非定常 浸透에 관한 實驗的 硏究

An Experimental Study on the Unsteady Seepage

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要 旨

非定常 浸透問題에서 가장 중요하고 어려운 점은 自由水面의 位置와 그 變化를 어떻게 解析하느냐이다.

그러므로 本 硏究는 특히 水位上昇에 따른 堤體內 浸透와의 상관계수를 찾았으며(즉, 수위상승에 대해 $V_R=\frac{dH}{dt}$, 침투속도에 대해 $V_S=Ki=k\frac{H-d_1}{d}$), 또한 旣 發表된 式들을 分析하여 實驗資料와의 비교 整理 分析하였다.

Abstract

The most important and difficult part in the problem of unsteady seepage is, how to analyze a position and variation of free water level.

Therefore, this paper found the relation between the rising water level and infiltration in the embankment by analyzing established equations and extrapolated the empirical equations from experimental data (For the seepage velocity; $V_S=Ki=K\frac{H-h_1}{d}$, For the rising velocity level; $V_R=\frac{dH}{dt}$)

With the aid of these data, the necessary equations were compared with the experimental analyses.

1. Introduction

A majority of analysis of unsteady seepage have been based on experimental methods because theoretical formulations are extremely difficult in a real system.

A most important and difficult part in the problem of unsteady seepage is to analyze the position and variation of free water level.

Water infiltration in an embankment soil has a large influence on the slope of the embank-

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ment. Therefore, the variations of seepage line have been examined due to rapid rising water level in the embankment as a function of time.

A simple arrangment of unequilibrium equations was introduced. Each time the variation of seepage line was plotted.

By analyzing the existing equations, relation between the rising water level and infiltration in the embankment was developed. The above relation between the rising water level and infiltration was then extrapolated to empirical equations using experimental data. With the seepage line, one can analyze the position of the effluence points. Stability in the seepage line, which is established when the water level rises, is the most important factor to be considered when a seepage line is examined. Free water level within the embankment does not as rapidly rise as the water level in a reservoir. Every time the water level suddenly rises, the state of the seepage line is also changed, hence the effectivc stress. Therefore, the analysis must vary.

The ratio of rise of seepage line between the experiment model and ordinary structures was calculated. The water level was calculated by using the simulation method. The were compared with the seepage lines based on established theories. Finally the equation was developed from experimental data using the above results.

2. Historical Background

Based on Darcy's law and using Dupuit's assumption, an extensive study has been done by numerous authors to solve the problems of seepage as well as to establish seepage lines. Of all these studies, the most widely used and accepted methods are being developed by Forchheimer(22), Schoklitsch(17), Schaffernak(18), Iterson(23), and Pavlovsky(4). The graphical solutions have been developed by R. Dachler(24), and A. Cassagrande(1). However, these solutions are only applicable to the steady seepage conditions with various boundary conditions.

Applications of Finite Element Methods to analysis of seepage were introduced first by Zienkiewicz(21). Finite Element Methods derived from Galerkin's method, was used in the analysis of the aquifer theory by Pinder and Frind.(19)

In steady state, Finite Element Methods of seepage with free water surface and seepage surface was studied by Finn(3), Kawamoto and Kono(5), etc.

In unsteady state of seepage, Skempton-Bishop analyzed failure of embankment with the change of pore water pressure by rapid drawdown. Schmied and Ujida studied the shapes of seepage line.

In the analysis of unsteady seepage, the Finite Element and Finite Differences Methods are extremely useful. These methods have the flexibility of using various boundary conditions. the methods can also be adapted to a desired degree of accuracy by changing the number of nodal points. Neumann and Witherspoon(8) studied steady and unsteady seepage with different free water surfaces which satisfied the respective boundary conditions.

Kochina(10) studied governing equations of temporary flow of free surfaces in porous materials. Using the similarity law Kono(5) and Yamamoto derived the unsteady seepage equations. Kaharada derived fundamental one-dimensional seepage equations. Tanaka derived fundamental equations neglecting acceleration terms. Akai derived the unsteady flow equation for quasi-one-dimensional unsteady flow. In addition many authors (e.g. Suresh P. Brahma, Milton E. Harr(4), D. Stephenson, Onaka, etc.) studied similar problems.

