〈研究論文〉

Journal of the Korean Institute of Industrial Engineers
Vol. 11, No. 2, Dec. 1985

A Two-Product Three-Facility Production Planning Model in a Combined Parallel and Serial System

C. S. Sung*
B. J. Lee*
Y. J. Lee*

Abstract

This paper considers a two-product three-facility production planning model, where facility 1 produces product 1 to satisfy its own market requirements and supplies input to facility 2, and facility 2 requires another input from facility 3 (outside supplier). The objective is to determine the optimal production amount in each period in order to satisfy the dynamic demands on time, which minimizes the total cost of production and storage. The set-up cost is incurred jointly from the multi-facility operations.

1. Introduction

This paper is concerned with a two-product three-facility production planning over a finite planning interval of n periods, where facility 2 needs two resources, one supplied from facility 1 and the other one ordered from outside (facility 3). Therefore, the production system is a kind of assembly system of three facilities in which facility 1 produces product 1 to satisfy its own market requirements and supplies input to facility 2, and facility 2 assembles semi-products from facility 1 and outside into product 2. That is, one unit of product 1 and one unit of the other material ordered from outside are assembled into one unit of product 2. This assembly system is depicted in Figure 1.

* Korea Advanced Institute of Science and Technology

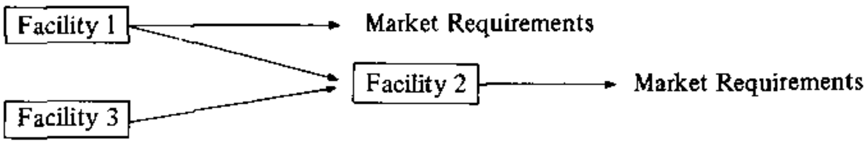


Figure 1. The assembly system of three facilities.

The associated problem is to determine the amounts to be produced in each period at each facility in order to satisfy each of the demands on time, which minimizes the situation of joint set-up cost.

A production model that manufactures a single product to satisfy known demands over a finite planning interval of n periods without backlogging was studied by Wagner and Whitin [4]. Zangwill [5] characterized the dominant set for a general multi-facility model, which is a linking together of certain single facility models to form an acyclic network. Kao [1] suggested a dynamic programming formulation for finding the optimal production policy which calls for a smaller state space than that proposed by Zangwill [5]. Recently, Luss [3] described a multi-facility capacity expansion model in which each facility represented each different quality level, where joint set-up was assumed in each period.

In this paper, the dominance property of an optimal solution and a modified version of Kao's shortest path algorithm to search a solution for the aforementioned model with joint set-up cost will be derived.

2. Model Formulation

Required notations and assumptions are described as follows: for each period i, $i = 1, 2, \ldots, n$, and facility j, $j = 1, 2, 3$ (Where facility 3 represents the source for procurement from outside),

r_i^j = known demand in period i, facility j, $j = 1, 2$

x_i^j == amount of product manufactured (ordered) at facility j in period i

I_i^j = inventory at the end of period i for product at facility j in period i

h_i^j = holding cost per unit for product at facility j

p_i^j = cost of producing (ordering) one unit of product at facility j

r_j = n period demand vector for facility j

H_i^j = linear holding cost function such that

$$H_i^j (I_i^j) = h_i^j I_i^j.$$

Assuming that the production (order) costs are constant through all the periods (i.e., $p_i^j = p^j$ for all i) for each facility, the linear portion of the production costs can be neglected, since the total production amount for each facility by the end of period n must satisfy exact market requirements: that is,

$$\sum_{i=1}^{n} x_i^1 = \sum_{i=1}^{n} (r_i^1 + r_i^2), \quad \sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} r_i^2 \text{ and } \sum_{i=1}^{n} x_i^3 = \sum_{i=1}^{n} r_i^2$$

In this system, we treat a production planning model in which the joint set-up cost is incurred from multi-facility operations. This means that no matter how many facilities are operated in each period, a fixed set-up cost is incurred. Thereby, the proposed model assumes that the joint set-up cost is incurred in each joint operation of facility 1 and facility 2. But the set-up cost in facility 3 is assumed to be independently incurred since it is originally concerned with the ordering cost from the outside supplier.

