Transaction of the Korean Society of Mechanical Engineers, Vol. 9, No. 5, pp. 683~690, 1985.

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Inverse Dynamic Analysis of Spatial Mechanical Systems with Euler Parameters†

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(Received July 30, 1985)

Euler 매개변수를 이용한 3차원 기계시스템의 역동력학 해석

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Key Words: Inverse Dynamic(역동력학), Mechanical System(기계시스템), Euler Parameter (Euler 매개변수), Constraint(구속조건), Lagrange Multiplier(Lagrange 승수)

초 록

본 논문에는 Euler 매개변수를 회전좌표계로 사용하여 구속된 3차원 기계시스템의 역동력학 해석을 수행한 연구결과가 수록되었다. 해석을 위해 문제에 등장하는 비선형 Holonomic 구속조건식들과 운동방정식들을 Cartesian 일반좌표계를 사용하여 표시하였으며, 일반좌표계를 구성하는 각 강체의 좌표계로는 변위를 나타내기 위한 3개의 좌표와 회전을 나타내기 위한 4개의 Euler 매개변수가 사용되었다.

구속조건식들과 미분방정식 형태의 운동방정식들을 결합하여 시스템 전체의 운동방정식을 유도하기 위해 Lagrange 승수 기법을 사용하였다. 각 강체의 주어진 시간에서의 위치, 속도, 가속도는 기구학적 해석(kinematic analysis)을 통해 얻어지고, 이 자료들을 전체운동방정식에 대입하여 Lagrange 승수의 값을 계산하며, 이로부터 기계시스템을 구동하기 위한 구동력을 계산하게 된다.

본 논문에 제시된 방법을 사용하여 6개의 자유도를 가진 로봇 기구를 원하는대로 운전하는데 필요한 각 관절의 토오크를 계산하였으며, 계산결과가 정확하다는 사실이 입증되었다. 연구결과 Euler 매개변수를 회전좌표로 사용할 경우 특이 경우(singular case)가 발생하지 않으며, 이 방법은 역동력학 해석용 다목적 전산프로그램 개발에 광범위하게 응용될 수 있음이 밝혀졌다.

1. Introduction

An effective method of formulating and solving differential equations of motion for general mechanical systems subject to holonomic constraints has recently been presented(1~3). Standard and user-supplied constraints are formulated yielding algebraic relations involving generalized coordinates of the bodies connected by each joint. A Lagrange multiplier technique allows coupling of the algebraic constraint equations and differential equations, yielding a large system of differential and algebraic equations wh-

† Presented at the KSME Spring Conference, 1985
Research Supported by Ministry of Science and Technology of Korea

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ich are solved iteratively using implicit numerical integration methods.

In a preliminary formulation and code development of three-dimensional rigid body mechanics, Euler angles were used to define rotational degrees of freedom. However, the use of Euler angles often causes numerical difficulties when one or more of the rigid bodies experiences large rotations. In particular, when the second Euler rotation angle is equal to $\pm k\pi$, $(k=0, 1, 2, \cdots)$, the axes of the first and third rotation angles coincide, so these two angles can not be uniquely determined. Therefore, some constraint equations become dependent at that instant and a unique solution does not exist. A method to circumvent this problem is to monitor the row rank of the constraint Jacobian matrix. The matrix loses rank when the second Euler rotation angles approach $k\pi$. Before this occurs, the computation can be interrupted and the body fixed coordinate systems rotated to new positions. This technique can be performed automatically by the algorithm. However, it is time consuming and in general can not be done easily. Euler parameters, in contrast to Euler angles or any other set of three rotational generalized coordinates have no such singular cases[4], thus they are attractive for formulating system constraints and differential equations of motion. Research result of successful application of Euler parameters to the dynamic analysis of three-dimensional constrained mechanical systems can be found in Ref. (5).

Inverse dynamic analysis technique of mechanical systems has been developed mainly for robot manipulator design. Lagrangian or Newton-Euler formulations are usually employed to calculate required joint forces or torques, to drive mechanical systems as specified[6~8]. In both methods kinematic analysis of the systems must be performed—facing the singularity problems of Euler angles or any other set of three generalized coordinates. Therefore, application of Euler parameters for inverse dynamic analysis of general mechanical systems is desirable.

