

## 音響傳達系の推定임펄스 應答의 精度評價에 關한 研究

### A Study of Method for Evaluating Accuracy of the Estimated Impulse Response in an Acoustics Transfer System

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#### 요 약

본 논문에서는 시스템의 전달계에 있어서 추정임펄스응답신호의 精度評價法으로서 새로운 방법을 제안한다. 이 방법의 효과에 대해서는 종래부터 이용되어 온 방법들과의 논의와 컴퓨터 시뮬레이션실험 결과를 비교하여 기술하고 있다.

推定임펄스응답의 精度는 다음의 절차에 의해 評價하고 있다. 전달계의 입력신호는 컴퓨터가 발생하는 화이트노이즈(White noise)로서 구동시키고, 그 전달계의 응답신호는 전달계를 통한 입력신호를 측정한다. 이때 전달계에서 추정응답신호를 얻기 위하여 推定임펄스응답신호와 구동입력신호인 화이트노이즈를 컨볼루션(Convolution) 계산을 행한다. 추정응답신호인 계산한 응답신호와 측정된 응답신호는 각 샘플링 시간에 있어서 差의신호의 파워의 합을 측정된 응답신호의 샘플값을 제곱한 신호의 합에 대해 비를 계산한다. 이 비율이 음향전달계의 推定임펄스응답신호의 精度評價法에 사용된다.

#### ABSTRACT

This paper proposes a new method for evaluating the accuracy of estimated impulse response of a system and its effectiveness which is shown by discussion and the computer simulation comparing it with the conventional methods.

The accuracy of the estimated impulse response is evaluated by the following procedure. The system is driven by computer generated white noise and the response of system to it is observed. Then the convolution between the estimated impulse response and the driving white noise is computed to obtain the estimated response of the system. The difference between the computed response and the measured response is squared at every sampling time. The ratio between the sum of the squared difference and the sum of the squared sample values of the measured response is computed. This ratio is used for the evaluation.

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### I. INTRODUCTION

It is usually difficult to find the impulse response of the complex acoustic transfer system that has the resonant mode over several hundreds or thousands of points and keeps up a fairly long time to the transient vibration of each mode within a bandlimited frequency for the structure forming an acoustic system such as a space indoors and a duct etc. In spite of its difficulties, an investigation to find this with high accuracy for this system have been done in the numerous studies such as [1], [2], [3], etc.

Several conventional methods for the accuracy evaluation have been proposed and used for comparing and investigating the estimated impulse response. But, it is hard to believe that these conventional methods can yield the satisfying evaluation with quantity because of the problems within a phase and a normalized amplitude.

This paper describes the new method for the accuracy evaluation of the estimated impulse response in the acoustic transfer system. It evaluates the accuracy with power of different signal that is subtracted the computed response from the observed response. The computed response is obtained by a convolution computing between the estimated impulse response and a white noise which is a input signal. The observed response is obtained by through the system which is formed by the same input signal.

When the sound is radiated by a loudspeaker in a room or by a vibrating a pannel, the value of accuracy evaluation shows the same effect that the inverse signal component by the loudspeaker or the vibration pannel is added to the output of a receiver microphone set up in the same room.

In this paper, according to conventional evaluation methods, their characteristics and

problems are suggested by the simulation experiment. Also, the new method proposes to examine the evaluation method of estimated impulse response experimentally. Therefore, the validity of this proposed method will be described in the experiment.

### II. THE CONVENTIONAL EVALUATION METHOD

#### A. Evaluation by the Coherence Function

The coherence function is a coherent relation showing how much the each frequency component of an input signal  $x(t)$  is included within that of an output signal  $y(t)$ , when the transfer function in an acoustic transfer system is estimated by the cross spectrum method, which is carried out by DFT (Discrete Fourier Transform).

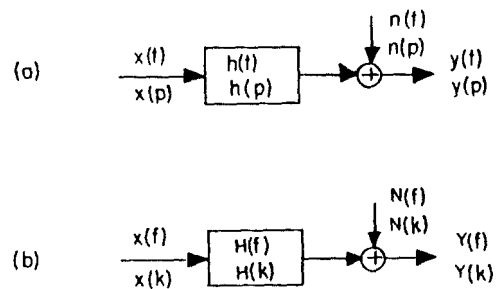


Fig. 1. The relation of input and output with transfer system.