Let s_i be the joint set-up cost in period i and s_i^3 be the set-up cost of facility 3 in period i. Also, let $C_i(x_i^1, x_i^2)$ be the production cost associated with the production amounts of facility 1 and facility 2, and $P_i^3(x_i^3)$ be the production cost of facility 3. Then it follows that

$$C_i(x_i^1, x_i^2) = s_i\, d(x_i^1 + x_i^2),$$

$$P_i^3(x_i^3) = s_i^3\, d(x_i^3),$$

where $d(x)$ is 1 if x is positive, or 0 otherwise.

Based on the above notations and assumptions the production planning model is formulated as follows:

$$\text{(OP)} \min F(X) = \sum_{i=1}^{n} \{C_i(x_i^1, x_i^2) + P_i^3(x_i^3)\} + \sum_{j=1}^{n} \sum_{i=1}^{n} H_i^j(I_i^j) \quad (1)$$

subject to

$$I_{i-1}^1 - r_i^1 + x_i^1 - x_i^2 = I_i^1 \quad (2)$$

$$I_{i-1}^2 - r_i^2 + x_i^2 = I_i^2 \quad (3)$$

$$I_{i-1}^3 - x_i^2 + x_i^3 = I_i^3 \quad (4)$$

$$I_0^1 = I_0^2 = I_0^3 = I_n^1 = I_n^2 = I_n^3 = 0 \quad (5)$$

$$I_i^1,\ I_i^2,\ I_i^3,\ x_i^1,\ x_i^2,\ x_i^3 \geqq 0 \quad (6)$$

for $i = 1, 2, \cdots\cdots, n$.

The constraints of the problem (OP) can be depicted as in figure 2 which forms a two-source network. Note that x_i^2 (p) and x_i^2 (o) denote the inflows from facility 1 and outside respectively, and that x_i^2 (p) = x_i^2 (o) = x_i^2 for all i.