In this research Euler parameters are employed to define rotational degrees of freedom of rigid bodies. Nonlinear holonomic constraint equations and differential equations of motion are written in terms of the parameters. Lagrange multiplier technique is used to couple the algebraic constraint equations and the differential equations. Kinematic analysis of a mechanical system is performed first, to calculate positions, velocities, and accelerations of the rigid bodies in the system. The data are then substituted to the coupled equation of motion to calculate Lagrange multipliers that are interpreted as required generalized joint forces to drive the system as specified. One exmaple is treated, to illustrate the method and to evaluate the effectiveness of numerical implementation. The example is a six d.o.f(degree of freedom) robot mechanism with a specifed motion. In the procedure the geneiralized coordinate partitioning technique[9] is employed for efficient numerical calculations.

2. Equations of Constraint

In order to specify angular orientation of a rigid body in an inertial(global) xyz coordinate system, it is sufficient to specify the angular orientation of a coordinate system $\xi\eta\zeta$ that is rigidly attached to the body. Let the $\xi_i\eta_i\zeta_i$ coordinate system be attached to body i as shown in Fig. 1, where the origin O_i is located at the center of mass. A point P_i on body i is located in the inertial xyz coordinate system by

$$\boldsymbol{r}_i^p = \boldsymbol{r}_i + \boldsymbol{A}_i \boldsymbol{S}_i'^p \qquad (1)$$

where $\boldsymbol{S}_i'^p = [\xi^p, \eta^p, \zeta^p]_i^T$ locates p_i in the $\xi_i\eta_i\zeta_i$ coordinate system, $\boldsymbol{r}_i = [x, y, z]_i^T$ locates O_i in the xyz coordinate system, and $\boldsymbol{A}_i$ is the rotational transformation matrix of body i. Superscript T denotes vector or matrix transpose. Matrix A_i expressed in terms of Euler parameters e_0, e_1, e_2 and e_3 is

$$A_i = 2\begin{bmatrix} e_0^2+e_1^2-1/2 & e_1e_2-e_0e_3 & e_1e_3+e_0e_2 \\ e_1e_2+e_0e_3 & e_0^2+e_2^2-1/2 & e_2e_3-e_0e_1 \\ e_1e_3-e_0e_2 & e_2e_3+e_0e_1 & e_0^2+e_3^2-1/2 \end{bmatrix}_i \quad (2)$$

where subscript i indicates transformation matrix for body i[4]. The four Euler parameters are required to satisfy the equation

$$e_0^2+\boldsymbol{e}^T\boldsymbol{e}=e_0^2+e_1^2+e_2^2+e_3^2=1 \quad (3)$$

Standard holonomic constraints between rigid bodies are taken as friction free(workless) joints Formulations for spherical, revolute, translational, universal, and cylindrical holonomic constraints, using the same notation and coordinate presentation, can be found in reference[5]. As

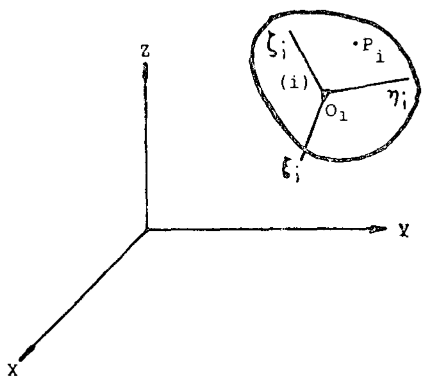

Fig. 1 Body fixed $\xi_i\eta_i\zeta_i$ and global xyz coordinate system

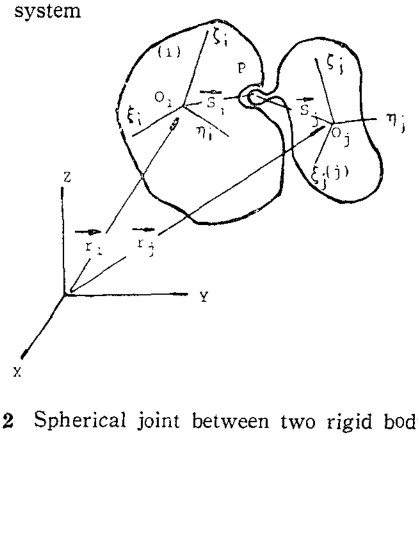

Fig. 2 Spherical joint between two rigid bodies

an example, the spherical joint formulation is described here. Fig. 2 shows two adjacent bodies, numbered i and j, connected by a spherical joint (ball joint). A vector loop equation can be written as

$$\boldsymbol{r}_i+\boldsymbol{S}_i-\boldsymbol{S}_j-\boldsymbol{r}_j=0 \quad (4)$$