3. Identification Procedures

3-1 Experiment Methods

3-1-1 Experimental Device

The experimental water tank was made of steel plates and glasses. The dimensions of the water tank was 150 cm wide, 50 cm high, 450 cm long and the glass plates were 0.3 cm thick. The tank was divided into three sections with 50 cm width, the other dimensions being the same for each section. The front section was made of specially-thermic-treated glass. A grid of silk printing of 1 cm wide and long was done on the glass to facilitate the visual observation of seepage line. The middle section was used to measure the seepage quantity. The back section was used to measure the potential head with piezometer setting of 2 cm wide and long. The piezometer was connected to 0.5 cm diameter vinyl pipe.

The inflow and outflow of the water was controlled through a storage tank and outlet. The outlet was placed at the downstream end at the bottom floor of the tank.

3-1-2 Experimental Methods

The sand model was saturated by raising the water level very slowly. Then water was allowed to drain through the outlet slowly. This method of saturating the sand also provided uniform hydraulic compaction of the sand model.

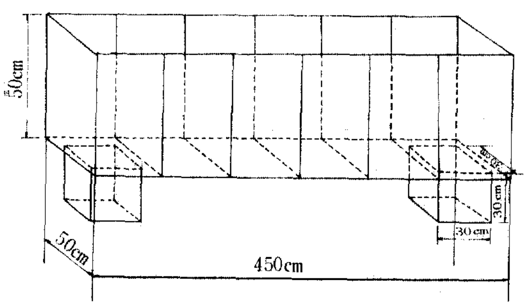

Fig. 1. Schematic sketch of experimental equipment.

The bottom floor of the tank and hence the base of the model was impermeable. Three experiments were performed with models 40 cm high and varying side slopes. The side slopes used were 1 : 2, 1 : 1.5 and 1 : 1 respectively.

A movie camera with a magnification photographic editor was used to record the variations of seepage lines. The variations of seepage line was observed at the intervals of 1, 3, 6, 15, 30 and 60 sec.

The details of experimental model is shown in Table 1.

3-2 Associated Work

The seepage line calculated from approximate analysis using theoretical equations are different from that obtained in experimental results. In this reseasch investigations, the experimental values were compared with some theoretical equations. The rising time of seepage line for each water level is different. Equation (4) and (5) [see appendix] have the same rising time. However, the height of seepage lines from these equations are little different. Equation (8) corresponds to the same experimental results in case of quick rising time of water levels.

The rising time of seepage line for different water levels is shown in table 2. This data were collected from the experiment. They are in good agreement with the theoretical analysis.

In appendix 1, the equation (5), (6), (8) refers to a linear relation between seepage height and distance. But in actuality it is not linear rather parabolic. That is why the results are different from experimental values. However, equation (4) is a parabolic equation, with some minor difference.

A parabolic relation between height of seepage line with distance was developed and is expressed in equation (2). The equations (2), (4) and (5) differ from theoretical equations

Table 1. Experimental conditions

Eep. Sign		Rising Time	B	B	m	n	H
EA	1	1 min 28 sec	10	170	2	2	40
	2	5 〃 41 〃					
	3	8 〃 42 〃					
	4	14 〃 00 〃					
	5	23 〃 27 〃					
	6	35 〃 56 〃					
	7	67 〃 45 〃					
EB	1	1 〃 34 〃	10	130	1.5	1.5	40
	2	7 〃 59 〃					
	3	13 〃 55 〃					
	4	25 〃 35 〃					
	5	57 〃 25 〃					
EC	1	1 〃 40 〃	10	90	1	1	40
	2	8 〃 50 〃					
	3	9 〃 10 〃					
	4	17 〃 47 〃					
	5	30 〃 38 〃					

Model

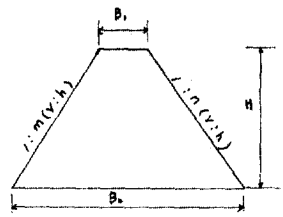

Table 2. Approximate time between theoretical equation and experimental result at a rising water level

