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A Mixed 0-1 Linear Program for the Inspection Location Problem

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Abstract

An economic model is developed for determining optimal locations of screening inspection stations in a multistage production system. The effect of screening inspection on the production rate is explicitly considered, and a fixed cost for maintaining an inspection station is assumed. The product is allowed to have multiple defects, each of which may be inspected at any inspection station after the defect-generating operation. The problem is formulated as a mixed 0-1 linear program which offers the advantage of versatility in handling various system constraints.

1. Introduction

For a sequential production system as shown in Figure 1, the problem of determining optimal locations of screening inspection stations has been considered by many authors. Screening inspection, inserted into such a system, may reduce the unnecessary processing as well as the external failure cost by taking corrective actions on the defective items during intermediate production stages. On the other hand, it may affect the overall economy of production due to additional costs needed for inspection and internal failures.

Considering these positive and negative aspects of screening inspection, previous authors formulated the inspection location problem as a dynamic program [1, 2, 3], a shortest route model [5, 7], a warehouse location problem [6], etc. In the previous works, however, an important aspect of screening inspection is ignored, namely, its effect on the overall production rate. If many quality characteristics are inspected at a certain location, the accumulated inspection time may break the existing balance of the production line resulting in a decrease in the production rate (or, equivalently, an increase in the cycle time which is defined here as the time the final product is available). Any screening inspection program which is designed without this consideration is most likely to be criticized as a bottleneck to production.

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The purpose of this article is to present an economic model for the inspection location problem for serial production systems considering the effect of screening inspection on the production rate explicitly. The problem is formulated as a mixed 0-1 linear program which offers the advantage of versatility in handling various system constraints. The present formulation is also applicable to a certain class of nonserial production systems (e.g., see Figure 2) through obvious generalization.

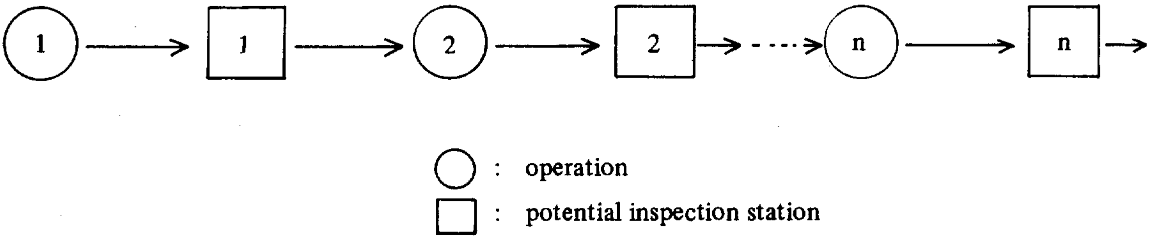

Figure 1. A Serial Production System.

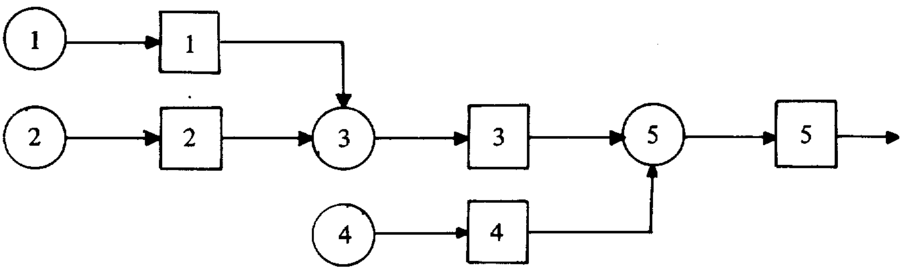

Figure 2. A Tree-like Nonserial Production System.