The scalar equations for this joint, determined by the use of Eq. (1), are

$$\boldsymbol{r}_i+\boldsymbol{A}_i\boldsymbol{S}_i'^p-\boldsymbol{r}_j-\boldsymbol{A}_j\boldsymbol{S}_j'^p=0 \quad (5)$$

3. Equations of Motion

Denote the vector of generalized coordinates of body i by $\boldsymbol{q}_i$, $\boldsymbol{q}_i=[x, y, z, e_0, e_1, e_2, e_3]_i^T$, and the composite vector of all system generalized coordinate by $\boldsymbol{q}=[\boldsymbol{q}_1^T, \boldsymbol{q}_2^T, \cdots\cdots, \boldsymbol{q}_n^T]^T$, where n is the number of rigid bodies. Holonomic constraint equations of the kind introduced in Section 2 (including Eq. (3)) are

$$\Phi(\boldsymbol{q}, t)=0 \quad (6)$$

where $\Phi(\boldsymbol{q}, t)=[\phi_1(\boldsymbol{q}, t),\ \phi_2(\boldsymbol{q}, t), \cdots\cdots, \phi_m(\boldsymbol{q}, t)]^T$ are assumed to be independent; *i.e.*, the Jacobian matrix for Eq. (6), defined as

$$\Phi_q=\frac{\partial\Phi}{\partial q}\equiv\left[\frac{\partial\Phi_i}{\partial q_j}\right],\quad i=1,\cdots,m,\quad j=1,\cdots,7n \quad (7)$$

has full row rank. It is presumed that $\Phi(\boldsymbol{q}, t)$ are twice continuously differentiable (in fact, they are normally analytic).

Differentiation of Eq. (6) gives the velocity relation

$$\Phi_q\dot{\boldsymbol{q}}+\Phi_t=0 \quad (8)$$

where $\dot{\boldsymbol{q}}$ is the vector of generalized velocities.

Before preceding to the equations of motion, Eq. (8) may be differentiated with respect to time to obtain

$$\Phi_q\ddot{\boldsymbol{q}}+(\Phi_q\dot{\boldsymbol{q}})_q\dot{\boldsymbol{q}}+2\Phi_{tq}\dot{\boldsymbol{q}}+\Phi_{tt}=0 \quad (9)$$

Then, the generalized acceleration equations may be written in the form

$$J\ddot{q}=\Phi_q\ddot{\boldsymbol{q}}=-(\Phi_q\dot{\boldsymbol{q}})_q\dot{\boldsymbol{q}}-2\Phi_{tq}\dot{\boldsymbol{q}}-\Phi_{tt} \quad (10)$$

Note that the coefficient of $\ddot{q}$ in Eq. (10) is just the system Jacobian $\boldsymbol{J}$, and the right side is a function of only $\boldsymbol{q}$ and $\dot{\boldsymbol{q}}$.

Virtual displacement δq that are consistent with constraints must satisfy differential form of Eq. (8), with time suppressed; i.e.,

$$J\delta q=\Phi_q\delta q=0 \tag{11}$$

In order to determine the equations of motion, let $\boldsymbol{w}_i'=[w_\xi, w_\eta, w_\zeta]_i^T$ be the projection of the angular velocity vector for body i on the local coordinates axes, $\boldsymbol{r}_i=[x, y, z]_i^T$ the global location of the center of mass, m_i the mass, and $I_{\xi\xi}^i, I_{\eta\eta}^i, I_{\xi\zeta}^i, I_{\xi\eta}^i, I_{\eta\zeta}^i$, and $I_{\zeta\xi}^i$ the moments and products of inertia about the $\xi_i\eta_i\zeta_i$ axes respectively. The kinetic energy of the ith body can thus be written as

