Reckoning (or Computing) Rods and a Suan-pan in Japan

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I'd like to explain in this paper why Chinese mathematicians couldn't understand the tien-yüan-shu (天元術), while on the other hand Japanese mathematicians could renew the tien-yüan-shü in the 17th century. I begin with a brief history of old Japanese mathematics.

We don't know when old Chinese mathematics was imported to Japan. Probably some Japanese studied it before the seventh century and it came to Japan through Korea. In the beginning of the eighth century the Japanese government established a school called Daigakuryō (大学寮) which made a model of the school Kuo-tzu-chien (国子監) of the Tang (唐) dynasty. Many youths, who were 13 to 16 years old and were lower nobles' sons, studied some subjects there. Some of them majored in mathematics using mostly Chinese texts and a few Korean ones. When they calculated, they used Chinese reckoning rods (算子,算木) as calculating tools. But we cannot know what sorts of rods they used.

In mathematical course the regular staff consisted two teachers and the student capacity was 30. The mathematical texts were Suntzu (孫子), Wu-tsao (五曹), Chiu-chang (九章), Hai-tao (海島), Liuchang (六章), Chui-shu (綴術), San-kai-Chang-chai (三開重差), Chiushu (九司). After graduating they worked as clerks in tax offices, surveyors and other occupation which required calculation.

Daigakuryō was still open after the 14th century; but it was not important to the nobility. In about the tenth century each noble family established a school of theirown, and their sons entered it. Some of them hired tutors or private teachers for their children. We have found one of the texts which was edited by a tutor called Kuchizusami (口遊, 970). There are some mathematical matters in it. For example, there was the multiplication verse, how to find number of a sticks in a bamboo bundle, how to sex an embryo, and how to know whether a patient is alive or dead. These matters were no doubt influenced by the Chinese text Sun-tzu-suan-ching.

This multiplication verse began from 9·9=81, and ended at 1·1 =1. Pronouciation of 9 is"ku", therefore, this verse has been called "ku-ku" in japan. We find many "八十一"(81) in the oldest Japanese poetry Manyōshu (万葉集 , ca.the 8th c.), and we must pronounce it "ku-ku". At that time, educated people were expected to learn the verse by heart.

Several centuries passed, and for this period we have only descriptions of mathematics. Some essays, literary works, and diaries show us that noble people and priests understood mathematics. In the end of the 16th century many Jesuit priests, their followers, and European marchants came to Japan. They were surprised to find that most Japanese could write and calculate. At the time Japanese used soroban (suan-pan,算盤 in a Japanese style). Some Japanese used still sangi (算木,算子). We cannot decide when Japanese began to use suan-pan (算盤) and when they improved it to a Japanese form.

There are some pictures which show a man is using sangi in the 14th or the 15th century. One of them is a poor person who is in a very small shanty like a kennel, and he is calculating with sangi on the ground. In my opinion, he was not a mathematician, but a fortuneteller. When a man practiced divination, he calculated with sangi.

In the beginning of the 17th century we can see the oldest Japanese mathematical text, Sanyōki (算用記). Of course there must be several old texts in the end of the 16th century, but we have not been able to find them as yet. Sanyōki begins to show us a verse of division (hassan 八算, ken-ichi 見一 in Japan; chiu-kuei 九歸, tung-kuei 撞歸 in China). Therefore, this text was for people who could use soroban in adition, subtraction, and multiplication.

After publication of Warisansho (割算書, 1622, edited by Shigeyoshi MŌRI 毛利重能), Mitsuyoshi YOSHIDA (吉田光由, 1598-1672), one of the MŌRI's followers, published the famous text Jingōki (or Jinkōki 塵劫記, 1627). This text explained all the problems at that time. It was made in a de lux edition paid for by the author, because he was a member of the wealthy Suminokura family. It includes many illustrations, and the audience can understand mathematics easily by looking at them.