- $x(t), x(p)$  : input signal (time function, sampling value)
- $h(t), h(p)$  : impulse response (time function, sampling value)
- $n(t), n(p)$  : external noise (time function, sampling value)
- $y(t), y(p)$  : output signal (time function, sampling value)
- $X(f), N(f)$  : Fourier series of  $x(t), n(t)$
- $X(k), N(k)$  : DFT of  $x(p), n(p)$

Block diagram in Fig. 1 shows the relation of the input  $x(t)$  and the output signal  $y(t)$ . They are signals of the stochastic process which

is a stationary strongly. The power spectrum for the input and output signal indicates  $W_{xx}(f)$  and  $W_{yy}(f)$ , respectively. The cross spectrum for them indicates  $W_{xy}(f)$ . Therefore the coherence function  $r^2_{xy}(f)$  between them indicates as follows.

$$r^2_{xy}(f) = \frac{|W_{xy}(f)|^2}{|W_{xx}(f) \cdot W_{yy}(f)|} \quad (1)$$

The coherence function  $r^2_{xy}(f)$  of equation (1) is normalized by dividing the square of the absolute value of the cross spectrum by product of each power spectrum. So that, the range of the value can be expressed in equation (2).

$$0 \leq r^2_{xy}(f) \leq 1 \quad (2)$$

If  $x(t)$  and  $y(t)$  are statistically independent and  $r^2_{xy}(f)$  equals one for the observed frequency, then it can be stated that  $x(t)$  and  $y(t)$  are in the complete correlation relation [4]. The estimated coherence function by the digital signal processing denotes as  $\hat{r}^2(k)$ . This is computed by  $X(k)$  and  $Y(k)$  which have done by DFT with the input and output sequence, that is, showing in the following equation (3).

$$\hat{r}^2(k) = \frac{|\sum_{i=1}^N X_i^*(k) Y_i(k)|^2}{\sum_{i=1}^N |X_i(k)|^2 \sum_{i=1}^N |Y_i(k)|^2} \quad (3)$$

Where the  $K$  is discrete frequency,  $*$  is the conjugate complex number and  $N$  is the number of segments of the input and output sequence. The segments have been taken by the length of time window which is a weight function and the  $X_i(k)$  and  $Y_i(k)$  designate for the  $i$ -th segment when the input  $x(p)$  and the output  $y(p)$  are carried out by DFT.

After the products between the conjugate complex number for the input and output

sequence which are carried out by DFT, those power and cross spectrum are obtained by summing their elements. These data are weighted by the suitable time window which is taken by moving the initial point on the time axis. For these reasons, the accuracy of the coherence function depends on a length and a shape of the time window, and the carried DFT number  $N$ . The estimated accuracy by  $r^2(k)$  shows coherent relation between each frequency of  $x(p)$  and  $y(p)$  sequence. If the value of  $\hat{r}(k)$  may yield 1 approximately in the bandlimit frequency, the estimated impulse which is founded by the inverse Fourier Transform of the transfer function can be obtained accurately except for the computing error when they are carried out by IDFT.

There is another method which takes the averaged value of the coherence function of time axis such as the following equation (4) [5].

$$r^2 = \frac{\sum_{k=1}^N (\sum_{j=1}^N |X_i(k) Y_i(k)|^2)}{\sum_{k=1}^N (\sum_{j=1}^N |X_i(k)|^2 \sum_{j=1}^N |Y_i(k)|^2)} \quad (4)$$

Where  $r^2$  specified the coherence coefficient in order to distinguish from the coherence function  $\hat{r}^2(k)$

In the following, two simulation experiments will be described for the accuracy of the estimated impulse response by  $\hat{r}^2(k)$  and  $r^2$ .

