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A Design Method for Suboptimal Control of Flexible Structure Vibration

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유연한 구조물의 진동 제어를 위한 부최적제어기의 설계방법에 관하여

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초 록

유연한 구조물의 진동은 이론적으로는 무한개의 진동모우드를 포함하고 있기 때문에 진동을 감소시키기 위한 제어시스템을 설계할 때에는 유한개의 진동모우드로 진동하고 있다는 가정하에서 제어기를 설계하는 것이 보통이다. 그러나 이러한 방법으로 구조물 진동을 제어할 때에는 설계시에 제외된 잔류 진동모우드에 의해서 오히려 구조물의 진동이 더욱 심하게 되어 안정성을 해치는 결과를 낳는 수가 있다. 본 연구에서는 잔류 모우드에 의한 불안정성을 제거하기 위해 최적제어 이론을 바탕으로 제어이론을 개발하였다. 구조물 진동이 안정화되기 위한 필요조건을 제시하였고 외팔보 진동의 예를 시뮬레이션하여 제어이론을 증명하였다.

1. Introduction

The control of a flexible structure requires control of an infinite dimensional system, since the flexible system is essentially a distributed parameter system. To design a realizable control system however, the controller dimension must be substantially smaller, which implies that a finite dimensional system must be considered for the controller design. To avoid this design difficulty many authors(1~10) have used a reduced-order model, truncating some unimportant high frequency modes that do not greatly influence the control system performance. This approach greatly reduces the system complexity but Balas(1) has shown that the residual modes effects could lead to the instability problem in the closed loop system due to the control and observation spillovers: Control spillover is the excitation of uncontrolled mode by the actuator, and observation spillover is the sensing of uncontrolled mode responses by the sensor. Recently Balas(1~5), Skelton and Likins(6), Sesak

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and Coradetti[7] and Sesak and Likins[8] have proposed a variety of methods to eliminate such spilover effects. These works provide a significant contribution to the solution of control and observation spillover problem.

In most of the previous approaches they used a linear dynamic observer to estimate the modal responses which are not available from the sensors. When the output feedback scheme is used for the controller implementation, the same spillover problem can also arise. This problem has not been fully investigated and thus, the objective of this paper is to provide a method to eliminate the spillover effects for the output feedback control of a flexible structure. A general procedure is presented for the design of an optimal output feedback controller that suppresses the controlled modes, while stabilizing the residual modes which are not random variables. As shown, a basic requirement to achieve this is that the measurement sensors must be added, depending upon the number of residual modes to be included for the controller design. This requirement is somewhat similar to Balas' result[3], although the controller form and the design technique used in his paper is different from this approach. Illustrative numerical results are presented for the control of a simply supported beam.

2. Control Problem Description

The theoretical development is concerned with the output feedback control of a class of the flexible system whose dynamics are governed by a generalized wave equation. The control force distribution is provided by P point force actuators, located at some spatial points $z_i^a (i=1, 2, \cdots p)$. Measurements for feedback are taken by point sensors, consiting of M displacement measuring devices and M velocity measuring devices located at various spatial points z_i^s, z_j^s, respectively.

Theoretically, mechanically flexible systems require an infinite number of elastic modes to completely describe their behavior. To control all these modes as desired one must design an infinite dimensional controller. Since it is not practically possible to design such controller, a finite dimensional model must be used by restricting the control system to a few significant modes that are critical to the system performance. If the neglected modes are present by some means, they interact with those controlled modes and thus degrade the system performance. To take into consideration the effects of these residual modes for the controller design, let the system dynamics be represented by state equations of the form:

$$\begin{bmatrix} \dot{\underset{\sim}{x}}_N(t) \\ \dot{\underset{\sim}{x}}_R(t) \end{bmatrix} = \begin{bmatrix} A_N & 0 \\ 0 & A_R \end{bmatrix} \begin{bmatrix} \underset{\sim}{x}_N(t) \\ \underset{\sim}{x}_R(t) \end{bmatrix} + \begin{bmatrix} B_N \\ B_R \end{bmatrix} u(t) \tag{1 a}$$

and the output equations are given by

$$y(t) = [C_N \vdots C_R] \begin{bmatrix} \underset{\sim}{x}_N(t) \\ \underset{\sim}{x}_R(t) \end{bmatrix} \tag{1 b}$$