The author, M.YOSHIDA, learned Suan-fa-t'ung-tsung (算法統宗, ca. 1593), from the book given to him and explained by Soan SUMINOKURA(角倉素庵, 1570-1632). With the help of the text, M.YOSHIDA could edit Jingōki. Soon after the first publication of Jingōki, the author revised and enlarged it. Then he issued it several times. People who wanted to learn mathematics amused themselves studying these issues. After death of the author, Jingōki was still a bestseller. Many publishers revised and sold it until the beginning of the 20th century. We cannot decide how many versions there were even in the author's lifetime. There were more than ten different versions in his life.

Jingōki showed not only how to use soroban, how to solve mathematical problems, and how interesting mathematical recreations were, but also how important illustrations were. At first in 1627, Jingōki was four volumes and a large format. The next edition was five volumes. Then, it was published in three volumes and large format in 1631. The 1631 edition had several colored illustrations. They were the beginning of color printing in Japan, which was the ancestor of nishikie (a colored woodblock print in Japan).

There was another of S.MORI's followers, Chishō IMAMURA (今村知商), who edited luxurious volumes, Jugairoku (竪亥録). It is a collection of mathematical formulae which show most mathematical problems. Therefore, it doesn't explain the reason why the formulae are given, however there are solutions to difficult problems of the time. He wrote it in old Chinese (the language of Japanese scholars as Latin was to scholors in Europe). Meanwhile Jingōki was written in colloquial Japanese, therefore, most Japanese read it, while Jugairoku was for mathematicians.

C.IMAMURA edited a text in next year, named Inkisanka (因帰算歌 1640). This text is consist of many formulae which expressed as poems. He said in the preface, "Nowadays most boys and girls play pranks often, and they usually sing trifling songs. Therefore, I have written it, and I'd like them to hum these useful verses." Jugairoku was for specialists, and Inkisanka was for either children or the youths.

M.YOSHIDA and C.IMAMURA were both S.MORI's followers. It is possible they felt some rivalry with each other.

After publication of Inkisanka, M.YOSHIDA edited a little different Jingōki (1641) from the former issues. It consists of three volumes and is a small format. There are several difficult problems which were not in the former issues. Of course his main aim in the issues was to explain how to use soroban, but this version includes two illustrations which show how to calculate multiplication and division using reckoning rods. One is for multiplication (876 x 2222 = 1946472), and the other is for division (1479852 : 2222 = 666). They are the oldest explanations of calculation using reckoning rods in illustrations in Japan. We can find his explanation of the extraction of square root and cube root in all of his issues about Jingōki. The explanation of extraction was one of the uses for soroban. But when the author extracted square root and cube root, he used probably reckoning rods. This method of using reckoning rods was changed by usinig soroban. Because he used many soroban when he extracted.

In this special issue in 1641, M.YOSHIDA include many problems about 2^n. They characterised the volume. And then the most special thing was the following. He asked mathematicians twelve difficult problems in his book. He explained in the preface why he wrote these twelve problems expect solutions in the end of the volume. "There are many mathematicians around the town. But most of them can solve only easy problems, and some of them have read my Jingōki merely. In this way they teach mathematics. If you want to distinguish real mathematicians or not, you'd better ask them these problems. When they can solve the problems, they are excèllent mathematicians." One of the most famous mathematicians, M.YOSHIDA, asked, therefore, almost all mathematicians probably wanted to find their solutions earnestly. The problems of this style were called idai (遺題).

In 1653, Tomosumi ENAMI edited Sanryōroku (参両録), and he showed the solutions of M.YOSHIDA's idai. He said that YOSHDA's problems were very easy and worthless, and if he had been YOSHIDA, he would not show such worthless problems. He asked mathematicians eight problems in the end of his book. The style of showing solutions of former's idai-problems and asking one's own idai, was called idai-keishō (遺題継承).

Many mathematicians were concerned with idai-keishō. Some of them asked more than 100 idai-problems, and these idai became more and more difficult. At last they couldn't solve idai-problems only using soroban. They thought that the method of how to solve those difficult problems might be written in Chinese texts. They found Suan-hsüeh-ch'i-mêng (算学啓蒙, 1299) which was reprinted in Korea.