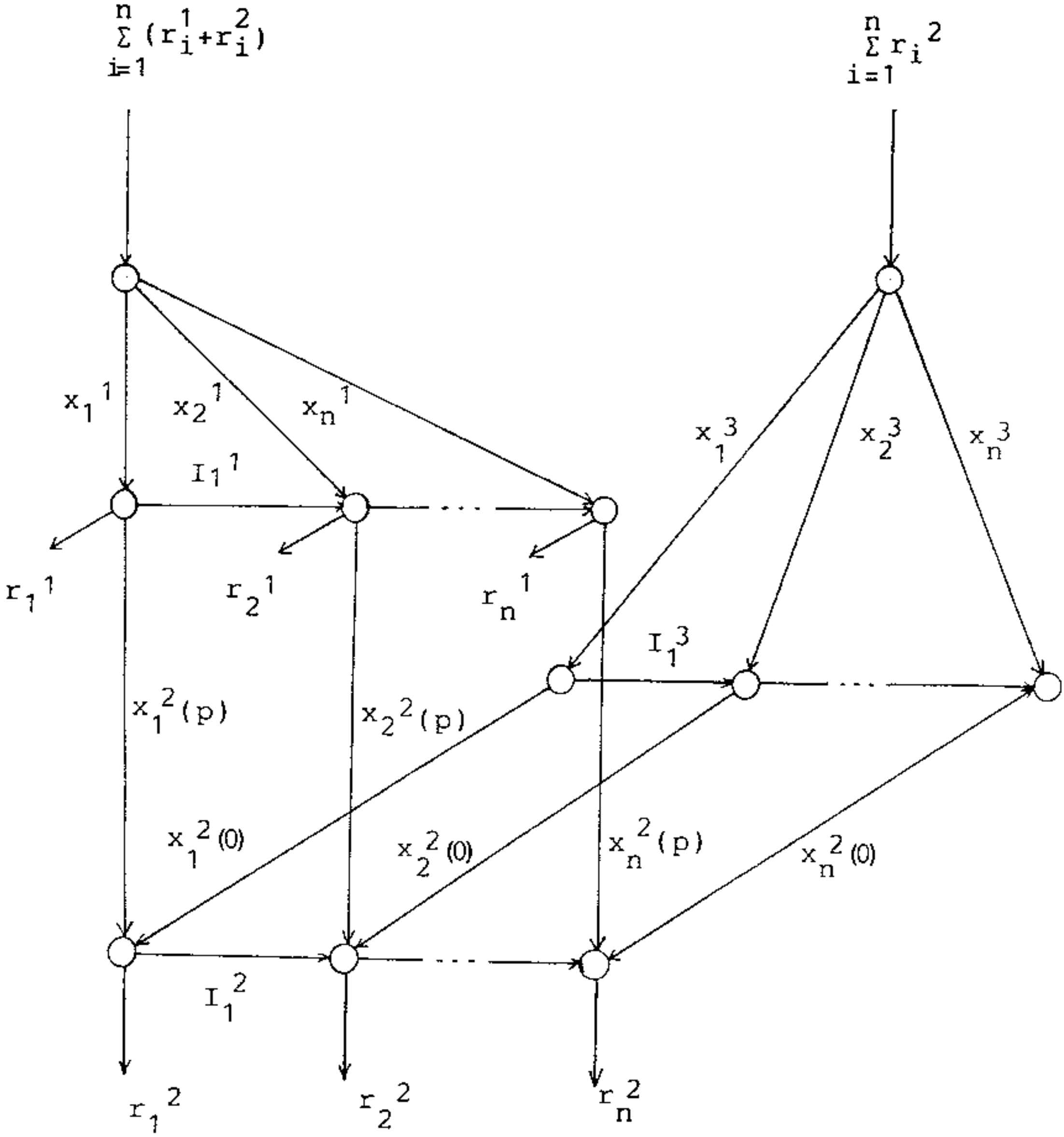


Figure 2. The two-source network

In the following sections we will consider the production planning model under the assumptions that no backlogging is allowed and lead time is negligible.

3. Characterization of Extreme Points

The constraints of the problem (OP) form a compact convex set and the objective function is concave. Therefore, the problem attains its minimum at an extreme point of the set.

Let D denote the set of all extreme points of the solution set. We introduce a production sequence satisfying exact requirements which forms the basis for our characterization of D. A partial production vector $X^j = (x_1^j, x_2^j, ..., x_n^j)$, $j = 1, 2, 3$, of entire production plan $X = (X^1, X^2, X^3)$ is said to satisfy "exact requirements" if there exist integers a_i $(i = 1, 2, ..., n)$ satisfying $0 = a_0 < a_1 < \dots < a_n = n$ such that $x_t^j = R_{a_t}^j - R_{a_{t-1}}^j$ $(j = 1, 2, 3; t = 1, 2, ..., n)$, where $R_m^1 = \sum_{i=1}^{m} (r_i^1 + r_i^2)$ (this equation can be justified in Theorem 2) and $R_m^2 = \sum_{i=1}^{m} r_i^2 = R_m^3$. Furthermore, if X^j satisfies exact requirements, then each x_t^j $(t = 1, 2, ..., n)$ is said to satisfy exact requirements.

Theorem 1.

All partial production vector X^j's $(j = 1, 2, 3)$ of a feasible entire production vector $X = (X^1, X^2, X^3)$ satisfy exact requirements iff it is in D.

Proof.

Its proof is straightforward in view of the nested schedule of Love [2].

The result of Theorem 1 implies that an optimal solution is composed of the so-called exact requirement sequences derived in Love [3]. This is more specifically described in Corollary 1.1.

Corollary 1.1.

The optimal production vector $X = (X^1, X^2, X^3)$ consists only of the components x_i^j's such that $I_{i-1}^j \cdot x_i^j = 0$ for $j = 1, 2, 3; i = 1, 2, ..., n$.

Theorem 1 and Corollary 1.1 indicate that even if the problem forms a two-source multi-destination network, the nested schedule property still holds for all the three facilities.

