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Analysis of the Effect of Strain Hardening on Central Bursting Defects in Strip Drawing

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판재 인발에서 내부결함에 대한 변형경화의 영향에 관한 해석

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Key Words : Central Bursting Defect(내부결함), Strip Drawing(판재일반), Strain Hardening(변형경화), Upper-Bound Theorem(상계정리)

초 록

극치해석인 상계정리를 기초로 하여, 평면변형 인발에서 발생하는 내부결함(central bursting)을 예측하기 위해 중심에서 空洞(voids)을 가진 금속에 대해 비례흐름(proportional flow)과 내부결함의 흐름을 비교하여 해석하였다.

이 결함을 촉진시키는 工程조건에 대한 판정식(criterion)을 변형경화 금속에 대해 유도하였다. 空洞을 가진 금속은 空洞들을 축소시키기 위해 정상적인 재료(sound material)의 흐름과 동일 방법으로 흐를 수 있으며, 경우에 따라서는 내부결함을 확장하기 위해 흐를 수도 있다.

본 연구에서는 다이의 경사각, 단면감소율 및 마찰 등의 어떤 공정변수 영역에서 중심축 상에 많은 空洞을 가진 변형경화 금속에서도 내부결함이 발생할 수 있다는 것이다,

Nomenclature

A, B, C, P : Function of process parameters
m : Shear factor
t_f : Final thickness
t_0 : Original thickness
ε : Crack size
α : Inclined angle of die
β_1, β_2 : Angles representing deformation zone
V_0, V_f : Original and final velocity
V : Velocity
σ_0 : Effective flow stress
σ_{xb}, σ_{xf} : Back and front pull stress
Γ_i : Boundary of velocity discontinuity
X_0 : Distance of apex from exit
J^*, J_s^* : External power of strain hardening material and non-strain hardening material
β : Coefficient of strain hardening

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1. Introduction

The previous studies in reference(1) deal with velocity fields and applied stresses corresponding to drawn sections without defects. Proportional flow of sound metal through inclined planes is shown in Fig. 1. A strip of initial thickness(t_0)

is pulled through the inclined plane. While the metal is passing through the die, the plastic deformations take place and the metal is reduced in thickness, emerging with final thickness(t_f) which is the exit gap of the die.

The main independent variables in the process

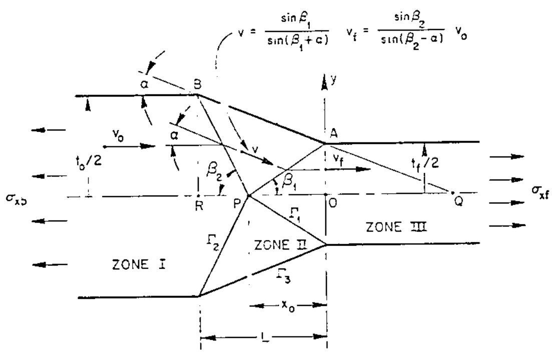
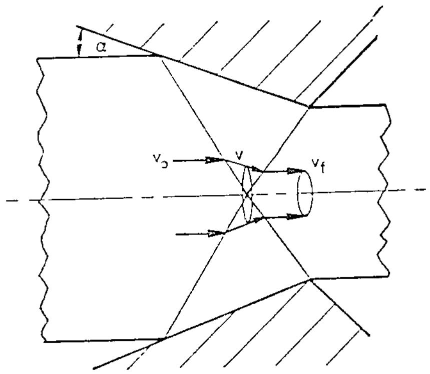
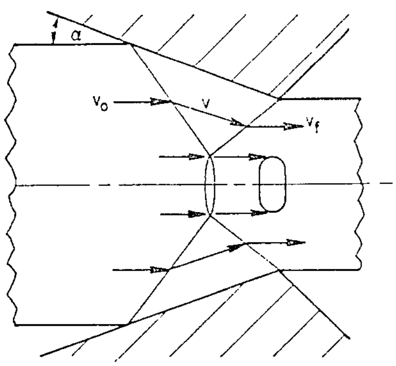