Around 1650, Seisū HASHIMOTO (橋本正数) understood Chinese algebra, tien-yüan-shu (天元術), only reading Suan-hsüeh-ch'i-mêng. (This text was reprinted in 1658, Japan.) He and his followers used tien-yüan-shu and could solve many difficult problems.

In 1660s many mathematicians could use tien-yüan-shu. Therefore, beginners wanted to know how to use it. One of S.HASHIMOTO's followers, Kazuyuki SAWAGUCHI (沢口一之), answered the need. He edited Kokonsanpōki (古今算法記, 1671), in which he explained how to operate tien-yüan-shu. It was the first edition which showed tien-yüan-shu in Japan. As you know, Suan-hsüeh-ch'i-mêng didn't explain the operation of tien-yüan-shu.

But tien-yüan-shu was used only to solve simultaneous linear equations with numerical coefficients, or an equation of the n-th degree with one unknown and numerical coefficients. Consequently, K.SAWAGUCHI's idai-problems were the simultaneous equations of n-th degree with many variations. He asked 15 idai-problems in the end of his book, Kokonsanpōki. The first solver was Takakazu (or Kowa) SEKI (関孝和 , ca.1640-1708), who could solve them using the new algebra which was almost the same as present algebra. He showed the solution in his edition, Hatsubisanpo (発微算法 , 1674). And then he explained a determinant which was used for simultaneous equations with two unknowns and literal coefficients, i.e., he made them an equation of the n-th degree with one unknown using a determinant.

Idai-keishō (relay system of A&Q) raised Japanese mathematics to a higher level in a very short time. Another reason for the splendid Japanese mathematics in Tokugawa period (or Edo period) was the national isolation. From the end of the 16th century to the beginning of the 17th century many Europeans came to Japan, who were merchants, priests, and others. Some of them recorded that Japanese liked learning very much. At the time Japanese mathematics was scarcely influenced by European mathematics (except in the field of surveying). Most Japanese people didn't want to study higher mathematics. In adition, soroban (Japanese abacus) was a very useful instrument to solve the mathematical problems. They could easily get accurate real values of roots for a cubic or a biquadratic equation. Therefore, they didn't want to know European mathematics very much. In the beginning of the 17th century, Tokugawa shogunate began the national isolation policy. Therefore, Japanese studied only Chinese texts or Japan's own texts.

In Japan most people could use soroban, and a few people could use reckoning rods in the beginning of the 17th century. On the other hand, in the 14th century, Chinese people used both suan-pans and reckoning rods. At the time, mathematicians could operate tien-yüan-shu. But, in the 16th century, they forgot to use reckoning rods. Many texts for using a suan-pan were issued. For example, some of them were Suan-fa-t'ung-tsung, Pan-chu-suan-fa (盤珠算法, 1573), and others. When they wanted to study higher mathematics, Europeans came to China. (One of them was Matteo Ricci,利瑪竇, 1552-1610). They taught Chinese mathematics. Tien-yüan-shu was not able to be revived. After a long time, Chinese mathematicians noticed that tien-yüan-shu was a kind of algebra, which was made by the Chinese themselves.

REFERENCE

Japan Academy: Meiji-zen-Nihon-sūgakushi (明治前日本数学史, A History of Japanese Mathematics Before the Meiji Era) 5 vols. 1954-60, Iwanamishoten Co. Tokyo.

T.Endō, Y.Mikami, A.Hirayama and others: Zoshū-Nihon-sūgakushi (増修日本数学史, Revised and Enlarged History of Japanese Mathematics) 1981, Kōseisha-kōseikaku Co. Tokyo.

Shinichi Ōya: Wasan-izen (和算以前, A History of Japanese Mathematics Before Edo Period) 1980, Chūō-kōron-sha Co. Tokyo.