Eq.	W.L	Time(t)	Eq.	W.L	Time(t)	Eq.	W.L	Time(t)
6	10	t<10 sec	9	10	52 sec <t<60 min	12	10	6 min<t<8 min
	15	〃		15	64 〃 <t<69 〃		15	3 〃 <t<4 〃
	20	13 min<t<15 min		20	71 〃 <t<73 〃		20	t=2 min
	25	25 〃 <t<30 〃		25	74 〃 <t<76 〃		25	t=90 sec
	30	36 〃 <t<40 〃		30	77 〃 <t<78 〃		30	t=40 sec
8	10	52 〃 <t<60 〃	10	10	31 〃 <t<35 〃			
	15	64 〃 <t<69 〃		15	19 〃 <t<23 〃			
	20	71 〃 <t<73 〃		20	14 〃 <t<16 〃			
	25	74 〃 <t<76 〃		25	12 〃 <t<14 〃			
	30	77 〃 <t<78 〃		30	10 〃 <t<11 〃			

* W.L; Water Level

in inverse proportion of X. On the otherhand equations (6) and (8) differ in inverse proportion of time and in proportion to the water level. The first sets of equations i.e. equations (2), (4) and (5) result in a higher effluence point in proportion of time. The other equations (6) and (8) result in lower effluence points.

To compare the results in equations (2), (4) and (5), a longer time should be used whereas in equation (6) and (8) time should be shorter. (For 30 cm rise it takes 36 minutes when equation (2) was used and 18 minutes when (4) and (5) was used).

The author felt it necessary to use equations (2), (4) and (5) in association with seepage time, and equations (6) and (8) with the time of water surface. Also it is required that one finds the seepage quantity using experimentally derived relationships.

3-3 Analytical Methods

A relation between seepage velocity and rate of rise of water level was also established. This equation is

$$V_R=\alpha V_S=\alpha K\frac{H-h}{d}$$

where V_R=rate of rise of water level a rising velocity$=\frac{dH}{dt}$

V_S=seepage velocity
α=constant of proportionality
H=upstream water level
h=height of seepage face
d=length of base of the embankment
K=permeability of soil

The constant of proportionality, α can be calculated provided all other dimensions including permeability is known. The relationship between the rising time T_R and the seepage time T_S, was expressed in the linear equation

$$T_S=A+BT_R$$

where A & B are constants. Seepage time T_S was computed from dividing the seepage length by seepage velocity. Seepage length is considered to be a straight line distance from inflow point to outflow point.

A correlation coefficient R between the rising time T_R and seepage time T_S was also calculated using the computer. This was done with respect to different water levels.

Based oe the above theoretical basis, methods of trial and error as well as least square methods were used to formulate the equation for position of seepage line. The experimental data generated the following equation.

$$Z=H-\alpha K\frac{1}{Ae^{BX}}\ \frac{x}{\sqrt{t}}$$

But this did not correlate with the theoretical equation very well. The least square method was called and the equation of the form

$$Z=H-\alpha K\frac{x}{\frac{1}{\frac{10}{H}e^{BX}}\cdot\sqrt{t}}$$

were derived. It was compared with the established theoretical equation based on experimental data from experiment EA_2. It was not satisfactory. The seepage time 't' was substituted equations (2), (4) and (5) by T_S which is $(A+BT_R)$. The theoretical and experimental equations were used in finding the seepage height and were compared with experimental data.

The experimental data disclosd a B value of -0.005 to -0.05. But a B value of -0.010 showed the best results. At the same time analysis was made to set up a relationship between theoretical and experimental equation. To facilitate this rigorous process, a computer program was used. Also the variables, H, t and x and z were changed each time. The α value was found to be within 0.12 to 0.50. Hence the term $\frac{x}{\sqrt{t}}$ in the experimental equation were substituted by

$$\frac{x^2}{t},\ \frac{x}{t},\ \frac{x^{1.5}}{t},\ \frac{x^{1.5}}{\sqrt{t}},\ \sqrt{\frac{x}{t}},\ \frac{x}{t/H}$$

and $\frac{x}{\sqrt{t/H}}$ etc.