The joint set-up cost assumption may lead to the result that operating two facilities simultaneously is better than that of one facility. This will be verified in Theorem 2.

Theorem 2.

Consider an extreme flow of the network for the joint set-up cost system with $r_i^1 > 0$ and $r_i^2 > 0$ for all i. Then it holds that $I_i^1 = 0$ iff $I_i^2 = 0$.

Proof.

Let $C_{i+1}(x_{i+1}^1, x_{i+1}^2, x_{i+1}^3)$ be the production and inventory cost function at period i+1. Then it follows that

$$C_{i+1}(x_{i+1}^1, x_{i+1}^2, x_{i+1}^3)$$

$$= s_{i+1} d(x_{i+1}^1 + x_{i+1}^2) + s_{i+1}^3 d(x_{i+1}^3) + h_i^1 I_i^1 + h_i^2 I_i^2 + h_i^3 I_i^3.$$

Now, suppose that $I_i^1 = 0$, but $I_i^2 > 0$. Then, this implies that $I_i^2 > 0$ affects negatively on the total cost, since the joint set-up at period i+1 can take care of the associated inventory amount. Thus, the proof is completed.

4. Dynamic Programming Algorithm

Kao [1] suggested an efficient algorithm to solve the shortest path problem for the parallel facility model. To find an optimal policy for our model, we will construct an algorithm which is a modification of Kao's one. We formulate the problem as a shortest path network whose nodes correspond to the regeneration points. Dynamic programming is used to solve the network problem.

It follows from Theorem 2 that the optimal solution has common regeneration points for product 1 and product 2. Let m_1 denote a common regeneration point of the two products and m_2 denote a regeneration point for the ordered-from-outside product, where $m_i \in \{0, 1, ..., n\}$ for $i = 1, 2$. Let $m = (m_1, m_2)$ and $m' = \min\{m_1, m_2\}$ denote a node in the associated network and a stage index for the associated dynamic programming procedure, respectively. With N_t denoting the set of nodes to be generated at stage t ($t = 1, 2, ..., n$), we introduce the following node generation procedure.

⟨Node Generation Procedure⟩

Step 1: At stage 0, $N_0 = \{(0, 0)\}$

Step 2: At stage $t \in \{1, ..., n-1\}$, we generate N_t as follows:

(a) $N_t^1 = \{(t, t), (t, t+1), ..., (t, n)\}$

(b) $N_t^2 = \{(t+1, t), (t+2, t), ..., (n, t)\}$

(c) $N_t = N_t^1 \cup N_t^2$

(d) If $t < n$, go to (a) with $t+1 \to t$. Otherwise go to Step 3.

Step 3: At stage n, $N_n = \{(n, n)\}$

In a forward recursion, when a node k is generated at a given stage, we need to know the arcs leading into it. The next arc generation procedure can confirm a node set M_k, where M_k represents the set of nodes from which these arcs emanate to node k.

⟨Arc Generation Procedure⟩

Step 1: For node $k = (k_1, k_2)$ in which $k' = \min(k_1, k_2) = 1$,
we have $M_k = \{(0, 0)\}$. For other nodes, go to Step 2.

Step 2: For node $k = (k_1, k_2)$ in which $2 \leq k' \leq n$

$$M_k = \{m = (m_1, m_2) \mid m_i \in \{1, ..., k'-1, k_i\} \quad \text{for } i = 1, 2\} \cup \{(0,0)\} - \{(k_1, k_2)\}$$

Step 2 of the above procedure leads to that node m has an arc leading into node k only if $m' < k'$ and $m_i \in \{m', k_i\}$

〈Recursive Relation〉

Having constructed the above acyclic network, we need to compute each arc size in cost, say C_{mk}:

$$C_{mk} = C_{m_1}\left(\sum_{t=m_1+1}^{k_1} R_t,\ \sum_{t=m_1+1}^{k_1} r_t\right) + \sum_{j=m_1}^{k_1}\left(H_j^1\left(\sum_{t=j}^{k_1} R_t\right) + H_j^2\left(\sum_{t=j}^{k_1} r_t^2\right)\right)$$
$$+ P_{m_2}^3\left(\sum_{t=m_2+1}^{k_2} r_t\right) + \sum_{t=m_2+1}^{k_2} H_j^3\left(\sum_{t=j}^{k_2} r_t^2\right),$$

where C_{mk} is the cost associated with the arc from node m to node k and $R_t = r_t^1 + r_t^2$. Then, we can solve the shortest path problem using the forward recursion

$f_k = \min \ \{ f_m + C_{mk} \}$,

$f_{(0,0)} = 0$,

where f_k is the cost associated with the optimal solution from node 0 to node k.