In the above the controlled state variables x_N consist of the displacement and velocity of N vibration modes while the residual modes variables include those of R vibration modes

The system matrices, A_N and A_R are given by

$$A_N = \begin{bmatrix} 0 & I_N \\ -\Lambda_N & D_N \end{bmatrix}, \quad A_R = \begin{bmatrix} 0 & I_R \\ -\Lambda_R & D_R \end{bmatrix} \tag{2 a}$$

where Λ_N, Λ_R are diagonal matrices whose diagonal entries are the square of the controlled and residual mode frequencies, $\omega_1, \omega_2 \cdots \omega_N$ and $\omega_{N+1}, \omega_{N+2} \cdots \omega_{N+R}$. The matrices D_N, D_R include the system damping which can be neglected without loss of generality. The control force inputs $u(t)$ are of the form:

$$\underset{\sim}{u}(t)=[u_1(t), u_2(t)\cdots u_p(t)]^T \tag{2b}$$

where T denotes the transpose. The associated input matrices B_N and B_R are $2N\times P$ and $2R\times P$ matrices, respectively,

$$B_N=\begin{bmatrix} 0 \\ \hdashline \phi_1(z_1^a), \phi_1(z_2^a)\cdots\phi_1(z_p^a) \\ \phi_N(z_1^a), \phi_N(z_2^a)\cdots\phi_N(z_p^a) \end{bmatrix},$$

$$B_R=\begin{bmatrix} 0 \\ \hdashline \phi_{N+1}(z_1^a), \phi_{N+1}(z_2^a)\cdots\phi_{N+1}(z_p^a) \\ \phi_{N+R}(z_1^a), \phi_{N+R}(z_2^a)\cdots\phi_{N+R}(z_p^a) \end{bmatrix}$$

The output vector y is given by

$$\underset{\sim}{y}(t)=[y_1(t), y_2(t), \cdots y_M(t), y_{M+1}(t), y_{M+2}(t), \cdots y_{2M}(t)]^T \tag{2c}$$

where the first M outputs represent the displacement sensor signals and the other M outputs are the velocity sensor signals. The output matrices C_N and C_R in equation (1 b) are $2M\times 2N$ and $2M\times 2R$ matrices, respectively,

$$C_N=\begin{bmatrix} C_{Nd} & 0 \\ 0 & C_{Nv} \end{bmatrix}, \quad C_R=\begin{bmatrix} C_{Rd} & 0 \\ 0 & C_{Rv} \end{bmatrix} \tag{2d}$$

where C_{Nd} and C_{Rd} are the $M\times N$ matrices associated with the displacement sensor output, C_{Nv} and C_{Rv} are the $M\times R$ matrices associated with the velocity sensor output, and

$$C_{Nd}=\begin{bmatrix} \phi_1(z_1^s), \phi_2(z_1^s)\cdots\phi_N(z_1^s) \\ \vdots \quad\quad \vdots \\ \phi_1(z_M^s), \phi_2(z_M^s)\cdots\phi_N(z_M^s) \end{bmatrix},$$

$$C_{Nv}=\begin{bmatrix} \phi_1(z^s_{M+1}), \phi_2(z^s_{M+1})\cdots\phi_N(z^s_{M+1}) \\ \vdots \quad\quad \vdots \\ \phi_1(z^s_{2M}), \phi_2(z^s_{2M})\cdots\phi_N(z^s_{2M}) \end{bmatrix}$$

$$C_{Rd}=\begin{bmatrix} \phi_{N+1}(z_1^s), \phi_{N+2}(z_1^s)\cdots\phi_{N+R}(z_1^s) \\ \vdots \quad\quad \vdots \\ \phi_{N+1}(z^s_{2M}), \phi_{N+2}(z^s_{2M})\cdots\phi_{N+R}(z^s_{2M}) \end{bmatrix},$$

$$C_{Rv}=\begin{bmatrix} \phi_{N+1}(z^s_{M+1}), \phi_{N+2}(z^s_{M+1})\cdots\phi_{N+R}(z^s_{M+1}) \\ \vdots \quad\quad \vdots \\ \phi_{N+1}(z^s_{2M}), \phi_{N+2}(z^s_{2M})\cdots\phi_{N+R}(z^s_{2M}) \end{bmatrix} \tag{2e}$$