Fig. 1 The strip in plane strain drawing



(a) Void closing by proportional flow



(b) Void enlargement by bursting flow

Fig. 2 Kinematically admissible velocity fields for void enlarging and closing

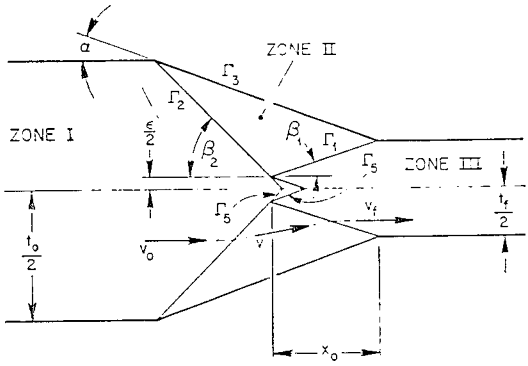


Fig. 3 Central burst flow associated with the triangular velocity field

of drawing are reduction, inclined angle of the die, friction, and coefficient of strain hardening. For severely cold worked metal with voids at the center point P, the proportional flow of Fig. 1 is a kinematically admissible velocity field, and it gives the same applied stresses to drawn sections as those for sound metal[1]. This proportional flow makes the size of voids decrease in height as shown in Fig. 2 (a). However, products become sometimes more defective as shown in Fig. 2(b). The central bursting flow of Fig. 3 is another kinematically admissible one for metal with voids at the center. This defect found in strip drawing, is in essence the two-diemnsional analog of the "cuppy" wire described by B. Avitzur[3], and is reported by Rogers and Coffin[2].

This defect appears only when a unique range of combinations of the process variables develops during processing, and this makes voids burst into axial direction as shown in Fig. 3. Generally, when central bursts occur, the conditions are varied on the basis of the common knowledge that central bursts are restricted to large inclined angles of the die and small reductions[2,3] Furthermore, central bursting can be prevented by a preliminary annealing treatment before the formation of voids. This study provides numerical boundaries for the range of variables which cause central bursts in drawing by comparing the solution of the proportional flow with that of the bursting flow for strain hardening metals with voids.

2. Derivation

2.1. Velocity Discontinuities

In the proposed triangular field(Fig. 1), the surfaces of velocity discontinuity Γ_1 and Γ_2 are plane surfaces meeting at a point with an arbitray distance, X_0, from the incoming material, zone I, moves as a rigid body at velocity $V=V_0$ until it contacts the surface Γ_2. At Γ_2, the material point undergoes a velocity discontinuity and continues to move through Zone II parallel to the inclined surface of the die.

Another velocity discontinuity occure at Γ_1, and the strip exits the die as a rigid body with velocity V_f. Zone III is already deformed strip. The optimal value of X_0 is found by minimizing J^* of the proportional flow by differentiating with respect to X_0, thereby becoming a pseudoindependent parameter. By continuing still further, it is possible to use the triangular field to model the flow of central burst. For the flow of central burst(Fig. 2), one simply assumes that the surface of velocity discontinuity, Γ_1 and Γ_2, intersect at an incremental distance, $\frac{\varepsilon}{2}$, from the center line where ε is the size of the existing void. Zone I is moving more slowly than zone III, so there the zones, at their common interface, Γ_5, must split and dript apart. The computed J^* becomes a function of a pseudoindependent parameter, namely, X_0. For the flow of central bursting, X_0 is fixed at the same position with the optimal value of the proportional flow of rigid plastic metals[1].

2.2. Power Consumed

A summary of the values of the velocities of zones I, II, and III, of the velocity discontinuities across the surfaces and the lengths of the surfaces of discontinuity and power consumed for each surface is provided in Table 1. In Table 1, friction losses are calculated for the friction factor m assumed to prevail along the tool-workpiece interface. The total power J^* consumed per unit width of strip is therefore

$$J^*/\sigma_0 V_f t_f = \frac{2}{\sqrt{3}}\Big[(1+0.5\beta\Delta\bar{\varepsilon}_2)\frac{\sin\alpha}{\sin(\beta_2-\alpha)}\cdot \frac{0.5}{\sin\beta_2}+\{1+\beta(\Delta\bar{\varepsilon}_2+0.5\Delta\bar{\varepsilon}_1)\}\frac{\sin\alpha}{\sin(\beta_1+\alpha)}\cdot \frac{0.5}{\sin\beta_1}+m(1+\beta\Delta\bar{\varepsilon}_2)\frac{\sin\beta_2}{\sin(\beta_2-\alpha)}\cdot\frac{0.5}{\sin\alpha} \Big\{1-(1-\frac{\varepsilon}{t_f})/(\frac{t_0}{t_f}-\frac{\varepsilon}{t_f})\Big\}\Big]\times(1-\frac{\varepsilon}{t_f}) \quad (1)$$