None of these substitutions gave satisfactory comparisons either. Thus, the form of the equation was changed to

$z=H-\phi(H,t,x,\beta,k)$
and expressed as:

$$Z=H-\alpha\sqrt{\frac{\beta}{2K}}\cdot H\cdot\sqrt{\frac{x}{t}}$$

$$Z=H-\alpha\sqrt{\frac{\beta}{2K}}\cdot H\cdot\frac{\sqrt{x}}{\log t}$$

$$Z=H-\alpha\sqrt{\frac{\beta}{2K}}\cdot H\cdot\frac{\sqrt{x}}{e^{t}}$$

$$Z=H-\alpha\sqrt{\frac{\beta}{2K}}\cdot H\cdot\frac{\sqrt{x}}{e^{-t}}$$

The approximate equation is

$$Z=H-\alpha\sqrt{\frac{3}{2K}}\cdot H\cdot\frac{x^{1/2}}{t^{1/4}}$$

By adjusting the above equation an experimental formula was obtained, which is

$$Z=H-0.3\,H\frac{\sqrt{x}}{t^{1/4}}$$

From experimental data, the equation was again modified according to the variations of H and t. The new equation is of the form

$$Z=H-0.067(0.0024\,t+3.0579)\cdot H\cdot(AH+B)\frac{\sqrt{x}}{t^{1/4}}$$

where A & B are constants their values are between 0.014 and 1.04 and between 0.015 and 1.065

4. Experimental Results

4-1. The Seepage Line for Each Model

The results of the measured height of the seepage line from the base was the distance printed in the computational coding sheet for each model. The models are with the slopes of 1 : 2, 1 : 1.5, & 1 : 1.0. The distance along x-axis was used 10 cm althrough. One of the results for model with slopes of 1 : 2 is shown in table 3 and Fig. 2. The scale used in

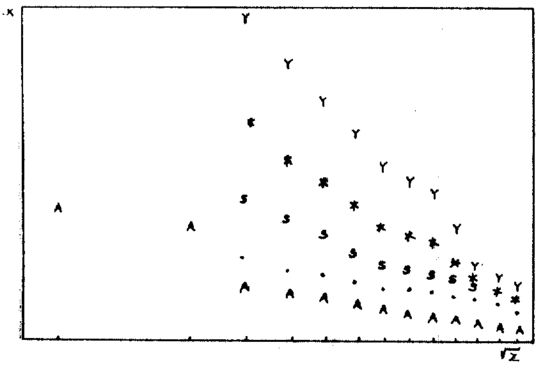

Fig. 2. The results of seepage lines for model 1 : 2.0.

Table 3. The height of saturation line in the model 1 : 2.0.

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037EA115.00 8.50 7.00 5.90 5.60 5.00 4.80 4.50 4.00 3.90 3.70 3.30 3.00 2.90 2.50  .00
128EA130.0017.1012.80 7.90 6.20 5.80 5.30 4.90 4.60 4.00 3.50 3.00  .00  .00  .00  .00

121EA210.00 7.70 6.80 6.00 5.50 5.00 4.60 4.30 4.00 3.80 3.50 3.00 2.90 2.50 2.00 1.70
215EA215.0011.40 9.40 8.00 7.00 6.50 5.70 5.20 4.70 4.10 3.80 3.30 2.80 2.40 2.10  .00
311EA220.0014.3012.00 9.80 8.70 7.40 6.50 6.00 5.80 5.50 4.60 4.00 3.50 2.60  .00  .00
413EA225.0017.2014.2012.3010.10 8.60 7.60 7.30 6.60 5.30 4.80 4.00 3.40  .00  .00  .00
541EA230.0020.6017.3014.8012.2010.40 9.00 8.50 6.90 5.50 5.00 4.50  .00  .00  .00  .00

2 4EA310.00 7.70 7.20 7.10 6.20 5.30 5.00 4.30 4.00 3.70 3.20 3.00 2.50 2.20 2.00 1.70
329EA315.0012.5011.6010.20 7.80 7.10 6.00 5.30 5.00 4.50 4.00 3.60 3.20 2.80 2.30  .00
516EA320.0017.2015.1010.7010.30 8.50 6.80 6.20 5.50 5.30 5.00 4.50 4.20 3.80  .00  .00
649EA325.0021.2016.6014.7012.3010.10 8.50 7.20 6.70 5.90 5.60 5.20 4.70  .00  .00  .00
842EA330.0023.1020.1017.3014.4012.1010.10 8.80 7.60 7.00 5.70 5.20  .00  .00  .00  .00