As stated previously, the control objective is to suppress the undesired vibration of the N controlled modes. This problem can be formulated as the well-known optimal regulator problem with quadratic performance index which represents the vibration energy of those modes.

$$J=\frac{1}{2}\int_0^\infty(\underset{\sim}{x}_N^T Q_N \underset{\sim}{x}_N+\underset{\sim}{u}^T R_N \underset{\sim}{u})dt \tag{3}$$

where Q_N and R_N are positive semidefinite and positive definite weighting matrix, respectively. If the system in equation (1 a) and (1 b) satisfies the controllability and observability conditions (Balas[1]), the output feedback control law is given by[11]

$$\underset{\sim}{u}(t)=G\underset{\sim}{y}(t) \tag{4}$$

Since all the state variables, $\underset{\sim}{x}_N(t)$ cannot be accessed directly. The output signal of the individual sensor contains the residual mode signal as well as that of the controlled mode. Then the output equation can be rewritten as

$$\underset{\sim}{y}(t)=G(C_N\underset{\sim}{x}_N+C_R\underset{\sim}{x}_R) \tag{5}$$

where the second term indicates the observation spillover. Substituting equation (4) into equation (1a) yields

$$\begin{bmatrix} \dot{\underset{\sim}{x}}_N \\ \hdashline \dot{\underset{\sim}{x}}_R \end{bmatrix}=\begin{bmatrix} A_N+B_NGC_N & B_NGC_R \\ B_RGC_N & A_R+B_RGC_R \end{bmatrix}\begin{bmatrix} \underset{\sim}{x}_N \\ \hdashline \underset{\sim}{x}_R \end{bmatrix} \tag{6a}$$

or in a simple form

$$\begin{bmatrix} \dot{\underset{\sim}{x}}_N \\ \hdashline \dot{\underset{\sim}{x}}_R \end{bmatrix}=H\begin{bmatrix} \underset{\sim}{x}_N \\ \hdashline \underset{\sim}{x}_R \end{bmatrix} \tag{6b}$$

Examination of the equation (6a) shows that the controlled and residual modes interact and excite each other. This implies that, although the controller is designed to have the controlled state $\underset{\sim}{x}_N(t)$ behave well with the control law obtained by minimizing the performance index (3), the controller have a potential to generate instability of the residual state, $\underset{\sim}{x}_R(t)$. This instability mechanism can seriously degrade the overall control system performance. Thus a modified controller design technique is needed to remove the residual mode effects.

3. Suppression of Residual Mode Effect

In this section a design procedure is developed, which stabilizes the residual modes while keeping the controlled mode performance near optimum. The method is based upon the consideration that the suboptimal feedback gain G determined by minimizing J in equation (3) cannot destabilize the residual modes, only if it satisfies,

$$GC_R=0 \tag{7}$$

This consequence can be seen from examination of the eigenvalues of the system matrix H. The method consists of two basic steps: first, determine the suboptimal gain G^* which minimize J in (3) subject to the constraint in equation (7). Second, take a small variation ΔG around the gain G^* obtained in the step 1, so that the residual modes can have a small stability margin.

Step 1 The equation $GC_R=0$ implies that the residual mode signals can be removed from the controller command signal (4) by adjusting the controller gain. A basic requirement to achieve this is that the number of displacement sensors to be added and the number of the velocity sensors to be added must be equal to or greater than the number of residual modes R, respectively. This condition can be derived by the fact that the rank of C_{Rd} in equation (2e) must be less than the number of element in a row vector of the gain matrix G_d for the solution of $G_dC_{Rd}=0$ to be nontrivial: The gain matrix G is partitioned into two submatrices, G_d and G_v which are associated with C_{Rd} and C_{Rv}, respectively. The same condition applies to the nontrivial solution of $G_vC_{Rv}=0$. If so, using constraint condition (7) the system equation (6a) can be rewritten by

$$\begin{bmatrix} \dot{\underset{\sim}{x}}_N \\ \dot{\underset{\sim}{x}}_R \end{bmatrix} = \begin{bmatrix} A_N+B_NGC_N & 0 \\ B_RGC_N & A_R \end{bmatrix} \begin{bmatrix} \underset{\sim}{x}_N \\ \underset{\sim}{x}_R \end{bmatrix} \tag{8}$$

