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A Note on S-closed Space and RC-convergence.

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S-closed 空間과 RC 收斂에 관하여

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요 약

Semi-open을 기초로 하여 만들어진 S-closed공간의 일반적인 성질을 살펴보고 S-closed공간과 (maximum) filterbase와의 관계를 조사하였다. 이를 바탕으로 regular closed된 cover C, regular open set인 族 C, rc-accumulation, (maximum) filterbase에서의 關係를 살펴 보았다. Mapping theory에서 almost-open almost-continuous map f가 almost continuous되는 것을 보였다.

I. Introduction

In this paper investigating the several new characterizations of S-closed spaces. These characterizations are based on semi-open in terms of a generalization of complete accumulation point. S-closed spaces can be characterized in terms of regular closed (rc) or regular open subsets of a space (X,T). And rc-convergence structure used to characterizations of S-closed space. And investigate almost-open almost-continuous map f is almost continuous.

II. Preliminaries

Let X be a space and $A\subset X$, $x\in X$, We denoted closure of A as $\bar{A}$, interior of A as A°, $RC(X)$ denotes family of regular colsed subsets of X, $SO(X)$ denotes family of semi-open subset of X and $O(A)$ denotes family of open subsets which contain A, and semi-closure of A denoted as $SC(A)$ and θ-semiclosure of A as θ-$sc(A)$.

Throughout this paper the topological space (X,T) denoted simply X and (Y,τ) denoted simply Y.

Def. 1) A sudset V of topological space is semi-open if and only if $V^{\circ}\subset V\subset \overline{V^{\circ}}$

Def. 2) A is regular closed if $A=\overline{A^{\circ}}$.

A is regular open if $X-A$ is regular closed.

Def. 3) x is in the the semi-closure of A, if each $V\in SO(x)$ satisfies $\bar{V}\cap A\neq\phi$. A is θ-semiclosed if $A-$θ-$sc(A)$.

Def. 4) A map $f: X\rightarrow Y$ is almost-open if for each $G\in RO(X)$, $f(G)\in\tau$. A map is almost-continuous if and only if for each $x\in X$ and open neighborhood V of $f(x)$, there exists an open neighborhood G of x such that

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$f(G)\subset(\bar{V})^{\circ}$ in Y.

Following properties are easy to see from previous definitions. No proofs are given for next properties.

Prop. 1) A is semi-open if and only if $\bar{A}=\overline{A^{\circ}}$.

A is semi-open then $\bar{A}$ is semi-open and $\bar{A}=\overline{(\bar{A})^{\circ}}=\overline{(\overline{A^{\circ}})^{\circ}}$.

Prop. 2) If $P\subset Q\subset X$ then $\theta\text{-}sc(P)\subset\theta\text{-}sc(P)$.

Prop. 3) If $A\in SO(X)$ then $\theta\text{-}sc(P)$ is θ-semicloesd for each $P\subset X$ since $\bar{A}\in SO(X)$.

Prop. 4) Since $RC(X)=\{\bar{V}: V\in SO(X)\}$, $x\in\theta\text{-}sc(A)$ if and only if each $R\in RC(X)$ satisfies $A\cap R\neq\phi$.

Prop. 5) A regular open subset of a space is θ-semiclosed.

Prop. 6) The following statements are equivalent for a function $\lambda: X\to Y$

i) the function λ is θ-continuous.

ii) for every $A\subset X$, $\lambda(\bar{A})\subset\theta\text{-sc}(\lambda(A))$.

iii) for every $A\subset Y$, $\overline{\lambda^{-1}(A)}\subset\lambda^{-1}(\theta\text{-sc}(A))$.

iv) for every θ-semiclosed $A\subset Y$, $\lambda^{-1}(A)$ is closed in X.

v) for every $R\in RC(\lambda(x))$ there exists a $V\in O(x)$ with $\lambda(V)\subset R$.

Ⅲ. Main Theorem

Thm. 1) For a topological space X the following are equivalent ;

i) X is S-closed.

ii) For each family of semi-closed sets $\{Fa\}$ (i.e. each Fa is the complement of a semi-open set) such that $\cap Fa=\phi$, there exists a finite subfamily $\{Fa_i\}_{i=1}^{n}$ such that $\cap_{i=1}^{n}(Fa_i)=\phi$.

iii) Each filter base on X has an s-accumulate to some point in X.

iv) every maximum filterbase F s-converges.