A length and a shape of time window, and carried DFT number  $N$  are taken as the parameters of the experiments. True impulse response is assumed in these experiment. The assumed impulse response is made from false random number which are multiplied by the weighting of the exponential function from the initial point to the final point having the 60 dB decay between them. Its length is 256 points sequence. The input signal which is used in

the experiment is false random numbers showing the Gaussian distribution which is generated by the electronic computer. The response signal is made up by the convolution between the input signal of system and the true impulse response that assumed.

The desired impulse response is obtained by the cross spectrum method with an input and output sequence. These impulse response are estimated by parameters which are the lengths and the shapes of time window when the DFT performance is carried out. Its accuracy is evaluated by the coherence function  $\hat{r}^2(k)$  and the coherence coefficient  $r^2$ .

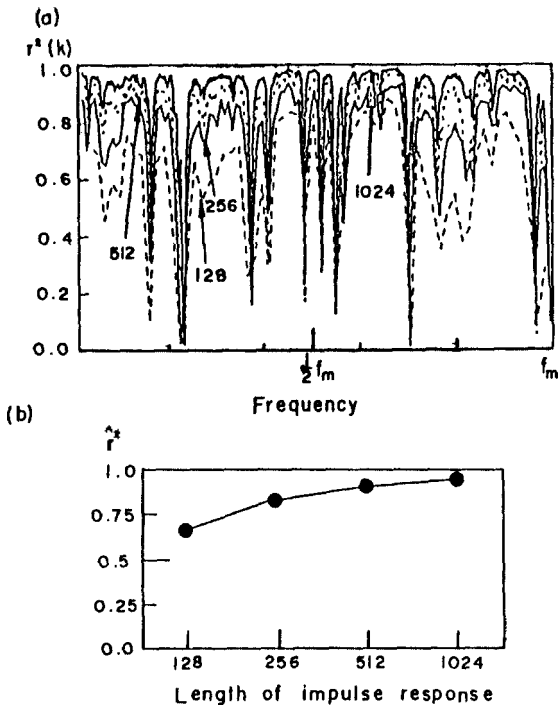


Fig. 2. The accuracy evaluation of estimated impulse response by coherence function (a) and coherence coefficient (b). (the shape of time window: rectangular, the length of time window: 128, 256, 512 and 1024 points)

Fig. 2(a) and (b) show the coherence function  $\hat{r}^2(k)$  and the coherence coefficient  $r^2$  respectively for various data lengths which is

time window when the DFT is performed. The lengths of the input and output data take  $\frac{1}{2}$ , 1, 2 and 4 times to those of the true impulse response. In this case, its shape window will be the rectangular.

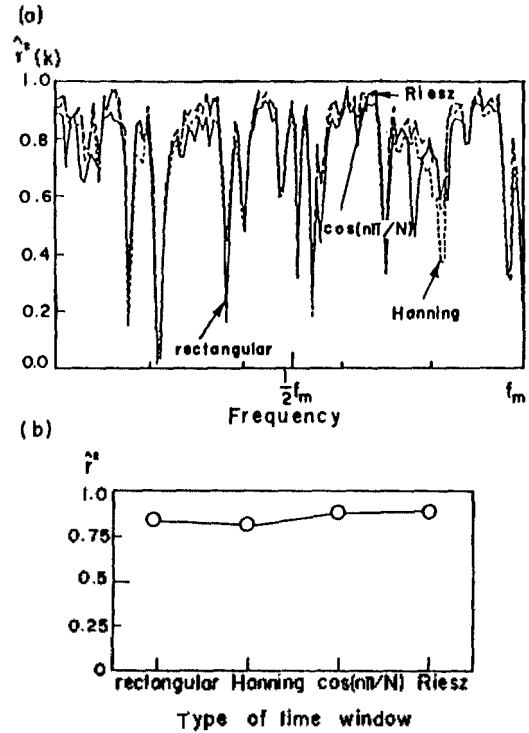


Fig. 3. The accuracy evaluation of estimated impulse response by coherence function (a) and coherence coefficient (b). (the length of time window: 256 points, the shapes of time window: rectangular,  $\cos(2\pi n/N)$ , Hanning and Riesz window)

Fig. 3(a), and (b) show the coherence function  $\hat{r}^2(k)$  and the coherence coefficient  $r^2$  respectively for the various shapes of the time window when the DFT is performed. The shapes of the time window are the rectangular, Hanning,  $\cos(2\pi n/N)$  and Riesz window [6]. In this case, the length of data will be the same as that of true impulse response.