Then the poles of the partitioned system matrix in equation (8) are those of $A_N+B_NGC_N$ and A_R due to block triangularity. The poles of $A_N+B_NGC_N$ are stable ones, since they are designed to suppress the controlled modes. The poles of A_R are essentially the original ones, which indicates that the residual mode eigenvalues remains unchanged by the design.

The problem is then: determine the suboptimal gain G^* which minimizes the J, while satisfying the constraint equation (7). To do this, J in equation (3) is modified by augmenting the equality constraint $GC_R=0$

$$\hat{J}=\frac{1}{2}\int_0^\infty(\underset{\sim}{x}_N^TQ_N\underset{\sim}{x}_N+\underset{\sim}{u}^TR_N\underset{\sim}{u})dt+\sum_i\sum_j\sigma_{ij}[GC_R]_{ij} \tag{9}$$

where $[GC_R]_{ij}$ is a ij^{th} element of the matrix, and σ_{ij} is a Lagrange multiplier. The necessary condition for G^* to minimize such a functional, $\hat{J}$ is that

$$\left.\frac{d\hat{J}}{dG}\right|_{G^*}=\frac{d}{dG}\left\{\frac{1}{2}\int_0^\infty(\underset{\sim}{x}_N^TQ_N\underset{\sim}{x}_N+\underset{\sim}{u}^TR_N\underset{\sim}{u})dt\right\} + \frac{d}{dG}\{\sum_i\sum_j\sigma_{ij}[GC_R]_{ij}\}=0 \tag{10}$$

The necessary condition for the first term has already been derived by Levine and Athans.[11] With the aids of their results the necessary condition can be summarized as follows:

$$(R_NG^*C_N+B_N^TK)LC_N^T+\sigma C_R^T=0 \tag{11}$$

$$G^*C_R=0 \tag{12}$$

where K and L matrices are the solution of the following two Lyapunov equations,

$$K\hat{A}_N+\hat{A}_N^TK+Q_N+C_N^TG^{*T}R_NG^*C_N=0 \tag{13}$$

$$L\hat{A}_N^T+\hat{A}_NL+I=0 \tag{14}$$

and $\hat{A}_N\equiv A_N+B_NG^*C_N$

These four equations constitute the necessary conditions for the controller to make the residual mode poles remain unchanged as well as to minimize the controlled mode vibrations.

Step 2 In practical implementation, it is more desirable to give a small stability margin to the residual modes than their poles remain in the imaginary axis of s-plane. This is because slightest variation of G may create an instability in the closed-loop system. If a small variation ΔG is taken as $G=G^*+\Delta G$ for a stability margin of the residual mode, then equation (6 a) can be rewritten as,

$$\begin{bmatrix} \dot{\underset{\sim}{x}}_N \\ \cdots \\ \dot{\underset{\sim}{x}}_R \end{bmatrix} = H^* \begin{bmatrix} \underset{\sim}{x}_N \\ \cdots \\ \underset{\sim}{x}_R \end{bmatrix} + \Delta H \begin{bmatrix} \underset{\sim}{x}_N \\ \cdots \\ \underset{\sim}{x}_R \end{bmatrix} \tag{15}$$

where

$$H^* = \left[\begin{array}{c:c} A_N + B_N G^* C_N & 0 \\ \hdashline B_R G^* C_N & A_R \end{array}\right]$$

and

$$\Delta H = \left[\begin{array}{c:c} B_N \Delta G C_N & B_N \Delta G C_R \\ \hdashline B_R \Delta G C_N & B_R \Delta G C_R \end{array}\right]$$