$$T_i=\frac{1}{2}\dot{\boldsymbol{r}}_i^TN_i\dot{\boldsymbol{r}}_i+\frac{1}{2}\boldsymbol{w}_i'^TI_i\boldsymbol{w}_i' \tag{12}$$

where

$$N_i=\begin{pmatrix} m & 0 & 0\\ 0 & m & 0\\ 0 & 0 & m\end{pmatrix}_i,\quad I_i=\begin{pmatrix} I_{\xi\xi} & I_{\xi\eta} & I_{\xi\zeta}\\ I_{\eta\xi} & I_{\eta\eta} & I_{\eta\zeta}\\ I_{\zeta\xi} & I_{\zeta\eta} & I_{\zeta\zeta}\end{pmatrix}_i \tag{13}$$

Angular velocity $\boldsymbol{w}_i'$ can be expressed in terms of Euler paramters as[4]

$$\boldsymbol{w}_i'=2\boldsymbol{B}_i\dot{\boldsymbol{p}}_i \tag{14}$$

where $\boldsymbol{p}_i\equiv[e_0, e_1, e_2, e_3]_i^T$, $\dot{\boldsymbol{p}}_i\equiv[\dot{e}_0, \dot{e}_1, \dot{e}_2, \dot{e}_3]_i^T$ and

$$\boldsymbol{B}_i=\begin{pmatrix} -e_1 & e_0 & e_3 & -e_2\\ -e_2 & -e_3 & e_0 & e_1\\ -e_3 & e_2 & -e_1 & e_0\end{pmatrix}_i \tag{15}$$

Using the Lagrange multiplier formulation of Lagrange's equations of motion, with kinematically admissible virtual displacements of Eq. (11), one can prove existence of multiplier $\boldsymbol{\lambda}(t)\in R^m$, with which the equations of motion for the ith body are written as[5].

$$\frac{d}{dt}(T_{\dot{r}_i})^T+\Phi_{r_i}^T\boldsymbol{\lambda}-\boldsymbol{f}_i=0,\quad \text{(3 equations)} \tag{16}$$

$$\frac{d}{dt}(T_{\dot{p}_i})^T-T_{p_i}^T+\Phi_{p_i}^T\boldsymbol{\lambda}-\boldsymbol{h}_i=0,\quad \text{(4 equations)} \tag{17}$$

where $\boldsymbol{f}_i$ and $\boldsymbol{h}_i$ are the vectors of generalized forces and torques corresponding to generalized coordinates $\boldsymbol{r}_i$ and $\boldsymbol{p}_i$, respectively. Substitution of Eq. (12) into Eq. (16) and (17) yields

$$N_i\ddot{\boldsymbol{r}}_i+\Phi_{r_i}^T\boldsymbol{\lambda}=\boldsymbol{f}_i \tag{18}$$

$$4\boldsymbol{B}_i^TI_i\boldsymbol{B}_i\ddot{\boldsymbol{p}}_i+\Phi_{p_i}^T\lambda=\boldsymbol{h}_i+8\dot{\boldsymbol{B}}_i^TI_i\dot{\boldsymbol{B}}_i\boldsymbol{p}_i \tag{19}$$

Defining $\boldsymbol{g}_i=[\boldsymbol{f}^T, (\boldsymbol{h}+8\dot{\boldsymbol{B}}^T\boldsymbol{I}\dot{\boldsymbol{B}}\boldsymbol{p})^T]_i$ and

$$\boldsymbol{M}_i=\begin{pmatrix} \begin{pmatrix} m & 0 & 0\\ 0 & m & 0\\ 0 & 0 & m\end{pmatrix}_i & 0\\ 0 & [4\boldsymbol{B}^T\boldsymbol{I}\boldsymbol{B}]_i\end{pmatrix}\ (7\times7 \text{ matrix}) \tag{20}$$