〈Solution Procedure〉

Step 1: Set m = (0, 0) and $f_{(0,0)} = 0$

Step 2: For stage k' in which $1 \leqslant k' \leqslant n$

(a) generate M_k for all nodes at stage k'

(b) compute C_{mk} and f_k, and check node m_k among M_k that minimizes f_k.

Step 3: $k' + 1 \rightarrow k'$ and go to Step 2 until f_n is obtained.

5. A Numerical Example

Consider the following three period situation. Let facility 1 have market requirement $r^1 = (2,3,4)$ and facility 2 have $r^2 = (1,2,1)$. The associated cost functions are given as follows:

$C_i(x_i^1, x_i^2) = s_i d(x_i^1 + x_i^2)$, $(i = 1, 2, 3)$,

where $s_1 = 3$, $s_2 = 4$, $s_3 = 2$, and

$P_i^3(x_i^3) = s_i^3 d(x_i^3)$, $(i = 1, 2, 3)$,

where $s_1^3 = 3$, $s_2^3 = 2$, $s_3^3 = 4$. Furthermore,

$H_i^1(I_i^1) = 2I_i^1$, $H_i^2(I_i^2) = 3I_i^2$, and $H_i^3(I_i^3) = 2I_i^3$, $(i = 1, 2, 3)$.

Then, based on the results of section 4, we can summarize the calculations as follows:

(1) At stage 0

m = (0, 0) and $f_{(0,0)} = 0$.