### B. Evaluation by the Reverberation Curve

The reverberation curve suggested by

Schroder etc [6], [7], [8] can be obtained by integrating the squared wave form from the final point to the initial point inversely according to time axis. The final point is spot where the characteristics of decay of the estimated impulse response in the acoustic transfer system decays almost enough. The slope of this curve moves comparatively slow so that the characteristics of reverberation can be recognized easily. Therefore, this is used widely.

It can be shown analytically that the ensemble average  $\langle S^2(t) \rangle$  of the squared decaying sound pressure at a receiving point in a room excited by filtered white noise is equal to a certain integral over the squared impulse response  $h^2(t)$  for the room. Here the response includes effects associated with the transducers and filters for shaping the spectrum of the noise.

Mathematically the relation can be written as follows.

$$\langle S^2(t) \rangle = N \int_t^{\infty} h^2(\tau) d\tau \quad (5)$$

Here, the range of integral of impulse response  $h(\tau)$  between the sound source and received sound point is from a certain time( $t$ ) to an infinite time (usually three second or so).  $N$  is proportional to the power-spectral density of the noise in the frequency range measured.

When the value of the squared integral is taken as a logarithm, the ideal reverberation curve will be a straight line. The closer to a straight line the logarithm curve is, the higher the accuracy of estimation has. The reverberation time means one which is measured on the time axis according the oblique line from the initial point to the 60 dB decayed point if the reverberation curve is almost straight line. It is strictly difficult to use the reverberation curve for accuracy evaluation of the impulse response. However, in the acoustic transfer system, it is

used for the evaluation to examine whether the reverberation curve taken by logarithm in a approximate way is a straight line or not.

In other word, this is based on the concept that the reverberation curve will be a straight line if the impulse response of a system is obtained very precisely [10].

In the next, the simulation experiment using the reverberation curve for a evaluation of estimated accuracy of impulse response will be described. The parameter of this experiment is a change of time window length.

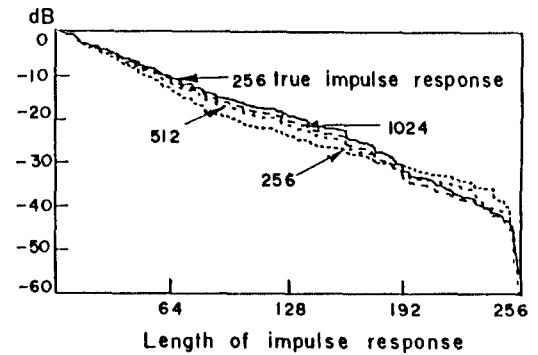


Fig. 4. The accuracy evaluation of estimated impulse response by reverberation curve. (The shape of time window: rectangular, the length of time window: 256, 512 and 1024 points)

Fig. 4 shows the reverberation curve for the true impulse response and the estimated one. In this case, the shape of time window is rectangular. Its lengths are 1, 2 and 4 times of the true impulse response.

## B. Evaluation by the Misalignment

This method is used in case of simulation experiment in order to investigate the elements which affect the estimated error in the problem of the system identification. The accuracy evaluation of the identification model will be carried by equation below by calculating the errors between the assumed true impulse re-

sponse of the system  $h(p)$  and the estimated one  $\hat{h}(p)$ .

$$Q = 10 \log_{10} \left[ \frac{\sum_{p=0}^{M-1} [h(p) - \hat{h}(p)]^2}{\sum_{p=0}^{M-1} h^2(p)} \right] \quad (6)$$

The quantity  $Q$  is called the "misadjustment" or "misalignment" between  $h(p)$  and  $\hat{h}(p)$  [11].