Eqs. (18) and (19) can be written as

$$\boldsymbol{M}_i\ddot{\boldsymbol{q}}_i=\boldsymbol{g}_i-\Phi_{q_i}^T\boldsymbol{\lambda} \tag{21}$$

where $\boldsymbol{q}_i=[\boldsymbol{r}^T, \boldsymbol{p}^T]_i^T\equiv[x, y, z, e_0, e_1, e_2, e_3]_i^T$. The total system of equations of motion for n rigid bodies is then

$$\boldsymbol{M}\ddot{\boldsymbol{q}}=\boldsymbol{g}-\Phi_q^T\boldsymbol{\lambda} \tag{22}$$

where $\boldsymbol{M}\equiv\text{diag.}[\boldsymbol{M}_1, \boldsymbol{M}_2, \cdots, \boldsymbol{M}_n]$, $\boldsymbol{g}\equiv[\boldsymbol{g}_1^T, \boldsymbol{g}_2^T, \cdots, \boldsymbol{g}_n^T]^T$. When an external force $\boldsymbol{f}_i$ acts at a point p on body i, the force components for body i can be calculated from

$$\begin{pmatrix}\boldsymbol{f}\\ \boldsymbol{h}'\end{pmatrix}_i=\begin{pmatrix}\boldsymbol{f}\\ \tilde{\boldsymbol{S}}'^pA^T\boldsymbol{f}\end{pmatrix}_i^* \tag{23}$$

where $\boldsymbol{h}_i'$ is the vector of components of torque about ξ_i, η_i, ζ_i axes, $\boldsymbol{S}'^p$ is the vector of coordinates of p in the $\xi_i\eta_i\zeta_i$ coordinate system. Transformation of the components of the torque $\boldsymbol{h}_i'=[h_\xi, h_\eta, h_\zeta]_i^T$ to $\boldsymbol{h}_i=[h_{e_0}, h_{e_1}, h_{e_2}, h_{e_3}]_i^T$ can be obtained by multiplying both sides of Eq. (14) $\boldsymbol{h}_i'^T$ to get

$$\boldsymbol{h}_i'^T\boldsymbol{w}_i'=2\boldsymbol{h}_i'^T\boldsymbol{B}_i\dot{\boldsymbol{p}}_i \tag{24}$$

The instantaneous power, $\boldsymbol{h}_i'^T\boldsymbol{w}_i'$ is independent of coordinate system representation, thus $\boldsymbol{h}_i'^T\boldsymbol{w}_i'=\boldsymbol{h}_i^T\dot{\boldsymbol{p}}_i$ and Eq. (24) becomes

$$\boldsymbol{h}_i=2\boldsymbol{B}_i^T\boldsymbol{h}_i' \tag{25}$$

Therefore, the generalized force components for body i by the external force $\boldsymbol{f}_i$ are written as

$$\begin{pmatrix}\boldsymbol{f}\\ \boldsymbol{h}\end{pmatrix}_i=\begin{pmatrix}\boldsymbol{f}_i\\ 2\boldsymbol{B}_i^T\boldsymbol{h}_i'\end{pmatrix} \tag{26}$$

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* $\tilde{\boldsymbol{S}}'^p$; the skew symmetric matrix of $\boldsymbol{S}'^p$

4. Inverse Dynamic Analysis

Inverse dynamic analysis may be defined as force analysis of kinematically driven systems. In a kinematically driven system, all of the d.o.f. are specified, i.e., $\boldsymbol{J}$ is non-singular. Therefore, positions, velocities, and accelerations can be computed uniquely. Since the kinematic data are available, the equations of motion of Eq. (22) may be solved to determine Lagrange multiplier vector $\boldsymbol{\lambda}$, to obtain required generalized actuator forces or torques.

