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Decentralized Stabilization of a Class of Uncertain Large Scale Continuous-Time Systems

(시스템 파라미터가 不確實한 大規模 線型連 續時間 시스템의 非集中 安定化)

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要 約

본 논문에서는 시스템 파라미터가 불확실한 일련의 연속시간 대규모 선형 시스템을 안정화하는 문제가 다루어졌다. 제안된 비집중 적응 제어기는 새로운 적응궤환 제어와 기존의 선형궤환 제어를 결합한 형태로서, 전체 폐루프 시스템의 안정을 보장하기 위한 충분조건이 유도되었다. 또, 제안된 방식의 유용성을 보이기 위하여 컴퓨터 모사를 통한 수치예가 제시 되었다.

Abstract

This paper considers the problem of stabilizing a class of continuous-time large scale linear systems when the system parameters are uncertain. The proposed local adaptive controls are a combination of a new adaptive feedback control and the conventional linear feedback control. A condition of stability is derived, under which the overall closed-loop system is assured to be globally stable. Also, a numerical example is illustrated via computer simulation.

Nomenclature

$|r|$: Absolute value of a real number r

A^T : Transpose of a matrix A

A^{-1} : Inverse of a square matrix A

$\|a\|$: Euclidean norm of a finite dimensional vector a

I_n : n-dimensional identity matrix

R^n : n-dimensional vector space

$\lambda_m(A)$: Minimum eigenvalue of a square matrix A

$\lambda_M(A)$: Maximum eigenvalue of a square matrix A

$\|A\|_s$: Spectral norm of a matrix A defined as

$$\|A\|_s=\sqrt{\lambda_M(A^TA)}$$

$\|A\|_E$: Euclidean norm of a matrix A defined as

$$\|A\|_E=\sqrt{\sum_i \sum_j a_{ij}^2}$$

where a_{ij} is the (i, j)-th element of A

diag $(r_1, \ldots, r_N)$: N-dimensional diagonal matrix which has real numbers r_i, i=1, ..., N on the diagonal and zeros elsewhere

block diag $(A \;, \ldots, A_N)$: Block diagonal matrix which has matrices A_i, i=1, ..., N on the diagonal and zeros elsewhere

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I. Introduction

In the study of large scale systems, the problem of stability has received a great deal of attention. A lot of work on this problem has been carried out under the basic assumption that the dynamics of each subsystem as well as their interconnections are fairly well known [1] frequently, the possibility of stabilizing such an interconnected system by using local feedback has been investigated. However, in practice, there are many situations in that such an exact model-based stabilizing control is not feasible either due to difficulties in estimating some or all of the system parameters or due to inaccurate modelling of the complex dynamic system. Power system, [2] process control system, [3] and robotic manipulation system [4] are but a few examples for which any exact model-based control scheme would often fail to exist.

As an approach to treat the system parameter uncertainties in large systems, Hmamed and Radouane [5] recently considered the stabilization problem of a class of interconnected continuous systems and proposed a new type of local adaptive controllers. Also, an alternative simple approach based on Lyapunov's Direct Method was developed for the same problem in. [6] However, these results were restricted to the class in which each subsystem has a single-input and moreover is assumed to be given in a controllable form.

In this paper, we extend the results of [6] to a wider class of systems in which each subsystem may have multi-input and have some relaxed assumptions on the system structure. A decentralized adaptive scheme is devised by combining an adaptive feedback control based on Lyapunov design [8] for compensating some effects by unknown system parameters and the exact model-based linear feedback control for overriding the unfavorable effects by interconnections.

II. Problem Statement

Consider the large-scale interconnected linear system described by

$$\mathring{x}_i = A_i x_i + B_i u_i + \sum_{j=1, j\neq 1}^{N} A_{ij} x_j, \; i = 1,2, \ldots, N \quad (1)$$

where $x_i \in R^{ni}$ is the state of the i-th subsystem, $u_i \in R^{mi}$ is its control input, and A_i, B_i and A_{ij} are constant matrices of appropriate dimensions. It is assumed that :

(A-1). Each of the decoupled subsystems is completely controllable and of known dimensions (n_i, m_i).