333EA410.00 8.30 6.80 5.80 5.30 4.70 4.40 3.90 3.50 3.00 2.90 2.50 2.40 2.10 2.00 1.80
549EA415.0011.8010.00 7.90 6.50 5.80 5.50 5.00 4.70 4.50 4.30 4.00 3.80 3.50 3.20  .00
817EA420.0015.1012.4010.60 9.10 8.00 6.90 6.30 5.90 5.60 5.00 4.70 4.50 4.20  .00  .00
1057EA425.0018.4015.4013.3011.7010.20 8.80 8.30 7.70 6.20 5.00 4.60 4.30  .00  .00  .00
14 0EA430.0022.6019.6016.9014.7012.8011.6010.00 8.30 5.90 5.80 4.60  .00  .00  .00  .00

749EA510.00 8.50 7.50 6.60 5.80 5.30 5.00 4.60 4.40 4.20 4.00 3.90 3.80 3.60 3.20 3.[illegible]
1144EA515.0013.3 11.5010.00 8.80 8.00 0.00 6.00 5.60 5.50 5.20 5.00 4.60 4.20  .00  .00
153[illegible]EA520.0016.5014.5012.9011.3010.00 8.60 8.00 7.50 7.00 6.00 5.00 4.60  .00  .00  .00
1933EA525.0020.5017.8016.2014.8012.7011.3010.80 9.60 8.00 6.20 5.30  .00  .00  .00  .00
2327EA530.0024.5022.4020.0018.0015.9014.8012.5410.50 7.20 6.00  .00  .00  .00  .00  .00

1159EA610.00 8.70 8.00 7.10 6.50 5.80 5.50 5.00 4.70 4.40 4.20 4.00 3.90 3.80 3.70  .00
1758EA615.0013.3011.5010.20 9.00 8.10 7.50 6.80 6.10 6.00 5.80 5.00 4.50 4.20  .00  .00
2357EA620.0017.0015.0013.5012.2011.0010.00 8.90 8.50 8.00 6.50 5.20 5.00 4.80  .00  .00
2957EA625.0020.9018.8017.0015.2013.5012.2011.8010.50 8.50 6.20 5.20  .00  .00  .00  .00
3556EA630.0025.0023.0020.8018.8016.6015.5014.0010.80 7.00 6.00  .00  .00  .00  .00  .00

2235EA710.00 9.20 8.20 7.50 6.80 6.20 6.00 5.50 5.00 4.50 4.20 4.00 4.00 3.90 3.00  .00
3353EA715.0013.0012.0010.70 9.80 9.00 8.20 7.50 7.00 6.50 5.80 5.00 4.00 3.30  .00  .00
4510EA720.0017.3016.0013.8012.8011.8010.50 9.70 9.00 7.80 6.50 5.00 5.00  .00  .00  .00
5628EA725.0022.0019.0017.5016.0014.5013.0012.0010.70 8.50 6.00 4.50  .00  .00  .00  .00
6745EA730.0025.2023.0021.0019.0017.0015.7014.0010.50 7.00 6.00  .00  .00  .00  .00  .00
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graphical representation by computer is (1 : 1, 1 : 1), (1 : 1, 1 : 1.5), (1 : 1, 1 : 10), (1 : 1.5, 1 : 1.5), (1 : 2, 1 : 2), (1 : 2, 1 : 4), and (1 : 10, 1 : 10) for (z, x) axis respectively. However, the results of other models as the graphical display will be presented in an investigation paper (the author is expressing apology for not printing table 4 and 5).

4-2. Seepage Line with Varying Water Level

The height of seepage line for varying water levels with distance is shown in table 2. The model had a slope of 1 : 1.5. The results of other models with slopes 1 : 2.0 and 1 : 1.0 are not shown here. The reason is the same as in 4-1. The scale used for the computer graphical display was (1 : 1, 1 : 1), (1 : 1, 1 : 1.5), (1 : 1, 1 : 10), (1 : 1.5, 1 : 1.5), (1 : 2, 1 : 2), (1 : 2, 1 : 4) and (1 : 10, 1 : 10) for (z, x) axis respectively.

5. Discussion of Results

5-1. Analysis of Experimental Results

To approximate relationship between infiltration phenomenon in embankment and rates of rise of upstream water level, an equation of the form

$$f(x, h, z)=0$$

was obtained. Here x, h and z are horizontal distance, water level and vertical height of seepage line respectively. This has been approximated by using the results of the experiment, movie camera, and analytical methods.