Since the matrix norm $\|\Delta H\|$ is bounded for a small $\|\Delta G\|$, and thus the eigenvalues λ_i depends continuously on the ΔH,[12,13] then $\Delta\lambda_i$ is bounded by

$$\Delta\lambda_i = \frac{v_i^T \Delta H w_i}{v_i^T w_i} \tag{16}$$

where v_i, w_i are the row and column eigenvectors of H^* corresponding to λ_i. Furthermore, from equation (16) and Siriesena and Choi,[14]

$$\frac{\partial\lambda_i}{\partial G} = \frac{\begin{bmatrix} B_N \\ \cdots \\ B_R \end{bmatrix}^T v_i w_i^T [C_N \vdots C_R]^T}{v_i^T w_i} \tag{17}$$

The gradient matrix in equation (17) enables us to relocate some of the undesirable system poles by varying the feedback gain G slightly from the optimally designed G^*. In particular, the residual mode poles can be moved slightly to left half of the s-plane from the imaginary axis. The trade-off is that the controlled mode poles may be shifted from their optimally designed ones to small variation of the feedback gain. As long as the stability margin of the controlled modes is large, as usually the case for an optimal control problem, the pole shifting of the controlled mode due to this design change cannot degrade the control performance.

4. Numerical Example: active control of a simply supported beam

The numerical example considers an active control of a simply supported beam vibration, which is exactly the same example taken by Balas.[1,3] The beam dynamics are modeled by the Euler-Bernoulli equation

$$m\frac{\partial^2 v(z,t)}{\partial t^2} + EI\frac{\partial^4 v(z,t)}{\partial z^4} = f(z,t),$$
$$0 \le z \le 1 \tag{18}$$

where $v(z,t)$ is the transverse displacement of the beam and $f(z,t)$ is the control force input. The boundary conditions for simple support are

$$v(z,t)=0 \qquad \text{at } z=0,1$$
$$\frac{\partial^2 v(z,t)}{\partial z^2}=0 \qquad \text{at } z=0,1 \tag{19}$$

The solution which satisfies equations (18) and (19) is

$$v(z,t) = \sum_{n=1}^{\infty} \sqrt{2}\, q_n(t) \sin n\pi z \tag{20}$$

The beam is controlled by a single force actuator at z^a,

$$f(z,t) = \frac{1}{\sqrt{2}} \delta_f(z-z^a) u(t)$$

and one displacement and one velocity sensors are assumed to be positioned at $z=z_1^s$ and $z=z_2^s$, respectively.

If the controlled modes are the first three modes ($N=3$) and the residual mode is the fourth mode, the state variables can be defined as,

$$\underset{\sim}{x}_N = \{q_1(t), q_2(t), q_3(t)\}^T, \quad x_R = q_4(t)$$

If system matrices defined in equations (1 a) and (1 b) are given by

$$A_N=\left[\begin{array}{ccc|c} & \underset{\sim}{0} & & I \\ \hline -\omega_1^2 & & \underset{\sim}{0} & \\ & -\omega_2^2 & & \underset{\sim}{0} \\ \underset{\sim}{0} & & -\omega_3^2 & \end{array}\right],$$

$$A_R=\begin{bmatrix} 0 & 1 \\ -\omega_4^2 & 0 \end{bmatrix} \qquad (21\text{ a})$$

$$B_N=\sqrt{2}\begin{bmatrix} 0 \\ \sin \pi z^a \\ \sin 2\pi z^a \\ \sin 3\pi z^a \end{bmatrix},$$

$$B_R=\begin{bmatrix} 0 \\ \sin 4\pi z^a \end{bmatrix} \qquad (21\text{ b})$$

$$C_N=\left[\begin{array}{c|c} \sin \pi z_1^s,\ \sin 2\pi z_1^s,\ \sin 3\pi z^s & 0 \\ \hline 0 & \sin \pi z_2^s,\ \sin 2\pi z_2^s,\ \sin 3\pi z_2^s \end{array}\right], \qquad (21\text{ c})$$

$$C_R=\sqrt{2}\begin{bmatrix} \sin 4\pi z_1^s & 0 \\ 0 & \sin 4\pi z_2^s \end{bmatrix}$$

Simulation results

For convenience the beam parameter, m, E and I are set to unity, and then the natural frequencies of the lowest four modes are:

$\omega_1=9.87,\ \omega_2=39.48,\ \omega_3=88.83,\ \omega_4=151.91$

The weighting matrices used for J are given by

$Q_N=DIAG\{1, 1, 1, \omega_1^2, \omega_2^2, \omega_3^2\}$

$R_N=0.1$

The actuator and sensor locations are given by

$z^a=\frac{1}{6};\ z_1^s=\frac{5}{6},\ z_2^s=0.3$

When the residual mode(fourth mode) is not considered for the controller design, the terms associated with the residual mode in equations (21) are all zero, i.e., $A_R=0$, $B_R=0$, $C_R=0$. Using the necessary conditions (11), (13) and (14), the suboptimal gain G for this case, was computed via Boggs' method[15]. The suboptimal gain thus obtained may influence stability of the residual mode, if it is present. This effect is investigated by computing the eigenvalues of the system matrix given in equation (6 a), and they are listed in Table 1. As shown in the table, the fourth mode become unstable, although the controlled mode poles are suboptimally determined and stable.

To remove this instability, the fourth mode is included for the controller design. In this case, one displacement sensor was added at $z_3^s=0.3$ and one velocity sensor at $z_4^s=0.2$ to satisfy the number of sensor condition stated earlier. The schematic of this arrangement is shown in Fig. 1. Following the method stated in "Step 1" in section 3, the suboptimal gain matrix G^* was numerically solved from the

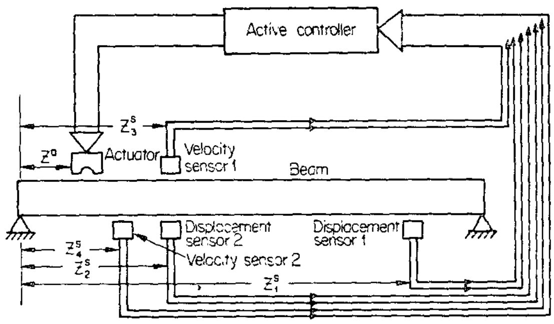

Fig. 1 Active control of a simply-supported beam

equations (11)~(14). The result given in Table 1, indeed, shows that with the addition of the constraint $GC_R=0$ the positive part of the pole of the fourth mode becomes zero. But the poles of the controlled modes are shifted with the negative values of the poles all decreased.

Next, the design procedure followed "Step 2" in section 3. The ultimate design objective is to have the imaginary pole of the fourth mode shifted to the left half plane of the s-plane by adjusting the gain G^* determined in "Step 1". Therefore, it is necessary to examine the gra-

Table 1 Eigenvalues of the controlled and residual modes

	Uncontrolled	Residual modes is not considered:	$GC_R=0$
Controller gain G^T	0 0	−24. 39965 −11. 18565	1. 465165 −3. 259755 −2. 158731 −3. 259755
1 st	$\pm j$ 9. 8696	−1. 623715 $\pm j$ 10. 0334	−0. 8067089 $\pm j$ 9. 8786
2 nd	$\pm j$ 39. 4784	−3. 235834 $\pm j$ 38. 9195	−1. 903202 $\pm j$ 39. 4098
3 rd	$\pm j$ 88. 8264	−1. 208554 $\pm j$ 88. 7351	−1. 445710 $\pm j$ 88. 7075
4 th	$\pm j$ 157. 9137	2. 002140 $\pm j$ 158. 0338	0 $\pm j$ 157. 9137

dient matrix in equation (17). Let the i th eigenvalue λ_i be denoted by

$$\lambda_i=\alpha_i\pm j\beta_i \quad (i=1, 2, 3, 4) \tag{22}$$

Since $G=\{g_1, g_2, g_3, g_4\}$, the gradient matrix $\frac{\partial \alpha_i}{\partial G}$ can be computed from equation (17) and given by

$$\frac{\partial \alpha_i}{\partial g_j}=\left\{\frac{\partial \alpha_i}{\partial g_1}, \frac{\partial \alpha_i}{\partial g_2}, \frac{\partial \alpha_i}{\partial g_3}, \frac{\partial \alpha_i}{\partial g_4}\right\}$$

$$=\begin{bmatrix} -0.06138, & 0.14462, & 0.54018, & 0.10513 \\ 0.90013, & 0.29332, & 0.29124, & 0.29383 \\ -0.54912, & 0.10638, & -0.64527, & 0.33347 \\ -0.28963, & -0.18082, & -0.18614, & 0.17890 \end{bmatrix} \tag{23}$$