2. Assumptions

Consider a n-stage serial production system as shown in Figure 1. Each stage consists of an operation and a potential inspection station. Given that raw meterials are continuously available at operation 1 and the material flow rate between successive operations is unity, consider the following assumptions:

1. An operation i generates a type-i defect in an item according to an independent Bernoulli process with known probability e_i.
2. Type-i defect can be inspected at most once at any inspection station j ($i \leqslant j \leqslant n$) with inspection cost and time u_{ij} and t_{ij}, respectively.
3. Inspection is assumed to be perfect. At the j-th inspection station, if an item is found to have type-i defect, it is repaired at a cost r_{ij} and returned to the production process.
4. A fixed cost u_j per unit time is assumed to maintain the j-th inspection facilities.
5. Type-i defect in the final product incurs an external failure cost f_i.
6. The cycle time of the production process is T_o when there is no screening inspection. A penalty

cost Q is assumed per unit time increase in the cycle time due to screening inspection.

Then, the problem is to determine which defect types to inspect at station j such that the total inspection, repair, external failure, and penalty cost are minimized.

3. An Economic Model

Define a set of decision variables $\{X_{ij}, i \leqslant j \leqslant n, i=1,2,...,n\}$ such that Xij = 1 if type-i defect is inspected at the j-th inspection station and 0 otherwise. Since each defect type is inspected at most once (assumption 2),

$$\sum_{j=i}^{n} X_{ij} + S_i = 1 \quad \text{for } i = 1, 2, ..., n \qquad (1)$$

where S_i = 0 if type-i defect is inspected at a certain station and 1 otherwise.

An expression for the total expected cost is developed per unit of the final product produced. First, from the definitions of X_{ij} and S_i, the following costs are easily obtained.

$$\text{total inspection cost} = \sum_{i=1}^{n} \sum_{j=1}^{n} u_{ij} X_{ij} \qquad (2)$$

$$\text{total expected repair cost} = \sum_{i=1}^{n} \sum_{j=i}^{n} e_i r_{ij} X_{ij} \qquad (3)$$

$$\text{total expected external failure cost} = \sum_{i=1}^{n} e_i f_i S_i \qquad (4)$$

At the j-th inspection station, the total inspection time is given by

$$T_j = \sum_{i=1}^{j} t_{ij} X_{ij} \quad \text{for } j = 1, 2, ..., n \qquad (5)$$

Then, the cycle time associated with the policy $\{X_{ij}\}$ is given by

$$T = \max_{1 \leqslant j \leqslant n} \{T_o, T_j\} \qquad (6)$$

Where T_o is the cycle time when no screening inspection is performed. The penalty cost for the increase in the cycle time is then given by $Q(T - T_o)$. Note that when $T_j \leqslant T_o$ for all j, the penalty cost is 0.

Finally, the fixed charge for maintaining the j-th inspection facilities can be represented by $u_j Y_j T$ where a 0-1 indicator variable Y_j is given by

$$Y_j = \max_{1 \leqslant i \leqslant j} \{X_{ij}\} \quad \text{for } j = 1, 2, ..., n \qquad (7)$$

Note that if no defect type is inspected at the j-th station, Y_j = 0 and no fixed cost is incurred.

The problem of optimally determining the locations of screening inspection stations is then represented by the following optimization problem:

$$
(\text{PO}) \begin{cases} \min Z = \sum_{i=1}^{n} \sum_{j=1}^{n} (u_{ij} + e_i r_{ij}) X_{ij} + \sum_{i=1}^{n} e_i f_i S_i + Q(T - T_o) + \sum_{i=1}^{n} u_j Y_j T \\ \text{subject to} \\ \quad \text{Eqs. (1), (5), (6), (7)} \\ \quad X_{ij}, Y_j = 0, 1 \text{ for } i = 1, 2, \ldots, j; j = 1, 2, \ldots, n \\ \quad S_i = 0, 1 \text{ for } i = 1, 2, \ldots, n \end{cases}
$$

Each of the nonlinear constraints (6) and (7) can be easily modified to a set of linear constraints. For instance, constraint (7) is equivalent to: $Y_j \geqslant X_{ij}$ for i = 1,2,..., j and j = 1,2,..., n. For the nonlinear elements $\{Y_j T\}$ in the objective function, we introduce a new variable $W_j = Y_j T$, together with the following set of constraints:

$$
\left.\begin{array}{l} W_j \leqslant MY_j \\ T - W_j + MY_j \leqslant M \\ T - W_j \geqslant O \\ W_j \geqslant O \end{array}\right\} \quad \text{for } j = 1, 2, \ldots, n \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (8)
$$

where M is a "big" positive constant. Note that constraint (8) is linear and $Y_j = 0$ and 1 respectively imply $W_j = 0$ and T, which is the desired result.