(A-2). The state x_i is available for measurements only at the i-th subsystem.

(A-3). The elements of A_i and B_i are unknown, while an upper bound on elements of A_{ij} is known such that

$$|a_{ij}^{pq}| \leqslant \mu_{ij}, \quad \begin{matrix} p = 1, \ldots, n_i \\ q = 1, \ldots, n_j \end{matrix} \quad (2)$$

where a_{ij}^{pq} is the (p, q) - th element of A_{ij}, and μ_{ij} is a known constant.

For convenience, an additional assumption will be made later.

Now, the problem is to determine a local control for each subsystem (decentralized control) which stabilizes the overall interconnected system (1). For this, we first present a method to design a local adaptive feedback control, and then show that the resultant closed-loop system is assured to be globally stable.

Remark 1.

The assumption (A-3) is not unrealistic in the sense that in many situations, a designer usually has information on the bounds on the interconnection elements which correspond to gains determining the magnitude of information flow among the various subunits, while the dynamics of these processes themselves may not be certain.

III. Design of Local Adaptive Controllers

The system (1) can be rewritten as

$$\mathring{x}_i = A_i^{\circ} x_i + B_i^{\circ} u_i + (A_i - A_i^{\circ}) x_i + (B_i - B_i^{\circ}) u_i + \sum_{j\neq 1}^{N} A_{ij} x_j, \; i=1,2, \ldots, N \quad (3)$$

where (A_i^o, B_i^o) is a predetermined controllable pair.

In order to stabilize the system (3), we propose the following local adaptive controllers:

$$u_i = -(I_{m_i} + G_i(t))^{-1}(K_i + F_i(t))\, x_i, \quad i = 1, 2, \ldots, N \tag{4}$$

where

$$K_i = B_i^{oT} P_i. \tag{5}$$

The symmetric positive definite matrix P_i is the solution of the following algebraic riccati equation:

$$(A_i^o + \alpha_i I_{n_i})^T P_i + P_i(A_i^o + \alpha_i I_{n_i}) - P_i B_i^o B_i^{oT} P_i + Q_i = 0 \tag{6}$$

for a symmetric positive definite Q_i and non-negative constant α_i. The closed-loop system is then

$$\dot{x}_i = (A_i^o - B_i^o K_i)\, x_i + (A_i - A_i^o - B_i^o F_i(t))\, x_i + (B_i - B_i^o - B_i^o G_i(t))\, u_i + \sum_{j \neq i}^{N} A_{ij} x_j, \quad i = 1, 2, \ldots, N. \tag{7}$$

It is noted that $(A_i^o - B_i^o K_i)$ is stable matrix with degree of stability α_i.

It is further assumed here that

(A-4). there exist the matrices F_i^* and G_i^* satisfying the following relations:

$$\left.\begin{aligned} A_i - A_i^o &= B_i^o F_i^* \\ B_i - B_i^o &= B_i^o G_i^* \end{aligned}\right\} \tag{8}$$

These conditions actually imply that the column vectors of the matrices $(A_i - A_i^o)$ and $(B_i - B_i^o)$ should be linearly dependent on the column vectors of the matrix B_i^o [7]. The typical cases where these conditions are satisfied are that:

(i) The number of state variables is not greater than that of input.

(ii) The state equation is written in partitioned phase variable canonical form [8].

It is remarked that the matching conditions (8) are necessary for deriving the generating schemes of $F_i(t)$ and $G_i(t)$.