For inverse dynamic analysis purpose, one can partition Eq. (6) as

$$\Phi=\begin{pmatrix}\Phi^K\\ \Phi^A\end{pmatrix} \tag{27}$$

where Φ^K and Φ^A denote vectors of the kinematic constraint equation and the actuator constraint equation, respectively. Likewise, Eq. (22) can be rearranged and partitioned as

$$[\Phi_q^{KT},\ \Phi_q^{AT}]\begin{pmatrix}\boldsymbol{\lambda}^K\\ \boldsymbol{\lambda}^A\end{pmatrix}=\boldsymbol{g}-\boldsymbol{M}\ddot{\boldsymbol{q}} \tag{28}$$

where $\boldsymbol{\lambda}^K$ and $\boldsymbol{\lambda}^A$ are vectors of Lagrange multipliers associated with Φ^K and Φ^A, respectively. Once the Lagrange multiplier vectors λ^K and λ^A are calculated from Eq. (28) one can obtain the generalized actuator forces by

$$\boldsymbol{g}=-\Phi_q^{AT}\boldsymbol{\lambda}^A \tag{29}$$

where $\boldsymbol{g}=\sum_{i=1}^{l}\boldsymbol{g}_i$, $\lambda^A=[\lambda_1, \lambda_2, \cdots, \lambda_l]^T$, $\Phi^A=[\Phi_1{}^A, \Phi_2{}^A, \cdots, \Phi_l{}^A]^T$, $g_i=-\Phi_{iq}{}^{AT}\boldsymbol{\lambda}_i$, and l is the number of actuators in the system. Consider the kth actuator connecting rigid bodies i and j. Let $\boldsymbol{g}_{ik}$ and $\boldsymbol{g}_{jk}$ be the generalized forces that must be applied to the bodies i and j, respectively, by the actuator. Transforming the generalized forces into the forces in the global coordinate system and the torques in the local coordinate system, one obtains

$$\begin{pmatrix}\boldsymbol{f}_i\\ \boldsymbol{h}_i'\end{pmatrix}=\begin{pmatrix}\boldsymbol{g}_{ik}{}^{(r)}\\ \frac{1}{2}\boldsymbol{B}_i\boldsymbol{g}_{ik}{}^{(p)}\end{pmatrix} \tag{30}$$

where $\boldsymbol{g}_{ik}=[\boldsymbol{g}_{ik}{}^{(r)}, \boldsymbol{g}_{ik}{}^{(p)}]^T$, $\boldsymbol{f}_i=[f_x, f_y, f_z]_i{}^T$ and $\boldsymbol{h}_i'$ is the vector of torque components about $\xi_i\eta_i\zeta_i$ axes. In Eq. (30), superscriptsr and p denote that the variables are related to coordinates $\boldsymbol{r}$ and $\boldsymbol{p}$ respectively. Using the actuator force vectors $\boldsymbol{f}_i$ and $\boldsymbol{h}_i'$ in Eq. (30), one can calculate the actuator control forces corresponding to actuator types, i.e., torsional and translational actuators.

4.1. Torsional Actuator

Torsional atuator elements may be defined between adjacent bodies i and j that are connected by a revolute joint, as shown in Fig. 3. Two vectors $\boldsymbol{S}_i$ and $\boldsymbol{S}_j$, embedded in bodies i and j respectively, define a plane perpendicular to the revolute joint axes. In addition, the two vectors define the torsional spring, damper attachment points on the two bodies. The angle between $\boldsymbol{S}_i$ and $\boldsymbol{S}_j$ is denoted by θ and is initially assumed to be $0\leqq\theta\leqq\pi$. The angle θ can be calculated from the equation

$$\theta=\cos^{-1}\frac{\boldsymbol{S}_i{}^T\boldsymbol{S}_j}{|\boldsymbol{S}_i|\cdot|\boldsymbol{S}_j|},\quad 0\leqq\theta\leqq\pi \tag{31}$$

To determine all possible values of θ, a point k is initially defined on the revolute joint axis such that the direction of vector $\boldsymbol{S}$ is determined by the right hand screw law, rotating from $\boldsymbol{S}_i$ to $\boldsymbol{S}_j$ and sweeping angle θ(initially $0\leqq\theta\leqq\pi$). During the inverse dynamic analysis, the cross product of $\boldsymbol{S}_i$ and $\boldsymbol{S}_j$ yields a vector parallel to $\boldsymbol{S}$, having the same direction if $0\leqq\theta\leqq\pi$, and opposite direction if $\pi\leqq\theta\leqq2\pi$, i.e.,

$$\boldsymbol{S}^T\tilde{\boldsymbol{S}}_i\boldsymbol{S}_j\begin{cases}\geqq0\ ;\ 0\leqq\theta\leqq\pi\\ \leqq0\ ;\ \pi\leqq\theta\leqq2\pi\end{cases} \tag{32}$$