Especially, this method is useful for investigating the effect of lengths and shapes of time window, S/N, and average number to the estimated accuracy when the impulse response is estimated by the cross spectrum method. Here, the less the value of  $Q$  is, the higher accuracy it has. There are the values of  $Q$  which evaluate the accuracy in Fig. 5 according to the change of the length of time window as the parameter when DFT is carried out.

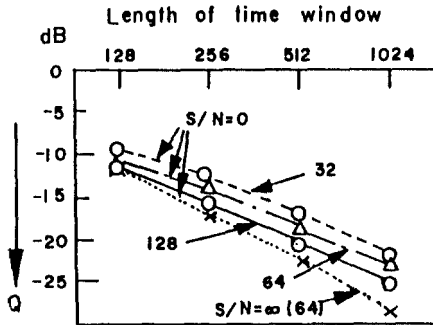


Fig. 5. The accuracy evaluation of the estimated impulse response according to the result of simulation experiment.

(the length of estimated impulse response: 256 points, a number of average for DFT: in case of S/N = 00 64 times in case of S/N = 0 32, 64 and 128 times)

The lengths of time window are 1/2, 1, 2 and 4 times of the true impulse response, and the shape of time window is rectangular. The numbers of DFT performance are 32, 64, and 128 times in either case, and S/N is ∞ and 0 dB.

### III. EVALUATION BY CANCELLING THE RESPONSE SIGNAL

This chapter describes the practical evaluation method on the base of eliminating the response signal in the system which is calculated by the estimated impulse response from the response signal which is observed experimentally.

This is profitable for evaluating the accuracy of the estimated impulse response without knowing the true one in the system. However, in other hand, there is a weak point (defect) that the effect of additional noise increases the error at the time of accuracy evaluation.

The outline of this method is as follows:

If the impulse response in acoustic transfer system could be estimated precisely, the convolution signal between the source signal and the estimated impulse response would be the same response signal through the system which is formed by the same input signal. So, a difference can be yield between the observed response signal and computed one at a certain input signal. According to the power of this difference signal, the accuracy for the estimated impulse will be evaluated. This method will evaluate the accuracy in quantity by normalizing the power of difference between them by the observed response signal.

This coefficient of evaluation  $P$  is as follows.

$$P = \frac{(\text{Computed response} - \text{Observed response})^2}{(\text{Observed response})^2}$$

$$= \frac{\sum_{p=0}^{L-1} \{ \sum_{r=0}^{L-1} x(p-r) \hat{h}(r) - y(p) \}^2 W^2(p)}{\sum_{p=0}^{L-1} \{ y(p) W(p) \}^2}$$

- Where  $x(p)$  : input sequence  
 $h(p)$  : the estimated impulse response  
 $y(p)$  : the observed response  
 $w(p)$  : time window  
 $R$  : the length of impulse response  
 $L$  : the length of time window

In equation (7) for getting the coefficient  $P$ , the numerator indicates the power of difference signal. This is changed depending on the spectrum and the power of input signal. So, in order to use  $\tilde{P}$  as a coefficient of evaluation, it is necessary to eliminate their effect. The power of the input signal of two elements which are changing the difference signal can be solved by normalizing the denominator, the summation of the estimated response signal, in equation (7). However, for the spectrum of the input signal, it is impossible to make the normalized coefficient of evaluation. So, instead of this, it is better to use white noise which has flat spectrum as a test signal. In the evaluation experiment by the coefficient  $P$ , the false white noise by electronic computer will be taken for the input sequence.

The following describes the effect of noise which is added to response signal in the accuracy evaluation of impulse response. The noise which is affecting the output signal can be ignored since it can be converged to zero by the average of a number of cross spectrum by the input and output signal, when impulse response is investigated in the acoustic transfer system [12]. However, the noise which is added to the response signal make the evaluation value worse, and its effect can not be ignored at the accuracy evaluation experiment for the estimated impulse response.