The following parameter adaptation laws are proposed based on the lyapunov design [7,8]:

$$\begin{aligned} \dot{F}_i &= K_i x_i x_i^T \Gamma_i^1 \\ \dot{G}_i &= K_i x_i u_i^T \Gamma_i^2 \end{aligned} \tag{9}$$

where Γ_i^1 and Γ_i^2, which are called adaptation gain matrices, are chosen to be symmetric positive definite. It is noted that the present parameter adaptation mechanism is simpler than that of. [7]

IV. A Condition of Stability

Substituting (8) into (7), we obtain

$$\begin{aligned} \dot{x}_i &= (A_i^o - B_i^o K_i)\, x_i + B_i^o (F_i^* - F(t))\, x_i + B_i^o (G_i^* - G_i(t))\, u_i + \sum_{j \neq i}^{N} A_{ij} x_j, \quad i = 1, 2, \ldots, N \\ &= (A_i^o - B_i^o K_i) x_i + B_i^o \theta_i(t) \psi_i + \sum_{j \neq i}^{N} A_{ij} x_j \\ & i = 1, 2, \ldots, N \end{aligned} \tag{10}$$

where

$$\begin{aligned} \theta_i(t) &= [\, F_i^* - F(t),\ G_i^* - G_i(t)\,] \\ \psi_i^T &= [x_i^T,\ u_i^T\,]. \end{aligned} \tag{11}$$

Also, using the definitions (11), the parameter adaptation laws (9) are simplified as

$$\dot{\theta}_i = K_i x_i \Psi_i^T \Gamma_i, \quad i = 1, 2, \ldots, N \tag{12}$$

where

$$\Gamma_i = \text{block diag}\ \{\Gamma_i^1,\ \Gamma_i^2\}.$$

The stability of the overall adaptive system given by eqns. (10) and (12) is then established through the following main result. For use in the following Theorem 1, let us first introduce the relations as:

(i) [9]

$$\lambda_m(S_i)\,\|x_i\|^2 \leqslant x_i^T S_i x_i \leqslant \lambda_M(S_i)\,\|x_i\|^2 \quad (13)$$

for a positive definite matrix S_i

(ii) [5];

$$\|A_{ij}\|_S \leqslant \|A_{ij}\|_E \leqslant \sqrt{n_i n_j\ \mu_{ij}} \quad (14)$$

where μ_{ij} is given in (2).

Also, let us define the terms

$$\|A_{ij}\|_E^M = \sqrt{n_i n_j\ \mu_{ij}} \quad (15)$$

$$D_i = P_i B_i^\circ B_i^{\circ T} P_i + Q_i \quad (16)$$

Theorem 1 (A sufficient condition of stability)

Under the assumptions (A-1) ~ (A-4) in section II and (A-4) in section III, the equilibrium state of the closed-loop system given by (10) and (12) is stable, and for each i = 1,2, ..., N, $x_i(t) \to 0$, to $t \to \infty$, if α_i (for each i) can be chosen such that the matrix $L = [\ell_{ij}]$, of dimension N,

$$\ell_{ij} = \begin{cases} -\lambda_m\,(D_i + 2\,\alpha_i P_i)/\lambda_M(P_i), & i=j \\ 2\|A_{ij}\|_E^M, & i\neq j \end{cases} \quad (17)$$

is negative definite, where D_i and $\|A_{ij}\|_E^M$ are defined in (15) and (16), respectively.

(Proof) Let us choose a lyapunov function candidate as follows:

$$V(x,\theta) = \sum_{i=1}^{N} V_i(x_i, \theta_i) = \sum_{i=1}^{N} \{x_i^T P_i x_i + \mathrm{tr}\,(\theta_i \Gamma_i^{-1} \theta_i^T)\} \quad (18)$$

where

$$x^T = [x_1^T, \ldots, x_N^T]\ ;\ \theta^T = [\theta_1^T, \ldots, \theta_N^T].$$

Then, the time derivative of V (x,θ) and eqn. (10) lead to

$$\mathring{V}(x,\theta) = \sum_{i=1}^{N} [\, x_i^T \{(A_i^\circ - B_i^\circ K_i)^T P_i + P_i\,(A_i^\circ - B_i^\circ K_i)\}\, x_i + 2\,x_i^T P_i \sum_{j\neq i}^{N} A_{ij}\,x_j\,]. \quad (19)$$