where

$$\beta_1=\tan^{-1}\Big\{(1-\frac{\varepsilon}{t_f})/2(\frac{X_0}{t_f})\Big\} \quad (2)$$

$$\beta_2=\tan^{-1}\Big\{\frac{t}{t_f}-\frac{\varepsilon}{t_f})/2(\frac{L-X_0}{t_f})\Big\} \quad (3)$$

$$L/t_f=0.5(\frac{t_0}{t_f}-1)\cot\alpha \quad (4)$$

$$\Big(\frac{X_0}{t_f}\Big)\text{opt.}=0.5\Big[-\cot\alpha+\sqrt{\frac{t_0}{t_f}\cdot\frac{1}{\sin^2\alpha}\Big\{1+m\frac{(1-t_f/t_0)}{(1+t_f/t_0)}\Big\}}\Big] \quad (5)$$

$$V_f=V_0\frac{(t_0/t_f-\varepsilon/t_f)}{(1-\varepsilon/t_f)} \quad (6)$$

$$\Delta\bar{\varepsilon}^1=\sin\alpha/\sqrt{3}\ \sin\beta_1\cdot\sin(\alpha+\beta_1) \quad (7)$$

$$\Delta\bar{\varepsilon}_2=\sin\alpha/\sqrt{3}\ \sin\beta_2\cdot\sin(\beta_2-\alpha) \quad (8)$$

Because $\Delta\bar{\varepsilon}_1$ and $\Delta\bar{\varepsilon}_2$ represent the total effective strain increment across the velocity discontinuity surfaces, Γ_1 and Γ_2, the average flow stress on the velocity discontinuily surfaces become:

$$\bar{\sigma}_1=\sigma_0\{1+\beta(\Delta\bar{\varepsilon}_2+0.5\Delta\bar{\varepsilon}_1)\} \quad \text{on } \Gamma_1 \quad (9)$$

$$\bar{\sigma}_2=\sigma_0(1+0.5\beta\Delta\bar{\varepsilon}_2) \quad \text{on } \Gamma_2 \quad (10)$$

$$\bar{\sigma}_3=\sigma_0(1+\beta\Delta\bar{\varepsilon}_2) \quad \text{on } \Gamma_3 \quad (11)$$

where β is the linear coefficient of strain hardening.

When $\varepsilon/t_f=0$, Eq. (1) represents the total power of the proportional flow of Fig. 1. For non-strain hardening material, $\beta=0$ and J^* of the proportional flow reduces to

$$J_s^*/\sigma_0 V_f t_f=\frac{1}{\sqrt{3}}\frac{1}{(t_f/2)\cot\alpha+X_0}\left[\left(\frac{2}{t_f}\right)\{X^2+(t_f/2)^2\}+\left(\frac{2}{t_0}\right)\{(L-X_0)^2+(t_0/2)^2\}+m\frac{L}{\cos\alpha}\cdot\frac{1}{\sin\alpha}\right] \quad (12)$$

which is given by reference (1).

From the next equation,

$$\frac{\partial}{\partial X_0}(J_s^*/\sigma_0 V_f t_f)=0 \quad (13)$$

the optimal value of X_0 in Eq. (5) can be derived, and held fixed to the same value for central bursting flow.

Eq. (1) gives us the drawing stress of proportional flow when ε/t_f becomes zero, and otherwise that of central bursting flow. The reasoning behind the technique employed in this study is as follows:

Computing internal power of deformation, shear and friction losses evaluates the power requirements for proportional flow and central bursting flow. If the flow with central bursts, as computed, requires less power than the proportional flow, then under the same conditions, actual flow will occur with central bursts.