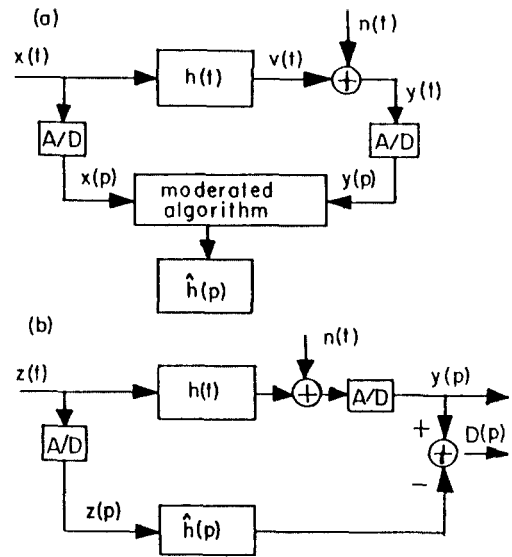


Fig. 6(a) Block diagram of impulse response measurement in acoustics transfer system such as real space.

(b) Block diagram of the accuracy evaluation of the estimated impulse response.

Fig. 6(a) is a block diagram to get the value of estimation  $h(p)$  of the true impulse response  $h(p)$  using the sequence  $x(p)$  and  $y(p)$  which are sample values of the input signal  $x(t)$  and the output signal  $y(t)$ .

Fig. 6(b) is a block diagram of the estimated accuracy evaluation using the same transfer system, where the input signal  $z(t)$  is a white noise and  $n_2(p)$  is a sequence of sampling value for the external noise  $n_2(t)$ . If a external noise is added to the transfer system in equation (7),  $P$  will be as follows.

$$\tilde{P} = \frac{\sum_{p=0}^{N-1} \{y(p) - \hat{y}(p)\}^2}{\sum_{p=0}^{N-1} \{y(p)\}^2}$$

$$= \frac{\sum_{p=0}^{N-1} [Z(p) * (h(p) - \hat{h}(p) + n_2(p))]^2}{\sum_{p=0}^{N-1} [Z(p) * h(p) + n_2(p)]^2} \quad (8)$$

The effect of external noise for  $\tilde{P}$  will be examined in equation (8)

In equation (8),

$$\begin{aligned} \text{if } \quad \Delta h(p) &= h(p) - \hat{h}(p) \\ V(p) &= Z(p) * h(p) \\ \Delta V(p) &= Z(p) * \Delta h(p) \end{aligned}$$

is put, it will become equation (9).

$$\begin{aligned} \tilde{P} &= \frac{\sum_{p=0}^{N-1} \{ \Delta V(p) + n_2(p) \}^2}{\sum_{p=0}^{N-1} \{ V(p) + n_2(p) \}^2} \\ &= \frac{\sum_{p=0}^{N-1} \{ \Delta V^2(p) + 2\Delta V(p)n_2(p) + n_2^2(p) \}}{\sum_{p=0}^{N-1} \{ V^2(p) + 2V(p)n_2(p) + n_2^2(p) \}} \quad (9) \end{aligned}$$

Generally, there is no correlation between the input signal  $z(p)$  and external noise  $n_2(p)$ . Therefore,  $2\Delta V(p)n_2(p)$  and  $2V(p)n_2(p)$  in equation (9) converge to zero by averaging a number of times. So, it will be the following equation.

$$\tilde{P} = \frac{\overline{\Delta V^2(p) + n_2^2(p)}}{\overline{V^2(p) + n_2^2(p)}} \quad (10)$$

where,

$$\overline{\Delta V^2(p)} = \frac{1}{N} \sum_{p=0}^{N-1} \Delta V^2(p)$$

$$\overline{n_2^2(p)} = \frac{1}{N} \sum_{p=0}^{N-1} n_2^2(p)$$

$$\overline{V^2(p)} = \frac{1}{N} \sum_{p=0}^{N-1} V^2(p)$$

Furthermore in equation (10),

$$\begin{aligned} \overline{\Delta V^2(p) + n_2^2(p)} &= A + N \\ \overline{V^2(p) + n_2^2(p)} &= B + N \end{aligned}$$

is put, it will become equation (11).

$$\tilde{P} = \frac{A + N}{B + N} \quad (11)$$

When the above equation has done differentiation by  $N$ , then it will be becomes as follow.