From the results of the analysis of the correlation coefficient R as stated above, we know that the case of EA_2, FA_3 and EA_4 for the slope 1 : 2.0 was a powerful negative. The case of EC_2, EC_3 and EC_5 for the slope 1 : 1.0 was negative and the case of EC_4 was a powerful positive, the case of EA_4, EA_6 and EA_7 for the slope 1 : 2.0 was in relation of a weak positive.

From the results of the analysis of the analysis of the correlation coefficient R for each model, we know that the case of the slope 1 : 1.0 was in relation of a powerful positive. The case of slope 1 : 2.0 and 1 : 1.5 was in relation of a weak positive. The results of the relation R was plotted at Table 6.

Table 4. The correlation coefficient for models

Model	R
1 : 2.0	−0.0664
1 : 1.5	−0.1782
1 : 1.0	0.9212

5-2. Relation Between Seepage Velocity (vs) **and Rising Velocity of Water**

5-2-1 Rising velocity of water

V_R, is given by:

$$V_R=\frac{dH}{dt}$$

To calculate this velocity, the difference in water level is divided by the time period to attain the new height of water level. If the rising water velocity is slower, then more time is available for seepage line to rise to a higher elevation. That is the height of the seepage line is inversely proportional to the rate of rise of the water level. It is directly proportional to the rising time.

5-2-2 Seepage Velocity

V_S, is given by:

$$V_S=k\frac{H-h}{d}$$

Seepage velocity is directly proportional to H and inversely proportional to d. It can be concluded from this that the height of the seepage line is inversely proportional to vs or seepage velocity. The higher the seepage velocity is, the lesser will be the height attained by the seepage line.

The relation between seepage velocity and rate of rise of water level was established as

$$V_R=\alpha V_S=\alpha k\frac{H-h}{d}$$

The value of α for different mode was found to be:

$\alpha=0.51437354$ for slope 1 : 2

$\alpha=0.36134154$ for slope 1 : 1.5

$\alpha=0.16240039$ for slope 1 : 1

Considering a linear relationship between V_S & V_R of the form

$$V_S=A+BV_R,$$

the following values of constants A & B were obtained (they are shown in table 7 & 8 in details for various experimental conditions):

1 : 2		1 : 1.5		1 : 1.0	
A	B	A	B	A	B
0.0473	−0.0566	0.0465	−0.019	0.0525	−0.0504

From this table it is possible to comment that as rate of rise of water level increases, there is a fall or decrease in seepage velocity i.e. the height of seepage line attained will be decreased.

Also from the first equations of this relation between velocity of rising water and seepage velocity, it is shown that the constant of proportionality α is inversely proportional to water level rise. α is inversely proportional to time.

Table 5. The relationship between water level H and hydraulic gradient i for experimental condions

Model 1 : 2.0

H \ M	EA 2	EA 3	EA 4	EA 5	EA 6	EA 7
i	i_2	i_3	i_4	i_5	i_6	i_7
10	0.057	0.057	0.056	0.049	0.045	0.051
15	0.095	0.094	0.088	0.083	0.082	0.090
20	0.138	0.130	0.128	0.128	0.126	0.134
25	0.191	0.184	0.186	0.181	0.178	0.186
30	0.252	0.249	0.252	0.243	0.245	0.247

Model 1 : 1.5

H \ M	EB 2	EB 3	EB 4	EB 5
i	i_2	i_3	i_4	i_5
10	0.029	0.033	0.034	0.037
15	0.042	0.043	0.045	0.046
20	0.048	0.051	0.053	0.056
25	0.056	0.067	0.080	0.083

Model 1 : 1.0

H \ M	EC 2	EC 3	EC 4	EC 5
i	i_2	i_3	i_4	i_5
10	0.046	0.056	0.065	0.071
15	0.066	0.068	0.077	0.091
20	0.076	0.083	0.093	0.108
25	0.102	0.109	0.130	0.171
30	0.173	0.182	0.208	0.275

* M : Model

Table 6. Constant α for experimental conditions

Model 1 : 2.0

H \ M	EA 2	EA 3	EA 4	EA 5	EA 6	EA 7
10	2.65	1.70	1.22	1.70	0.38	0.23
15	1.58	1.03	0.77	0.41	0.21	0.13
20	1.09	0.74	0.52	0.27	0.27	0.14
25	0.78	0.53	0.37	0.17	0.10	0.06
30	0.59	0.39	0.27	0.14	0.07	0.05