For non-strain hardening metals, Eq. (1) can be simplified neglecting higher order terms of ε/t_f as shown in Eq. (14)

$$J^*/\sigma_0 V_f t_f=\frac{2}{\sqrt{3}}\frac{\tan\alpha}{\left(1+\frac{2X_0}{t_f}\tan\alpha\right)}\cdot\left[A+P\frac{\varepsilon}{t_f}\right] \quad (14)$$

where

$$A=2\left(\frac{X_0}{t_f}\right)^2+\frac{1}{2}+2\left(\frac{t_f}{t_0}\right)\left\{\left(\frac{L-X_0}{t_f}\right)^2+\frac{1}{4}\left(\frac{t_0}{t_f}\right)^2\right\}+\frac{m}{\cos\alpha\cdot\sin\alpha\cdot}\cdot\left(\frac{L}{t_f}\right)$$

$$B=2+2\left(\frac{t_f}{t_0}\right)^2\left(1-\frac{t_f}{t_0}\right)\left\{\left(\frac{L-X_0}{t_f}\right)^2+\frac{1}{4}\left(\frac{t_0}{t_f}\right)^2\right\}+\frac{m}{\cos\alpha\cdot\sin\alpha}\cdot\left(\frac{L}{t_f}\right)$$

$$C=1+\left(\frac{2X_0}{t_f}\right)\tan\alpha$$

$$P=A\cdot C-B$$

For drawing, the flow of central burst will occur when

$$P<0 \quad (15)$$

Table 1 List of velocity discontinuities and power

$$V_0=V_f\frac{(1-\varepsilon/t_f)}{(t_0/t_f-\varepsilon/t_f)},\quad V=V_0\frac{\sin\beta_2}{\sin(\beta_2-\alpha)}$$

Surface	Length L_i	Tangential velocity discontinuity $\Delta V_i/V_f$	Power consumed(per unit width)
Γ_1	$\frac{0.5(t_f-\varepsilon)}{\sin\beta_1}$	$\sin\alpha/\sin(\alpha+\beta_1)$	$\frac{1}{\sqrt{3}}\bar{\sigma}_1\Delta V_1 L_1$
Γ_2	$\frac{0.5(t_0-\varepsilon)}{\sin\beta_2}$	$\frac{(1-\varepsilon/t_f)}{(t_0/t_f-\varepsilon/t_f)}\cdot\sin\alpha$ $/\sin(\beta_2-\alpha)$	$\frac{1}{\sqrt{3}}\bar{\sigma}_2\Delta V_2 L_2$
Γ_3	$\frac{0.5(t_0-t_f)}{\sin\alpha}$	$\frac{(1-\varepsilon/t_f)}{(t_0/t_f-\varepsilon/t_f)}\cdot\sin\beta_2$ $/\sin(\beta_2-\alpha)$	$\frac{1}{\sqrt{3}}m\bar{\sigma}_3\cdot\Delta V_3 L_3$

3. Results and Discussions

Proceeding with the analysis, we find

$$J^*/\sigma_0 V_f t_f=f(t_0/t_f, \alpha, m, \beta, X_0, \text{and } \varepsilon/t_f) \quad (16)$$

for both flows.

$J^*/\sigma_0 V_f t_f$ was plotted against ε/t_f, with all other parameters held constant as shown in Fig 4 and Fig. 5. These curvese indicate the slopes at the point $\varepsilon/t_f=0$. For the negative slope, the lowest point of J^* curve exists at positive value of ε/t_f, and thus flow with central burst will occur. A positive slope indicates that the minimum in J^* occurs for $\varepsilon/t_f=0$, and the proportional flow is expected. In the actual study, J^* was plotted with respect to ε/t_f by using "QIK-PLT" program(developed in Lehigh University)

In effect, the conditions leading to horizontal line in Fig. 4 and Fig. 5 sought, giving the conditions separating central burst flow and proportional flow. Resulting criteria are shown

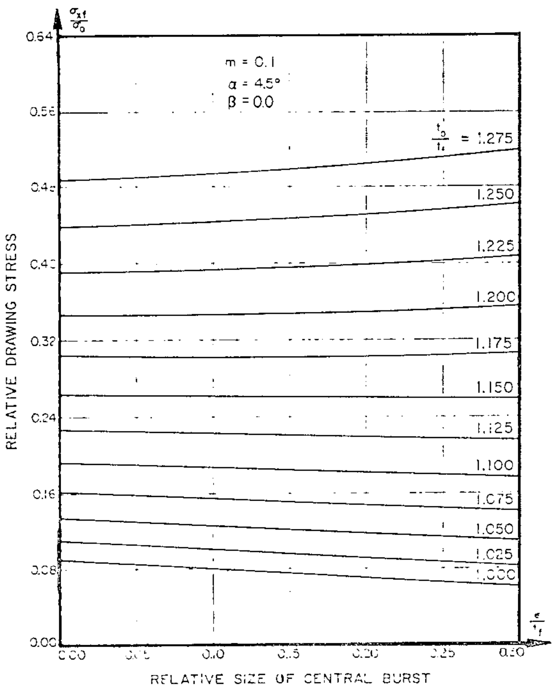


Fig. 4 Effect of internal burst on required energy for initially cracked materials