The constraint equation for a torsional actuator is given as

$$\phi=\boldsymbol{S}_i{}^T\boldsymbol{S}_j/|\boldsymbol{S}_i|\cdot|\boldsymbol{S}_j|-\cos\theta(t)=0 \tag{33}$$

Therefore, the driving torque for the kth torsional actuator connecting the two bodies i and j is expressed as

$$T_k = \boldsymbol{u}_j'^T \boldsymbol{h}_i' + k_t(\theta - \theta_0) + C_t \dot{\theta} \qquad (34)$$

where $\boldsymbol{u}_j'$ is the components of vector $\vec{u}$ in $\xi_j \eta_j \zeta_j$ coordinate system, $\boldsymbol{h}_j'$ is the torque components derived in Eq. (31) k_t and C_t are torsional spring constant and torsional damping coefficient, respectively.

4.2. Translational Actuator

A translational actuator between bodies i and j is shown in Fig. 4. The constraint equation corresponding to the physical meaning of the translational actuator is given as

$$\phi = \boldsymbol{l}^T \boldsymbol{l} - l^2(t) = 0 \qquad (35)$$

where $\boldsymbol{l} = \boldsymbol{r}_j^p - \boldsymbol{r}_i^p$ and $l(t) = (\boldsymbol{l}^T \boldsymbol{l})^{1/2}$

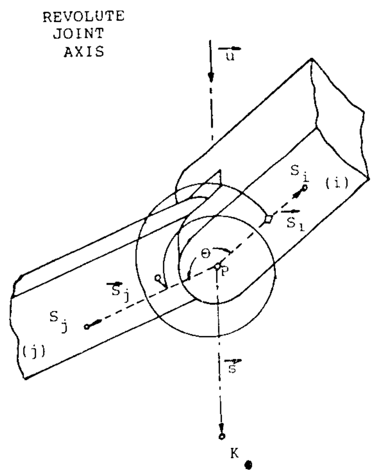

Fig. 3 Torsional spring-damper-actuator element

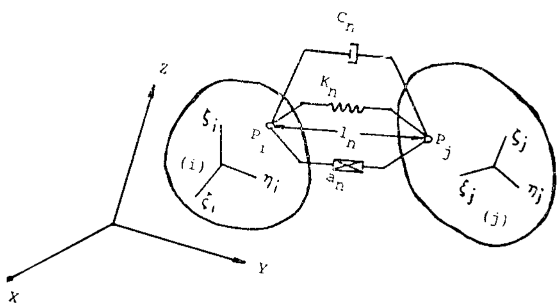

Fig. 4 Translational spring-damper-actuator element

Therefore, the driving force for the kth translational actuator connecting the two bodies i and j is expressed as

$$f = \begin{cases} (\boldsymbol{f}_j^T \boldsymbol{f}_j)^{1/2} + C\dot{l} + k(l - l_0), & \boldsymbol{f}_j^T \boldsymbol{l} \geqq 0 \\ -(\boldsymbol{f}_j^T \boldsymbol{f}_j)^{1/2} + C\dot{l} + k(l - l_0), & \boldsymbol{f}_j^T \boldsymbol{l} \leqq 0 \end{cases} \qquad (36)$$

where l_0, C and k are the undeformed length of the spring, damping coefficient, and spring constant.

5. Inverse Dynamic Analysis Algorithm

Inverse dynamic analysis algorithm for general three-dimensional mechanical systems using Euler parameters can now be stated in the following steps.

Step 1. Construct the mathematical model of the practical system, using the joints, rigid bodies, and actuators.

Step 2. Write the user supplied subroutines for the driving constraints corresponding to the desired driving plane.

Step 3. Set $t = t_0$ and solve Eqs. (6), (8) and (9) for the position, velocity, and acceleration analysis, respectively.

Step 4. Substitute the kinematic data obtained at Step 3 to Eq. (28), to calculate Lagrange multiplier vector $\boldsymbol{\lambda} = [\boldsymbol{\lambda}^{kT}, \boldsymbol{\lambda}^{AT}]^T$.