It is noted here that the parameter adaptation law (12) is derived during the calculation of $\mathring{V}(x,\theta)$, and $\mathring{V}(x,\theta)$ is not a function of θ any more. Using the relation (6) and the definition (16), eqn. (19) is simplified as

$$\mathring{V}(x,\theta) = \sum_{i=1}^{N} [\, -x_i^T (D_i + 2\,\alpha_i P_i)\, x_i + 2\,x_i^T P_i \sum_{j\neq i}^{N} A_{ij}\,x_j\,] \quad (20)$$

Applying the relations (13) and (14) to the first term and the second term of the right hand side of (20), respectively, we obtain

$$\begin{aligned} \mathring{V}(x) &\leqslant \sum_{i=1}^{N} \{-\lambda_m(D_i + 2\,\alpha_i P_i)\,\|x_i\|^2 + 2\,\|P_i\|_s\|x_i\| \sum_{j\neq i}^{N} \|A_{ij}\|_s\,\|x_j\|\} \\ &\leqslant \sum_{j=1}^{N} \{-\lambda_m(D_i + 2\alpha_i P_i)\,\|x_i\|^2 + 2\,\lambda_M(P_i)\,\|x_i\| \sum_{j\neq i}^{N} \|A_{ij}\|_E^M\,\|x_j\|\} \\ &= \sum_{i=1}^{N}\sum_{j=1}^{N} \lambda_M(P_i)\,\ell_{ij}\,\|x_i\|\,\|x_j\| \\ &= \bar{x}^T\,\mathrm{diag}\,[\lambda_M(P_1), \ldots, \lambda_M(P_N)]\,L\,\bar{x} \end{aligned} \quad (21)$$

where

$$\bar{x}^T = [\,\|x_1\|,\ \|x_2\|, \ldots, \|x_N\|\,].$$

It is noted that since P_i is positive definite, $\|P_i\|_s$ is equal to $\lambda_M(P_i)$.

Now, increasing α_i, if we can choose α_i

so as to guarantee the negative definiteness of L, then

$$\mathring{V}(x) \leqslant 0 \tag{22}$$

This means that the equilibrium state is stable in the lyapunov sense, i.e., $x_i(t)$, $F_i(t)$ and $G_i(t)$ for i=1, ..., N are bounded if $x_i(0)$, $Fi(0)$ and $G_i(0)$ for i=1, ..., N are bounded. furthermore, from (21) and (22), we obtain [10]

$$\mathring{V}(x) \leqslant - c \, \|\bar{x}\|^2 \tag{23}$$

for some positive constant c. Since the right hand side of (23) is monotonic decreasing function, it follows that $\mathring{V}(x) < 0$, unless $\|\bar{x}\| = 0$. Hence

$$\|x_i\| \to 0, \text{ i.e., } x_i \to 0, \text{ as } t \to \infty$$

for all i = 1,2, ..., N.

Q.E.D

Remark 2

From the parameter adaptation laws (9) and the fact that $x_i \to 0$, as $t \to \infty$, it follows that

$$\mathring{F}_i \text{ and } \mathring{G}_i \to 0, \text{ as } t \to \infty.$$

But, this does not necessarily imply that

$$F_i(t) \to F_i^* \text{ and } G_i(t) \to G_i^*, \text{ as } t \to \infty$$

V. An Example

Consider the unstable linear constant interconnected system with unknown system parameters described by

$$\mathring{x}_1 = \begin{bmatrix} -0.5 & 1.5 \\ 1 & 0 \end{bmatrix} x_1 + \begin{bmatrix} 0.8 \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 0.2 & 0.4 \\ 0 & 0 \end{bmatrix}$$

$$x_2, x_1(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\mathring{x}_2 = \begin{bmatrix} -0.5 & 1 \\ 0 & 1 \end{bmatrix} x_2 + \begin{bmatrix} 1 & 0.8 \\ 0 & 2 \end{bmatrix} u_2 + \begin{bmatrix} 0.4 & 0.1 \\ 0 & 0.2 \end{bmatrix}$$

$$x_1, x_2(0) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Here, upper bounds for the elements of interconnections are assumed to be known as

$$\mu_{12} = \mu_{21} = 0.4.$$

As a preliminary step, we first specify the exact model for each subsystem and choose the design parameters Q_i and α_i such that

$$A_1^\circ = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \quad A_2^\circ = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$B_1^\circ = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad B_2^\circ = I_2$$

$$Q_1 = I_2, \quad Q_2 = 3\, I_2$$

$$\alpha_1 = 0, \quad \alpha_2 = 2.$$

Then, the solution Pi of (6) are given by

$$P_1 = \begin{bmatrix} 1.7 & 1 \\ 1 & 1.7 \end{bmatrix} \quad P_2 = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