$$\frac{d\tilde{P}}{dN} = \frac{B - A}{(B + N)^2}$$

Generally the relation of  $\overline{\Delta V^2(p)} < \overline{V^2}$  is formed so that it is  $B > A$ . According to this, it becomes  $d\tilde{P}/dN > 0$ . Because of these reasons,  $P$  in equation (8) tends to increase according to increase of  $n_2(p)$  monotonously. The acoustic transfer system such as a indoor or a duct can not ignore the presence of external noise  $n_2(p)$ . In the following, the result of the electronic computer simulation experiment will be described for the evaluation method of estimating accuracy of impulse response by cancelling the response signal which is suggested in this time window is rectangular. Even though the coefficient of evaluation is  $\tilde{P}$  in equation (7), the result is denoted as dB to describe easily.

This equation is as follows:

$$\underline{P} = 10 \log_{10} \tilde{P} \quad (12)$$

As in the previous statement,  $P$  is affected by the external noise  $n_2(p)$  at the evaluation time.

The evaluation experiment set up  $S/N$  to  $\infty$ , 20dB, and 10dB and examine its effect, where  $S/N$  is defined as the ratio of the average value of power of  $y(p)$  and  $n_2(p)$ .

The input and output data for the simulation experiment are the same ones which are used in evaluation experiment (II-A) by coherence function.

The simulation experiment of estimated accuracy evaluation has been done according to the block diagram Fig. 6(b). Among the figures,

$$D(p) = y(p) - \sum_{r=0}^{R-1} x(p-r) \hat{h}(r) \quad (13)$$



this is used for calculating the numerators in equation (7).

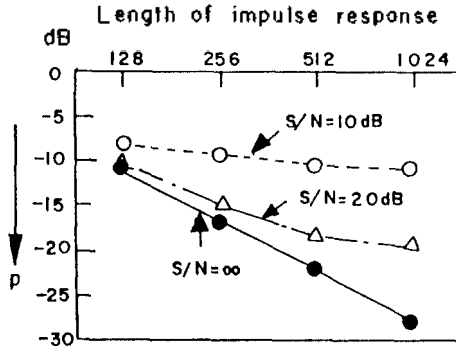


Fig. 7. The effect of the accuracy evaluation for the estimated impulse response by the external noise. (S/N = ∞, 20 dB and 10 dB)

Fig. (7) shows the results of estimated accuracy evaluation experiment. The value of this will be correspond to the evaluation value by the misalignment  $Q$  in Fig. 5, when  $S/N$  equals to  $\infty$ . However, even though the estimated accuracy is high, the value of  $P$  can not be over  $-20\text{dB}$  and  $-10\text{dB}$ , when  $S/N$  is  $20\text{dB}$  or  $10\text{dB}$  respectively.

#### IV. DISCUSSION

The accuracy evaluation for the estimated impulse response in the acoustic transfer system is investigated by the simulation experiment for the conventional methods and the new methods which is proposed in this paper by the electronic computer.

The conventional methods, which are the coherence function, the coherence coefficient

and the reverberation curve are regarded as an unsatisfying one because the phase is neglected in a dynamic system.

There are two methods to consider the phase characteristics for the accuracy evaluation, which are the misalignment and the for the accuracy evaluation, which are the misalignment and the cancellation of response signal. If the response signal would include the external noise when this accuracy evaluation has done, the new method will have problem because of increasing the error. The misalignment method can not evaluate the accuracy in the case of experiment, which could not tell the true impulse response of system. Consequently, for finding the accuracy evaluation by the experiment, the new method is useful by reducing the effect of external noise as much as it possible.

The white noise  $z(t)$  which is the input signal for the accuracy evaluation denoted in the Fig. 6(b) should have the higher level signal than the external noise. (Experimentally, the ratio between the signal and noise is over about  $40\text{dB}$ )

#### V. CONCLUSION

This paper studies the accuracy evaluation methods for an estimated impulse response, which is obtained by the experiment that has done observing the input and output waveform.

As a result, this new method which is obtained by the experiment is regarded as more practical method since it considers the phase characteristics comparing with the conventional methods.

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