(2) At stage 1

(a) $M_{(1,1)} = \{(0,0)\}$

$C_{(0,0)(1,1)} = 6$

$f_{(1,1)} = \min\{f_{(0,0)} + C_{(0,0)(1,1)}\} = 0 + 6 = 6$

(b) $M_{(1,2)} = \{(0,0)\}$

$C_{(0,0)(1,2)} = 10$

$f_{(1,2)} = \min\{f_{(0,0)} + C_{(0,0)(1,2)}\} = 0 + 10 = 10$

(c) $M_{(1,3)} = \{(0,0)\}$

$C_{(0,0)(1,3)} = 14$

$f_{(1,3)} = \min\{f_{(0,0)} + C_{(0,0)(1,3)}\} = 0 + 14 = 14$

(d) $M_{(2,1)} = \{(0,0)\}$

$C_{(0,0)(2,1)} = 12$

$f_{(2,1)} = \min\{f_{(0,0)} + C_{(0,0)(2,1)}\} = 0 + 12 = 12$

(e) $M_{(3,1)} = \{(0,0)\}$

$C_{(0,0)(3,1)} = 40$

$f_{(3,1)} = \min\{f_{(3,1)} + C_{(0,0)(3,1)}\} = 0 + 40 = 40$

(3) At stage 2

(a) $M_{(2,2)} = \{(0,0), (1,1), (1,2), (2,1)\}$

$C_{(0,0)(2,2)} = 20, C_{(1,1)(2,2)} = 6, C_{(1,2)(2,2)} = 4$

$C_{(2,1)(2,2)} = 2$

$f_{(2,2)} = \min\{0+20, 6+6, 10+4, 12+2\} = 12$

$m_{(2,2)} = (1,1)$

(b) $M_{(2,3)} = \{(0,0), (1,1), (1,3), (2,1)\}$

$C_{(0,0)(2,3)} = 26, C_{(1,1)(2,3)} = 8, C_{(1,3)(2,3)} = 4$

$C_{(2,1)(2,3)} = 2$

$f_{(2,3)} = \min\{0+26, 6+8, 14+4, 12+2\} = 14$

$m_{(2,3)} = (1,1)$ or $(2,1)$

(c) $M_{(3,2)} = \{(0,0), (1,1), (1,2), (3,1)\}$

$C_{(0,0)(3,2)} = 44, C_{(1,1)(3,3)} = 17, C_{(1,3)(3,2)} = 15,$

$C_{(3,1)(3,2)} = 2$

$f_{(3,2)} = \min\ \{0+44, 6+17, 10+5, 40+2\} = 23$

$m_{(3,2)} = (1,1)$

(4) At stage 3

$M_{(3,3)} = \{(0,0), (1,1), (1,3), (3,1), (2,2), (2,3), (3,2)\}$

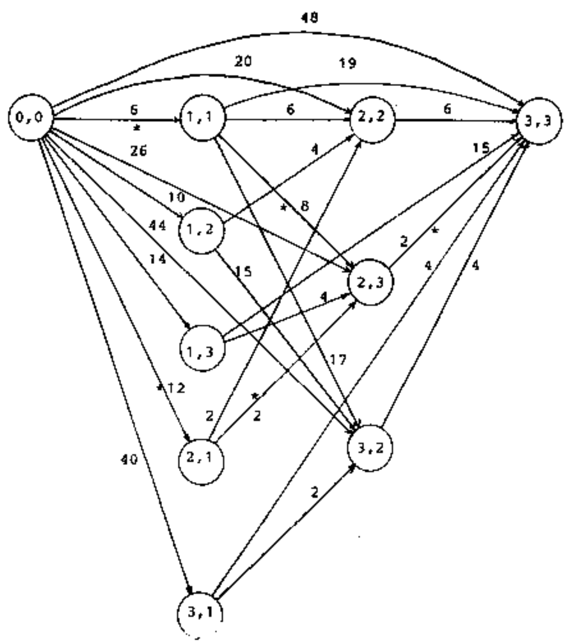
$C_{(0,0)(3,3)} = 48, C_{(1,1)(3,3)} = 19, C_{(1,3)(3,3)} = 15,$

$C_{(3,1)(3,3)} = 4, C_{(2,2)(3,3)} = 6, C_{(2,3)(3,3)} = 2,$

$C_{(3,2)(3,3)} = 4$

$f_{(3,3)} = \min\ \{0+48, 6+19, 14+15, 40+4, 12+6, 14+2, 23+4\} = 16$

$m_{(3,3)} = (2,3)$



* denote the shortest path

Figure 3. An acyclic network for n=3.

We see that the shortest path is (0,0) – (1,1) – (2,3) – (3,3) or (0,0) – (2,1) – (2,3) – (3,3), yielding the optimal production policy X = (3,5,5; 1,2,1; 1,3,0) or (8,0,5; 3,0,1; 4,0,0) and the minimum total cost excluding production cost f (3,3) = 16. The acyclic network and its solution are shown in figure 3.

6. Conclusion

In this paper we discussed how to find the optimal solution of production planning problem with the joint set-up cost. The problem was formulated as a shortest path problem in which nodes correspond to the regeneration points and a dynamic programming algorithm was presented based on Kao's approach of providing a smaller state space and fewer arcs.

References

[1] Kao, E.P.C., "A Multiproduct Dynamic Lot-size Model with Individual and Joint Set-up Costs", Operations Research, Vol. 27, p. 279-289 (1979).

[2] Love, S.F., "A Facilities in Series Inventory Model with Nested Schedules", Management Science, Vol. 18, p. 327-338 (1972).

[3] Luss, H., "A Multifacility Capacity Expansion Model with Joint Expansion Set-up Costs", Naval Research Logistics Quarterly, Vol. 30, p. 97-111 (1983).

[4] Wagner, H.A. and Whitin, T.M., "Dynamic Version of the Economic Lot Size Model", Management Science, Vol. 5, p. 89-96 (1959).

[5] Zangwill, W.I., "A Deterministic Multiproduct Multifacility Production and Inventory Model", Operations Research, Vol. 14, p. 486-507 (1966).