Step 5. Calculate the generalized force vector $\boldsymbol{g}$ using Eq. (29) and transform the generalized forces into the forces in the global coordinate system and the torques in the local coordinate system using Eq. (30).

Step 6. Calculate the actuator forces or torques corresponding to the actuator types.

Step 7. If t exceeds the final simulation time, terminate. Otherwise return to step 3 with $t = t + \Delta t$ where Δt is the predetermined step size.

6. Numerical Example

A six d.o.f. industrial robot, of which configuration is illustrated in Fig. 5, is taken as an example. The length, mass, and moment of inertia data of links of the robot are summarized in Table 1. The hand of the robot is required to start moving from the initial point and stop at the final point. Positions of the points and hand orientations at the points are given in Table 2. It is assumed that the hand moves on a straight line that connects the two points as a uniform velocity of 1.27m/sec except the acceleration and deceleration periods 0.25 second

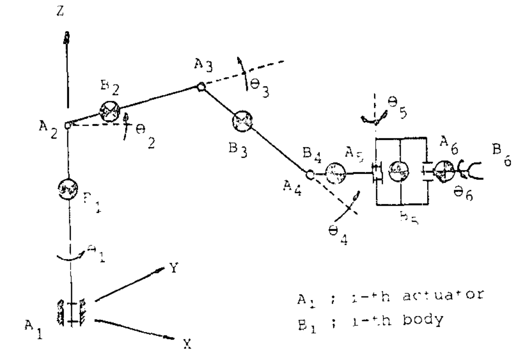

Fig. 5 Kinematic representation of an industrial robot

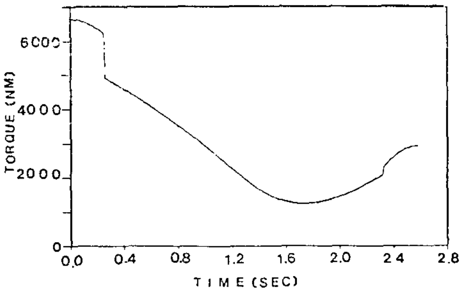

Fig. 6 Torque of actuator 2

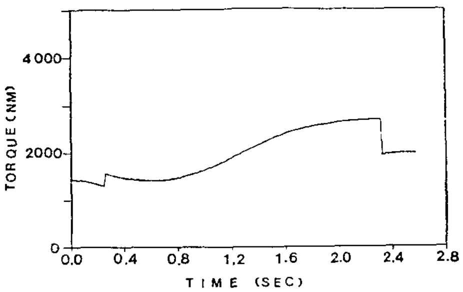

Fig. 7 Torque of actuator 3

Table 1 Inertia propreties of links (kg, m², kg, m)

Rigid body#	I_{xx}	I_{yy}	I_{zz}	Mass	Length
1	—	—	62.00	680.0	1.50
2	11.00	53.00	44.00	360.0	1.02
3	1.10	44.00	44.00	180.0	1.02
4	0.44	0.82	0.91	55.0	0.20
5	0.47	0.18	0.38	36.0	0.15
6	0.64	0.44	0.73	68.0	0.18

Table 2 Two end points and hand orientations (m, Deg.)

	Position			Hand Orientation (Euler Angles)		
	x	y	z	PSI	THE-TA	PHI
Initial	2.03	0.0	1.02	90.0	90.0	0.0
Final	0.0	1.52	2.54	180.0	90.0	90.0

each, near the points. It is also assumed that orientation change of the hand is proportional to its position change.

The required torques of the six torsional actuators, to drive the robot as specified, are calculated using the technique explained above. Among them, torque curves of two actuators, actuators 2 and 3 in Fig. 5, are shown in Fig. 6 and 7. Motion of the robot is reanalyzed regarding the calculated torques as external torques acting on the system, to check accuracy of the inverse dynamic analysis results. The DADS-3D Code[5] is used for the reanalysis and the maximum position error of the hand at the final point is 0.15mm. Considering that the hand moves about 2.96 meters and spline functions are used to approximate the torques for reanalysis, one can conclude that the error is negligible and that validity of the technique presented in this paper is fully demonstrated.

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