Hence

$$\lambda_M(P_1) = 2.7, \quad \lambda_M(P_2) = 3$$

$$\lambda_m(D_1 + 2\alpha_1 P_1) = 1, \quad \lambda_m(D_2 + 2\alpha_2 P_2) = 24.$$

Also, the off-diagonal terms of the L matrix in (17) are given by

$$\ell_{12} = \ell_{21} = 1.$$

Based on the above data, it can be easily shown that the condition of stability in Theorem 1 is satisfied.

Now, using the proposed adaptive scheme, computer simulations are carried out to stabi-

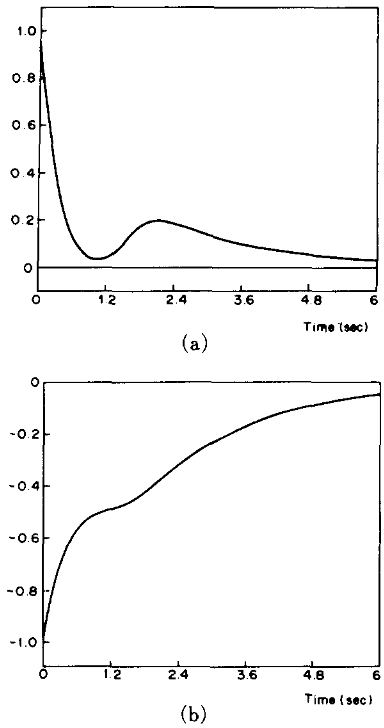

Fig. 1. Trajectory of $y_1(st)$.

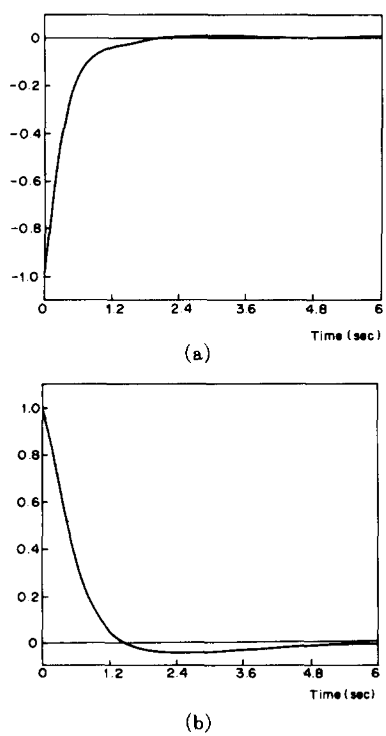

Fig. 2. Trajectory of $y_1 y_2(st)$.

lize the example system. The results with initial estimates

$$F_i(o)=0,\ G_i(o)=0, \text{ for } i = 1, 2$$

and with adaptation gains

$$\Gamma_1^1 = I_2,\ \Gamma_1^2 = 1,\ \Gamma_2^1 = I_2,\ \Gamma_2^2 = 0.2\, I_2$$

are presented in Figs. 1-2. It is noted that the notation s_i^j in the figures denotes the j-th component of the vector s_i.

As can be seen in the figures, the simulation results coincide with the expected ones given in section IV.

VI. Concluding Remarks

It has been shown that a class of uncertain continuous-time large scale interconnected linear systems could be stabilized using a decentralized adaptive scheme which combined an adaptive nonlinear feedback control and the exact model-based linear feedback control.

A further study of immediate interest is to develop some stabilization method which do not require the assumption of the existence of the matching conditions.

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