This matrix shows sensitivity of the real part of the eigenvalue λ_i to the variation of the feedback gain. As mentioned ealier, during the pole shifting process, it is desirable to make the variation of the controlled mode poles as small as possible, so that the new poles may not be greatly deviated from their optimal values. The great deviation of the new controlled mode poles makes the performance of controller worse. From the gradient matrix (23), it can be seen that variation of the gain g_2 makes the controlled mode poles deviated from their optimal values relatively less than the variations of the other gain elements (g_1, g_3, g_4) do. Therefore, in this simulation, the g_1, g_3 and g_4 are unchanged and only g_2 is varied so that the fourth mode can have a small stability margin δ; this δ value can be set equal to the negative real value of the fourth mode pole α_4. It is noted that, unless the δ value is chosen sufficiently small, the controlled mode poles may be undesirably shifted. Due to this pole shifting, the gain g_2 is changed by

$$g_2=g_2^*+\frac{g_2}{4}(-\delta) \quad \text{for small} \quad \delta>0 \tag{24}$$

Therefore, by specifying the δ value as desired, the new gain, g_2 can be computed from this equation. With this redesigned gain g_2 and the other gains g_1, g_3 and g_4 unchanged, the eigenvalues of the closed loop system were computed again from the system matrix of equation (6 a). In this computation δ was varied frcm 10^{-7} to 10^{-1}. In Table 2, the variation of the gain g_2 and the corresponding pole shifting of the controlled modes can be seen for the various values. Both the gain and the poles are barely changed. These results indicate that with only a small sacrifice of the controlled mcde performance, the residual mcde can be effectively stabilized by this design, otherwise unstable.

To demonstrate the effectiveness of this design further, time responses of the beam vibration are shown in Fig. 2(a) 2(b) and 3. These responses were taken at $z=0.3$ with the following initial conditions:

$$x(0)=\{1.0, 1.0, 1.0, 0.1\}^T, \quad \dot{x}(0)=\{0, 0, 0, 0\}^T$$

Table 2 Pole shiftings of the controlled and residual modes with δ value

	Stability margin, δ			
	0	1×10^{-7}	1×10^{-6}	1×10^{-5}
Controller gain g_2	−3.259755	−3.2597544	−3.2597494	−3.2596997
1 st	$-0.8067089\pm j\,9.8786$	$-0.8067087\pm j\,9.878590$	$-0.8067080\pm j\,9.878590$	$-0.8067008\pm j\,9.8786$
2 nd	$-1.903202\pm j\,39.4098$	$-1.903202\pm j\,39.40976$	$-1.903201\pm j\,39.40976$	$-1.903186\pm j\,39.4098$
3 rd	$-1.445710\pm j\,88.7075$	$-1.445710\pm j\,88.70754$	$-1.445710\pm j\,88.70754$	$-1.445704\pm j\,88.7076$
4 th	$\pm j\,157.9137$	$-0.99129\ 10^{-7}$ $\pm j\,157.9137$	$-0.99144\ 10^{-6}$ $\pm j\,157.9137$	$-0.99145\ 10^{-5}$ $\pm j\,157.9137$

	Stability margin, δ			
	1×10^{-4}	1×10^{-3}	1×10^{-2}	1×10^{-1}
Controller gain g_2	−3.2592019	−3.2542247	−3.2044522	−2.7067274
1 st	$-0.8066289\pm j\,9.8786$	$-0.8059098\pm j\,9.8786$	$-0.7987201\pm j\,9.8788$	$-0.7268988\pm j\,9.8808$
2 nd	$-1.903041\pm j\,39.4098$	$-1.901586\pm j\,39.4096$	$-1.887036\pm j\,39.4117$	$-1.741541\pm j\,39.4281$
3 rd	$-1.445651\pm j\,88.7076$	$-1.445124\pm j\,88.7077$	$-1.439847\pm j\,88.7091$	$-1.386958\pm j\,88.7230$
4 th	$-0.99145\ 10^{-4}$ $\pm j\,157.9137$	$-99.146\ 10^{-3}$ $\pm j\,157.9136$	$-0.99150\ 10^{-2}$ ± 157.9131	$-0.99198\ 10^{-1}$ $\pm j\,157.9078$