Proof)

i)$\to$iv). Let $F=\{Aa\}$ be a maximum filterbase. Suppose that F does not s-converges to any point : therefore F does not s-accumulate to any point. Then for every $x\in X$, there exists a semi-open set $V(x)$ containing x and an $Aa\in F$ such that $Aa\cap\overline{V(x)}=\phi$. Obviously $\{V(x): x\in X\}$ is a semi-open cover for X and there exists a finite subfamily such that $\cap_{i=1}^{n}V(x_i)=x$. Since F is a filterbase, there exists an $A_0\in F$ such that $A_0\subset\cap_{i=1}^{n}Aa_i$. Hence $A\cap V(x_i)=\phi$, $1\leqq i\leqq n$, which implies $A_0\cap(\cup_{i=1}^{n}V(x_i))=A_0\cap X=\phi$. Contradicting the essential fact that $A_0\neq\phi$.

iv)$\to$iii). Every filterbase is contained in a maximal filterbase.

iii)$\to$ii). Let $\{Fa\}$ be a collection of semiclosed sets such that $\cap Fa=\phi$. Suppose that for every finite subfamily, $\cap_{i=1}^{n}(Fa_i)^{\circ}\neq\phi$.

Therefore $F=\{\cap_{i=1}^{n}(Fa_i^{\circ}): n\in Z^{+}, Fa_i\in\{Fa\}\}$ forms a filterbase. From hypothesis, F s-accumulates to some point $x_0\in X$. This implies that for every semi-open set $V(x_0)$ containing x_0, $Fa^{\circ}\cap\overline{V(x_0)}\neq\phi$, for every $a\in A$. Since $x_0\notin\cap Fa$ there exists an $a_0\in A$. Since $x_0\notin Fa$. Hence X is contained in the semi-open set $X-Fa^{\circ}$.

Therefore

$(Fa_0)^{\circ}\cap\overline{(X-Fa_0)}=(Fa_0)^{\circ}\cap(X-(Fa_0)_0)=\phi$, contradicting the fact that F s-accumulates to x.

ii)$\to$i). Let $\{Va\}$ be a semi-open covering X. Then $\cap(X-Va)=\phi$. By hypothesis, there exists a finite subfamily such that

$$\cap_{i=1}^{n}(X-Va_i)^{\circ}=\cap_{i=1}^{n}(X-Va_i)=\phi.$$

Therefore $\cap_{i=1}^{n}\overline{Va_i}=X$, and consequently X is S-closed.

Next theorem is easily given from Thm. 1)

Thm. 2) For a topological space X, the following statements are equivalent.

i) X is s-closed.

ii) any cover C of X by regular-closed sets has a finite subcover.

iii) any family C of regular open sets such that $\cap C=\phi$ contains finite $B\subset C$ such that $\cap B=\phi$.

iv) every filterbase on X has an rc-accumulation point in X.

v) every maximal filterbase on X rc-converges.

Proof

i)→ii). Since regular closed sets included semi-open sets, this is obvious.

ii)→i). Assume X is not S-closed, then there exists a cover $C\subset SO(X)$ such that C has no finite proximate subcover. Thus $\{\bar{V} : V\in C\}\subset RC(X)$ has no finite subcover. Thus the result follows from the contradiction.

ii)↔iii). Since $RO(X)=\{X-x: x\in RC(X)\}$, the result follows.

iv)↔v). A maximal filterbase rc-converges if and only if it rc-converges.

v)↔i). Let a filterbase F s-converges to $x\in X$ and $R\in\{C(x)\}$. Then $R=\bar{G}_1\cap\bar{G}_2\cap\cdots\cdots\cap\bar{G}_n$ in X. Now for each $\bar{G}_i$ in X. $F\in H$ such that $F\subset\cap\{F_i\}\subset R$. On the other hand, assume that H rc-converges to $x\in X$ and $V\in SO(X)$ such that $x\in V$ there exists an $F\in H$ such that. $F\subset\bar{V}$ in X.

Corollary) The next two statements are equivalent ;

i) a filterbase on X rc-converges if and only if it θ-converges.

ii) if a filterbase on X converges with respect to the topology T, then it rc-converges.

Thm. 3) An almost-open almost-continuous map $f : X\to Y$ is almost-continuous.

Proof) Let $F\in RC(Y)$, then $f^{-1}(F)\in RC(X)$. Thus if $f(x)\in\bar{V}$ in Y for arbitrary $V\in\tau$, then $f^{-1}(V)=\bar{H}$, $H\in T$, V in Y, H in X. Consequently, $f^{-1}(\bar{V})\in C(x)$. Hence $C(f(x))\subset f(C(x))$ and the result follows.

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