With the above modifications, problem (PO) is now transformed to the following mixed 0-1 linear program:

$$
(\text{P}) \begin{cases} \min Z = \sum_{i=1}^{n} \sum_{j=1}^{n} (u_{ij} + e_i r_{ij}) X_{ij} + \sum_{i=1}^{n} e_i f_i S_i + Q(T - T_o) + \sum_{j=1}^{n} u_j W_j \\ \text{subject to} \\ \quad \sum_{j=1}^{n} X_{ij} + S_i = 1 \text{ for } i = 1, 2, \ldots, n \\ \quad T \geqslant T_o \\ \quad T \geqslant \sum_{i=1}^{j} t_{ij} X_{ij} \quad \text{for } j = 1, 2, \ldots, n \\ \quad Y_j \geqslant X_{ij} \quad \text{for } i = 1, 2, \ldots, j; j = 1, 2, \ldots, n \\ \quad \text{Eq. (8)} \\ \quad X_{ij}, Y_j, S_i = 0, 1 \end{cases}
$$

4. Example and Discussions

For a four-stage serial system the inspection location problem was formulated as (P) with parameter values as shown in Table 1. It was also assumed that $T_o = 5$ and $Q = 10$. The resulting 0-1 linear program was solved using an optimization package (LINDO) available at Texas Tech University. An optimal solution obtained consists of all x_{ij}'s at zero levels except that $x_{13} = x_{33} = 1$. That is, it is optimal to inspect defect types 1 and 3 at the third inspection station. The cost for maintaining the corresponding screening program is 1.66 per unit of the final product.

Table 1. Parameter Values for the Example.

i \ j	1	2	3	4
1	(0.1, 0.5, 1)*	(0.2, 0.5, 1)	(0.2, 1, 1.5)	(0.3, 1, 1.5)
2		(0.2, 0.4, 2)	(0.2, 0.5, 2)	(0.4, 0.5, 2.5)
3			(0.2, 1,2)	(0.3, 1.5, 2.5)
4				(0.3, 1, 1.5)

*(u_{ij}, r_{ij}, t_{ij})

i or j	e_i	f_i	u_j
1	0.1	4	0.15
2	0.04	5	0.2
3	0.1	8	0.1
4	0.06	6	0.2

Although formulation (P) assumes a serial system, it is also applicable to a tree-like production process as shown in Figure 2 (note that stages are numbered in such a way that no stage has a larger number than its successor).

Clearly, assumption (1) is not essential to the development of the present model, and can be relaxed to include the case when an operation may generate multiple defect types.

One advantage of the present formulation is its versatility in handling various system constraints. For instance, if there is a budget limitation in developing a screening program, then the following constraint may be included in (P):

$$\sum_{j=1}^{n} c_j Y_j \leqslant c \qquad (9)$$

where c_j is the cost for installing the j-th inspection station. Other types of constraints such as limitations on the availability of inspection resources(inspectors, space, etc.) and quality requirements may be handled without difficulty.

A rule of thumb which has been suggested by some authors (e.g., see [4]) is to inspect a certan defect type if the cost for inspecting it(inspection and internal failure cost) is cheaper than the cost of no inspection (external failure cost), and not to inspect otherwise. It can be shown that this rule is a solution to a very special case of the present model. That is, if we ignore the penalty and the fixed inspection cost in our present formulation (which is unrealistic in most situations), then the optimal solution is to inspect type-i defect at station j′if

$$e_i f_i \geqslant u_{ij}{'} + e_i r_{ij}{'} = \min_{i \leqslant j \leqslant n} \left\{ u_{ij} + e_i r_{ij} \right\} \quad \cdots\cdots \quad (10)$$

and not to inspect type-i defect otherwise.

In this article no attempt is made for developing an efficient algorithm to solve (P), although it can be solved using any general purpose mixed 0-1 linear programming code. A profitable area of future research may include an investigation of the special structure of (P) to develop an appropriate algorithm.

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