*The other gains remain unchanged as in Table 1 : $g_1=1.465165$, $g_3=-2.158731$, $g_4=-3.259755$

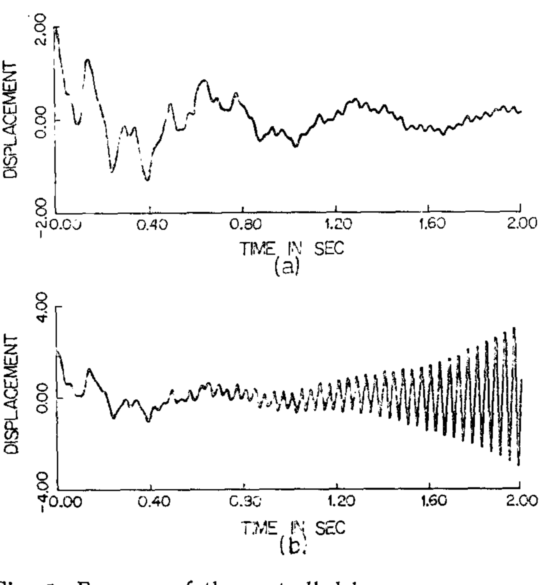

Fig. 2 Response of the controlled beam
(a) with the observation spillover
(b) with the residual mode removed from sensor signals ($GC_R=0$)

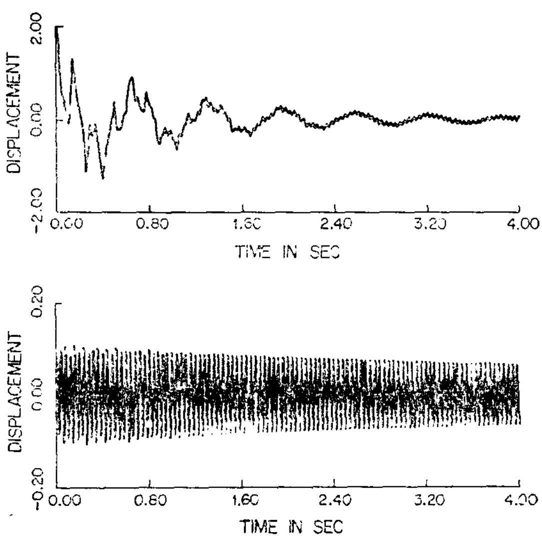

Fig. 3 Response of the controlled beam with the residual mode suppressed : $\delta=1\times10^{-1}$

Fig. 2(a) depicts the case when the residual mode (fourth mode) is not considered for the controller design. It shows an unstable response due to the slowly growing amplitude of the residual mode, although the controlled modes are decayed out. When the observation spilover term is removed, ($GC_R=0$) the response (Fig. 2(b)) does not decay out, oscillating with the residual mode frequency. At steady state only the residual mode oscillation will last, as can be observed from the eigenvalues in Table 1. However, with the slight variation of g_2 from -3.259755 to -2.7067274, this oscillatory response becomes stabilized, as shown in the lower figure of Figure 3. In this figure only the residual (fouth) mode response is redrawn for clarity, since comparision of the two figures (Fig. 2(b)) and (the upper figure of Fig. 3) shows no remarkable difference until about 2 sec. If the response time is taken longer, total controlled mode response of Fig. 3 will eventually die out, since the residual mode will be decayed out with the prescribed decay rate $\delta=1\times10^{-1}$.

5. Conclusions

A general method has been developed for the design of an optimal output feedback controller which suppresses certain selected residual modes as well as the controlled modes. This design method is based upon the pole shiftings of the residual modes otherwise unstable in such a way that those modes can have a small stability margin with only a small sacrifice of the suboptimally-determined controlled mode performance. Basic requirement to achieve this is that the measurement sensors have to be added, depending upon the number of residual modes to be suppressed. An illustrative numerical result has been presented for the control of a simply supported beam. The resuit shows that this controller design method can effectively remove the instability mechanism induced by the residual mcdes, thus achieving the control performance as desired.

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