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The Weight Distribution of Pellet Sizes of Carbon Black Products*

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Abstract

A model for the weight distribution of pellet sizes of carbon black products is proposed so that it may be utilized in establishing the product standard and a quality control scheme. The various grades of carbon black products from the Korea Steel Chemical Co., Ltd. are examined to validate the adequacy of the proposed model.

1. Introduction

The final products of carbon black are in the forms of pellets of various sizes. The sizes of pellets are believed to be one of the important factors that influence the quality of rubber products when the carbon black is mixed with natural rubber. Thus the proportion of certain sizes of carbon black products is an important characteristic considered by a rubber company in assessing a quality specification to its rubber products.

In this paper the weight distribution of pellet sizes is considered for the five different grades of carbon black products from the Korea Steel Chemical Co., Ltd. A large number of sample weight distributions of pellet sizes is examined using the regression method for ordinal data given by McCullagh (1980) and it is proposed that the Weibull distribution of pellet sizes of carbon black products. The parameters of the proposed model are estimated and using these estimates the percentiles of the weight distributions are determined.

It is hoped that the results and the methodology of this paper can be utilized in assessing the product standard of carbon black products and in establishing a quality control scheme in the production process.

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2. Model and Estimation

Let W(s) denote the percentage of weight of carbon black products whose pellet sizes are less than s, then a parametric model considered for W(s) is the Weibull distribution given by

$$(1)\quad W(s) = 1 - \exp(-\delta s^{\beta}).$$

The adequacy of the model is verified by a large number of replicates of independent samples of the five different grades of carbon black products and the parameters, δ and β, are estimated using the regression models for ordinal data given in McCullagh (1980). More specifically, when a random sample of carbon black is taken from a production process, the sample pellets are sorted according to their sizes by using the sieves of seven different mesh sizes. The percentage weights in each size category are measured in milligrams. The specified mesh sizes denoted by $s_1, s_2, ..., s_7$ are respectively

$$\frac{120}{120},\ \frac{120}{60},\ \frac{120}{35},\ \frac{120}{18},\ \frac{120}{14},\ \frac{120}{12},\ \frac{120}{10} \text{ in millimeters.}$$

Let $W(s_i)$ denote the cumulative relative weight of a sample whose pellet size are less than s_i,

$$Y_i = \ell n \ \{-\ell n\,(1 - W(s_i))\} \quad \text{, and } X_i = \ell n\, s_i,\ i = 1, 2, ..., 7.$$

A large number of replicates of independent samples is examined in order to validate the adequacy of the model proposed in (1). In all cases, the plots of Y_i's on X_i's show a clear linearity and the sample correlation between Y_i's and X_i's range 0.95 to 0.99. Thus assuming that the proposed model given in (1) is an adequate model for the weight distribution of pellet sizes, the following regression model is considered in estimating the parameters,

$$(2)\quad Y_i = \alpha + \beta x_i + \epsilon_i,\ i = 1, 2, ..., 7,$$

where $\alpha = \ell n\ \delta$ and ϵ_i's are the error terms. For the linear model given in (2), the ordinary least square estimates of α and β are given by, respectively,

$$\hat{\alpha} = \overline{Y} - \hat{\beta}\,\overline{X}$$
$$\hat{\beta} = \sum_{i=1}^{7} (Y_i - \overline{Y})(X_i - \overline{X}) \Big/ \sum_{i=1}^{7} (X_i - \overline{X})^2,$$

where

$$\overline{X} = \frac{1}{7} \sum_{i=1}^{7} X_i \quad \text{and } \overline{Y} = \frac{1}{7} \sum_{i=1}^{7} Y_i.$$

Since it may not be reasonable to assume that the error terms in the model (2) satisfy the usual assumptions of independence, normality, and the homogeneity of variances, the performance characteristics of the ordinary least square estimates of α and β are examined on the basis of a large number of independent estimates.

3. Data Analyses

The various sizes of independent samples from the five different grades of carbon black products are analyzed. Using the equation (3), the independent sets of estimates of α and β are computed. The means and standard deviations of these independent estimates are summarized in table 1.

Table 1: The means and standard deviations of estimates of α and β

Grades (Sample Size)	Mean of estimate of β (standard deviation)	Mean of estimate of α (standard deviation)
N 330 (52)	3.199 (0.1914)	– 6.386 (0.4880)
N 339 (38)	3.278 (0.2282)	– 6.656 (0.5523)
N 550 (37)	3.051 (0.2187)	– 6.412 (0.4586)
N 660 (40)	2.914 (0.2389)	– 5.964 (0.5450)
N 770 (39)	3.224 (0.1873)	– 6.471 (0.4149)

The percentiles of the weight distributions of pellet sizes are determined using the following equation

$$\hat{s}_p = \{-\ln(1-p)e^{-\bar{\alpha}}\}^{\frac{1}{\bar{\beta}}},$$

where p denote the pth percentile, $\bar{\alpha}$ and $\bar{\beta}$ are the means of the estimates of α and β respectively, given in Table 1. The summary of the percentiles of the weight distributions are given in Table 2.

Table 2: The percentiles of weight distributions

percentile % / grades	5	10	20	25	50	75	80	90	95
N330	2.91	3.64	4.61	4.99	6.54	8.15	8.54	9.55	10.37
N339	3.08	3.83	4.82	5.21	6.81	8.42	8.81	9.83	10.65
N550	3.09	3.91	5.00	5.44	7.25	9.10	9.56	10.75	11.72
N660	2.79	3.58	4.63	5.05	6.83	8.66	9.12	10.31	11.28
N770	2.96	3.71	4.67	5.06	6.64	8.24	8.63	9.64	10.46

The value in Table 2 may be used so as to determine the ranges of pellet sizes in which a certain percentage of carbon black are produced. For example, in grade N 330 the 50% of products have the pellet size range from 4.99 to 8.15, and 80% of products have the pellet size range form 3.64 to 9.55.

The weight distributions are determined at the specified mesh sizes of the sieves so that the proportion of carbon black products falling in the specified sizes may be obtained. Using the estimates $\bar{\alpha}$ and $\bar{\beta}$ given in Table 1 in the equation given in (1), the estimates of weight distribution are summarized in Table 3.

Table 3: The Estimates of Weight Distributions (%)

sizes / grades	120/120	120/60	120/35	120/18	120/14	120/12	120/10
N 330	.17	1.54	8.31	51.92	80.35	93.04	99.16
N 339	.13	1.23	7.04	47.58	77.05	91.28	98.81
N 550	.16	1.35	6.80	41.48	68.45	84.22	96.01
N 660	.25	1.91	8.89	47.63	73.95	87.85	97.23
N 770	.15	1.44	7.89	50.41	79.34	92.52	99.06

The entries of Table 3 may be used so that the percentage of products falling between the mesh sizes. For example, 72.04 (= 80.35 − 8.31) percent of carbon black N 330 falls between the mesh sizes from 120/35 to 120/14.

4. Conclusion

The types of data and problems considered in this paper are likely to be encountered in applied industrial situations. The data is summarized so that it can be utlized in setting up a quality control scheme and assessing the statistical stability of the production process of carbon black. By a well planned and timely sampling methods the weight distribution of pellet sizes may be used in order to detect the deviation from the controlled production standard. This is an inexpensive way of monitoring the production process.

The estimates of the parameters using the replicates of independent samples provide a production standard in terms of the weight distribution of pellet sizes for each grade of carbon black. These standards are the useful information to the users as well as producer of carbon black products. Once the weight distribution of pellet sizes is modeled among other things, the producer can investigate the other factors of input variables that may influence the parameters of the weight distribution.

References

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