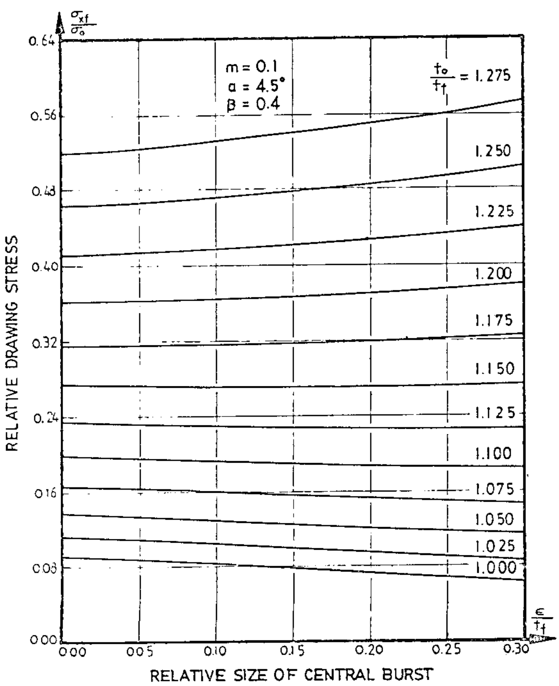


Fig. 5 Effect of internal burst on required energy for strain-hardening materials

in Fig. 6 and Fig. 7. When friction is sufficiently high, no safe zone exists at all. In reality, friction promotes the central burst defect in strip drawing.

The ability of a material to be strain-hardened is measured by β, the slope at any point on the true stress-true strain curve. This parameter was introduced into the analysis and the results are shown in Fig. 5 and Fig. 7. For non-strain hardening material, β is zero. For higher values of the strain hardening coefficient(β), the danger zone becomes smaller. Under the large inclined angle of the die and small reduction, the flow of central burst will occur. These facts agree well with the study of Rogers and Coffin[2] which shows that the larger the inclined angle

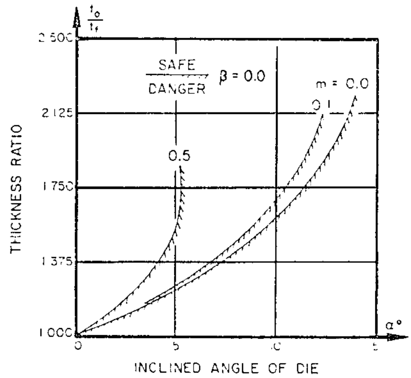


Fig. 6 Criterion for central burst in strip drawing (effect of friction factor)

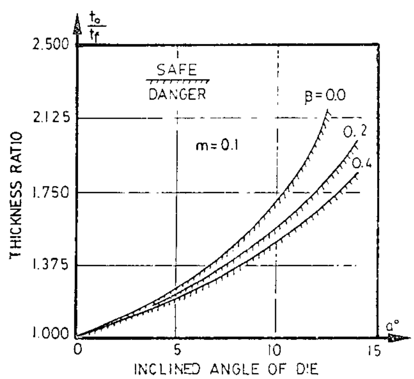


Fig. 7 Criterion for central burst for strain-hardening materials

of the die and the smaller reduction, the larger the inclined angle of the die and the smaller reduction, the higher the hydrostatic tension in the center line.

In the actual drawing of strip, voids would not open throughout the whole width, and the condition presented in Fig. 3 does not happen. Only one part of the width would be open as void.

Thus the flow of central burst is more difficult to occur in plane strain than in axisymmetric wire drawing in actual productions.

4. Conclusion

The major conclusion of this study is that, for a range of combinations of inclined angle of the die, reduction and friction, central bursting defects of plane strain drawing are expected in any strain hardening metal that shows voids at the central axis.

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