Model 1 : 1.5

H \ M	EB 2	EB 3	EB 4	EB 5
10	3.52	2.06	1.00	0.41
15	2.43	1.58	0.76	0.33
20	2.13	1.34	0.64	0.27
25	1.82	1.02	0.43	0.18
30	1.29	0.78	0.34	0.15

Model 1 : 1.0

H \ M	EC 2	EC 3	EC 4	EC 5
10	2.38	1.49	0.79	0.48
15	1.55	1.25	0.66	0.38
20	1.34	1.03	0.55	0.32
25	1.00	0.78	0.39	0.20
30	0.58	0.47	0.25	0.12

5-3. Relation Between Rising Velocity and Seepage Time

The relation between rising time and seepage time was expressed by a linear formula

$$T_S = A + BT_R$$

where T_S=seepage time

T_R=rising time of water surface

A & B are constants.

The seepage time is obtained from the seepage length divided by seepage velocity. The seepage length is considered to be a straight line distance from the influx point to effluence point.

The values of the constants A & B for T_S and T_R are shown in Table 9. The average values of these constants and the equations are shown in Table 10.

The results of the constants A and B showes that as time is increased constant B is decreased and A is increased.

Again for an increased slope the values of the constants are decreased. The seepage time is again longer for a flatter slope and also rising time is longer.

Table 7. Constant A and B related with rising time and infilteration time for experimental conditions

1 : 2.0			1 : 1.5			1 : 1.0		
M	A	B	M	A	B	M	A	B
2	4685.0664	−13.3440	2	3097.7285	−6.4315	2	1468.4907	−2.7711
3	4808.4238	− 8.7923	3	3281.5510	−3.9466	3	1697.6001	−3.1469
4	4977.6719	− 5.6984	4	3238.5464	−2.1286	4	1627.0354	−1.5895
5	6371.4093	− 4.4238	5	3057.5073	−0.9138	5	1777.4846	−1.0234
6	6824.0674	− 3.1559						
7	6106.6367	− 1.4643						

* M : Model

Table 8. Constant A and B for each model

1 : 2.0		1 : 1.5		1 : 1.0	
A	B	A	B	A	B
2636.8574	−0.5012	1804.5837	−0.5571	981.8782	−0.6179

5-4. Analysis and Consideration of Experimental Data

The experimental results showed that the formula of the form

$$Z = A + BX$$

and $$Z = Ae^{BX}$$

were not agreeable. So the other parabolic equations of the form

$$Z = H - \alpha K \frac{10}{H} e^{BX} \frac{x}{\sqrt{t}}$$

were tried. But this formula has the inherent weakness in that the 'x' is expressed as linear. In actulity the height of seepage line is not linear rather parabolic. So the idea to analyze it was dropped at this point.

The next trial was with the equation

$$Z = H - \alpha K \frac{x^{1/2}}{t^{1/4}}$$

and seemed to be closer but no exactly accurate.

The third trial was with the equation

$$Z = H - \alpha \sqrt{\frac{3}{2K}} \sqrt{H} \frac{\sqrt{x}}{t^{1/4}}$$

The results are much better but not upto the exact correctness.

Finally, the authors came up with the following two equations:

$$Z=H-0.067(0.0024\,t+2.0579)\cdot H\cdot (AH+B)\frac{\sqrt{x}}{t^{1/4}}$$

and values of A & B are (0.014, 1.01)and (0.015, 1.064) respectively.

6. Conclusions

1. Rising velocity of the water level and seepage velocity are approximately proportional to each other if the slope is slow.
2. The rising velocity of water level is equal to seepage velocity at the slope of 1 : 3.3 (Vih).
3. The rising time at which $V_R=Vs$ in each model is different for different water levels are

   $t=11.20H^2-158.5H+2006.31$ for the model 1 : 2.0

   $t=5.59H^2-110.2H+731$ for the model 1 : 1.5

   $t=7.4H^2-167H+1117$ for the model 1 : 1.0 respectively.
4. $V_R=V_S$ for each model is given by:

   $$V_R=\frac{t-200006.31}{t(11.20H-158.5)}$$ for the model 1 : 2.0

   $$V_R=\frac{t-731}{t(11.20H-110.2)}$$ for the model 1 : 1.5

   $$V_R=\frac{t-1117}{t(7.4H-167)}$$ for the model 1 : 1.0
5. Experimental formulas are:
   1. $Z=H-0.067(0.0024t+3.0579)\ H\ (0.014H+1.01)\frac{x^{1/2}}{t^{1/4}}$
   2. $Z=H-0.067(0.0024t+3.0579)\ H\ (0.015H+1.065)\frac{x^{1/2}}{t^{1/4}}$

These are obtained mostly in model 1 : 2.0 and they should be modified slightly for different slope.

