q}{2q}\varepsilon),\ F_{P_nz_n,S_nz_n}(\frac{1+q}{2q}\varepsilon),$$

$$F_{P_nz,T_nz}(\frac{1+q}{2q}\varepsilon),\ F_{P_nz_n,T_nz}(\frac{1+q}{q}\varepsilon),$$

$$F_{P_nz,S_nz_n}(\frac{1+q}{q}\varepsilon)\}$$

$$=\min\{F_{z_n,T_nz}(\frac{1+q}{2q}\varepsilon),\ F_{P_nz,T_nz}(\frac{1+q}{2q}\varepsilon)\},$$

since $P_nz_n=S_nz_n=z_n$,

$$\geq\min\{F_{z_n,z}(\frac{1+2q}{2q}\varepsilon),\ F_{Tz,T_nz}(\frac{1-q}{2q}\varepsilon),$$

$$F_{P_nz,Pz}(\frac{1+q}{4q}\varepsilon),\ F_{Tz,T_nz}(\frac{1+q}{4q}\varepsilon)\}$$

(5.2) $$\geq\min\{F_{z_n,z}(\frac{1+2q}{2q}\varepsilon),\ F_{T_nz,Tz}(\frac{1-q}{4}\varepsilon),$$

$$F_{P_nz,Pz}(\frac{1-q}{4}\varepsilon)\}.$$

Since P and T are pointwise limits of $\{P_n\}$ and $\{T_n\}$, corresponding to a point z there exists $\varepsilon, \lambda > 0$ such that

$$F_{P_n z,\, Pz}\left(\frac{1-q}{4}\varepsilon\right) > 1-\lambda \text{ and } F_{T_n z,\, Tz}\left(\frac{1-q}{4}\varepsilon\right) > 1-\lambda$$

for all $n \geq N = N(\varepsilon, \lambda)$.

So from (5.1) and (5.2) we have for all $n \geq N$,

$$F_{z_n, z}(\varepsilon) > 1-\lambda, \text{ since } F_{z_n, z}(\varepsilon) \leq F_{z_n, z}\left(\frac{1+2q}{2q}\varepsilon\right).$$

Hence $z_n \rightarrow z$.

6. Extension to Uniform Spaces

In all that follows we shall assume that X is set and $D = \{d_\alpha\}$ is a nonempty collection of pseudo-metrics on X. It is well known that the uniformity generated by D is obtained by taking as a subbase all sets of the form $U_{\alpha,\varepsilon} = \{(x,y) \in X \times X \colon d_\alpha(x,y) < \varepsilon\}$, where $d_\alpha \in D$ and $\varepsilon > 0$. In fact, the topology determined by this uniformity has all d_α-spheres as a subbase. For details refer to Kelley [10]. Cain and Kasriel [1] have shown that a collection of pseudometrics $\{d_\alpha\}$ can be defined which generates the usual structure for Menger spaces. Hence the following result is a direct consequence of Theorem 2.

THEOREM 6. Suppose X is sequentially complete Hausdorff space and $P, S, T \colon X \rightarrow X$ having the property that for every $d_\alpha \in D$, there is a constant $q_\alpha \in (0,1)$ such that

$$\begin{aligned} d_\alpha(Pu, Pv) \leq q_\alpha \max \{ & d_\alpha(Su, Tv),\ d_\alpha(Pu, Su), \\ & d_\alpha(Pv, Tv),\ \tfrac{1}{2} d_\alpha(Pu, Tv), \\ & \tfrac{1}{2} d_\alpha(Pv, Su)\} \end{aligned}$$

for all u, v in X. If P, S and T are continuous, P commuting with each of S and T, and $P(X) \subseteq S(X) \cap T(X)$, then P, S and T have a unique common fixed point.

In an anologous blend Theorems 1 and 4—5 may be extended to uniform spaces.

Theorem 6 includes a number of fixed point theorems in metric, Menger and uniform spaces, which may be obtained choosing P, S and T suitably. For example, if $S = T$ then it presents a nice generalization of Jungck's result [9]. Theorem 6 also includes a recent result of Khan and Fisher [11]. It may be mentioned that if $S = T$ and $T(X)$ a closed subspace of X then the mappings, in Theorem 6 (and hence in Theorems 1—3 also), need not be continuous (see Singh [18]).

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