Acknowledgement

Thanks are due to the Korean Science and Engineering Foundation for its financial support in the preparation of this paper.

Additional thanks are also due to Professor C.E. O'Bannon in Civil Engineering at Arizona State University, Professor A. Murota at Osaka National University and also all the student researchers who have assisted in the completion of this paper.

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Appendix : Development of Analysis of Unsteady Flow

1. Schmied

By means of model test using the manometer to measure the seepage state during gradual rising elevation of one-side water surface in model embankment two-dimensional analytical equations were published by Schmied.

$$x=\{(h^2-y^2)/\sqrt{(h^3-y^2)}\}\sqrt{(3/4)(\varepsilon/\mu)t} \qquad (1)$$

where, x is horizontal length
y is water level of manometer
h is water level of outside of embankment
t is seepage time
ε and μ constant for a given material

2. Ujida

For unsteady flow of rectangular embankment during rapid rising of the water level with free water surface, seepage analysis of sand with the impermeable bottom was published by Ujida. This method was used for numerical calculation with boundary conditions:

$$\frac{z_f}{H}=1-\left\{\frac{x_f/H}{\sqrt{8/3a}\sqrt{kt/H}}\right\}^{3/2}=1-\frac{3}{2}^{3/4}\left\{\frac{x_{f/l}}{2\sqrt{T}}\right\}^{3/2} \qquad (2)$$

$$T=\frac{kHt}{al^2}$$

$$\frac{Zf}{H}=1-\frac{\beta}{2kH}\ \frac{x_f{}^2}{t}\equiv 1-2\left\{\frac{x_{f/l}}{2\sqrt{T}}\right\}^2 \qquad (3)$$

where, β is effective porosity
a is porosity of soil
z_f is the height from the impermeable bottom to free water level
x_f is the length
t is the seepage time
h is the water level of outside of embankment.

3. Approximate analysis concerning variations of free water surface (rising)

$$\frac{z_f}{H}=1-\frac{\beta}{2KH}\ \frac{x_f{}^2}{t} \qquad (4)$$

where, β is storage factor
x_f is the length from boundary to the point of interest
z_f is the hieght from the point of consideration to free water surface
t is seepage time
and
$\beta\fallingdotseq n$
$\beta=0.87n-\frac{Y_d}{Y_w}\times\frac{w}{100}$
n is the porosity
Y_d is the dry density
Y_w is the unit weight of water
w is the water content

4. Approximate analysis with impermeable bottom (rapid rising)

The relation x_f and t;

$$x_f=\sqrt{\frac{2k\ H}{\beta}t}$$

$$\frac{z}{H}=1-\sqrt{\frac{\beta}{2kH}}\cdot\frac{x_f}{\sqrt{t}} \qquad (5)$$

where, β is the storage factor

x_f is the length from boundary to the point of consideration

t is the seepage time

k is the permeability

z is the height from the point of consideration to free water surface

h is the height of water levle of embankment

5. On the regulated rising of water level of embankment with impermeable bottom

$$z=\mu_0 t-x\sqrt{\mu_0/(k/\beta)} \quad \cdots\cdots(6)$$

where, μ_0 is the velocity of regulated water level

x is the length from end to the point of consideration

k is the permeability

β is the storage factor or the porosity

t is the rising time

But, above equation is applicate to the range

In case of $x_f \lneqq x$, $z=0$.

$$x_f=\sqrt{(k/\beta)}\ v_0 \times t \quad \cdots\cdots(7)$$

6. Approximate analysis in case of the regulated rising water level

$$z=(M\mu t-x)/(M-N) \quad \cdots\cdots(8)$$

where, $M=N+\sqrt{N^2+4P}$

$N=C_0\ t\ \alpha$

$P=(k/\beta)/\mu$

α is the angle of slope of gradient surface

μ is the velocity of regulation rising

(接受：1984. 11. 8)