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□論 文□

Self-Consistent Parameter Calibration of Combined Mode and Route Choice Model

交通手段 및 路線決定結合模型의 係數導出을 위한 研究

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目 次

要 約

本 研究는 「네트웍」 평형모형을 利用하여 一般費用函數(Generalized Cost function)의 模型係數를 決定하는 하나의 方法을 提示한다. 이러한 技法은 同一模型을 實現하기 위해서는 必然的인 過程이다. 既存의 大部分의 方法은 都市通行上 發生되는 通行者의 諸費用을 直接 觀測하고 蒐集하여 模型係數를 決定하였지만 本 研究에서는 이미 活用可能한 起終點別 通行實態調査를 利用하여 通行費用을 내생적으로 決定하고 이에 따른 模型係數를 抽出한다. 따라서 本 研究는 資料蒐集上에 惹起될 수 있는 諸般 問題點의 解決과 同 一目的의 産出過程을 단순화시키고 內的一貫性을 追求하였다는 데에 意義가 있다. 研究의 妥當性 檢討를 위해 「시카고」 地域에 本 模型과 方法論을 適用하고 結果를 分析하였다.

I. Introduction

During the last three decades, predicting an equilibrium of the transportation market has been regarded as necessary for the study of many transportation problems such as evaluation of operating procedures for urban and regional transportation systems, evaluation of traffic management in heavily congested urban road systems, decisions regarding public transportation fares and parking prices, evaluation of new transportation services, network design problems and so forth. This requirement reflects an increasing concern with traffic congestion, long regarded as one of the most difficult

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problems of urban settlements.

The research presented here is concerned with several issues that arise when such an equilibrium model is implemented. It concerns the estimation of generalized cost function coefficients from survey data under the hypothesis that the travel choices correspond to the assumptions of the network equilibrium model. A maximum likelihood approach to this problem is evaluated analytically and computationally for the Chicago region.

The classical view of an economic market for a certain good involves two competitive groups: the producers and the consumers. The behavior of the producers is represented by a supply function and the behavior of the consumers is characterized by a demand function. By competing with each other in terms of the amount of the good and its price, the market equilibrium is determined. In transportation markets, the producers include road service agencies, transit operators, traffic managers, and so on; their product may be regarded as the level of service in the transportation system. Transportation level of service can be measured in terms of travel time, cost, convenience, reliability, safety, comfort, and other factors.

Some of those factors are dependent upon traffic flows. Thus, the consumers, that is, the travelers in the transportation market, seek their optimal choices in a given situation. In the short-terms, users of the system reach an equilibrium over the network by rearranging their use of the available, but fixed alternatives supplied by producers. These phenomena give rise to the network equilibrium stated by Wardrop (1952) and Beckmann et al. (1956).

In the longer-term in highly congested networks, the level of service provided by the transportation producers may be improved. Then, travelers may be confronted with a re-evaluation of their long-term habitual choices such as locations of residence, destinations, modes, and activities at origins or destinations. In congested networks, moreover, the decision-making process may become very sensitive, and choices other than the least-cost route may materialize. The traditional transportation planning process does not recognize such complex feedback phenomena. The equilibrium conditions under these circumstances goes well beyond the formulation of Wardrop (1952) and Beckmann et al. (1956), since not only routes and their associated flows between given origin and destination, but also modes, destinations, and locations may be re-examined by an individual user.

Several recent models such as Florian et al. (1975), Evans (1973, 1976), Erlander (1977), Abdulaal and LeBlanc (1979), and Dafermos (1980, 1982) attempted to incorporate the effect of route costs on other choices such as destination, mode, and so on. The appearance of these models in the literature signalled an emergence of a new model type, namely combined urban location and/or travel choice models.

Implementation of such mathematical models requires the values of model parameters. Calibration, or choosing these parameter values, requires certain observed survey data such as travel cost information on each street or road, origin-destination-mode travel data, and so forth. Among the variables required for calibrating these parameters, one may find it very difficult to observe or survey travel cost data. Furthermore, the calibrated parameters estimated from these survey data may not reproduce observed behavior. The estimation procedure itself may not converge if the parameter estimation of interest is separately carried out from the model that produces the travel cost data, even though one repeats the process.

This research aims at reducing inconsistencies inherent in traditional approaches by calibrating the model parameters directly from the model that solves the equilibrium network problem. The model used for this study was the combined mode and route choice model (CMR). The reason of choosing the model was that the CMR model among many other combined models is the simplest to solve the research problems and the most directly related to generalized travel cost. The results obtained through this research may be readily applied to other models that have not been used.

In order to accomplish the overall objectives of this research, the following assumptions, models and data are used.

1. User choice behavior regarding mode and route choice is based on two major assumptions: (a) user-equilibrium choice of routes; (b) dispersion of mode choices, given by the entropy value.
2. The generalized cost function is defined as a linear function of the cost components. In particular, automobile travel time and operating cost are variables to be determined endogenously, while the other cost components are fixed.
3. The model parameters estimated in this study are: (a) the generalized cost coefficients; (b) the dispersion factor which is equivalent to the Lagrange multiplier associated with entropy constraint.
4. Constants required for solving the model are: (a) total number of trips, which is related to the proportion of work trips in the peak-hour; (b) auto occupancy factor; (c) value of the entropy function; (d) prior probabilities.
5. The trip table surveyed in 1975 for the Chicago region is used for estimating the parameters and evaluating the goodness-of-fit of the model.
6. Algorithms used in this study are: (a) Evans (1976) algorithm for solving combined urban location and travel choices model; (b) grid search for the maximum likelihood estimation of model parameters.

II. Network Equilibrium

The concept of an equilibrium in an urban transportation network stems from the dependence of link travel costs on link flows. Under the assumptions that (1) the number of travelers between a given origin and a given destination is known, and (2) these origin-destination pairs are connected by several possible routes, one problem of interest is to know how these travelers will be distributed among the possible routes. If all travelers choose the same minimum cost route in traveling from an origin to a destination, congestion would develop because the capacity of the road system cannot cope with such a high travel demand. Some travelers may use an alternative route so as to avoid the congestion. The alternative route can, however, also be congested, and so forth. This situation will continuously adjust until no user has any incentive to alter his travel route. The resulting solution is called a network equilibrium.

In the transportation literature, the flows which satisfy the above conditions are said to be a user-equilibrium or user-optimal flow pattern. The mathematical expression equivalent to user-equilibrium can be stated as follows:

[P1] For all $i \varepsilon I$, $j \varepsilon J$, $r \varepsilon R_{ij}$.

$$(c_r(h) - u_{ij})h_r = 0 \qquad (1)$$

$$c_r(h) - u_{ij} \geq 0 \qquad (2)$$

$$\sum_{r \varepsilon R_{ij}} h_r - D_{ij}(u) = 0 \qquad (3)$$

$$h_r \geqslant 0 \tag{4}$$

$$u_{ij} > 0 \tag{5}$$

where

$c_r(h)$ = travel cost from i to j by route r

u_{ij} = travel cost associated with a given O-D pair (i, j)

h_r , $r \varepsilon R_{ij}$ = traffic flows from i to j on route r

$D_{ij}(u)$ = total travel demand between i and j

The equations (1) and (2) represent Wardrop's user-equilibrium principle, requiring that (a) travel costs for all route with $h_r > 0$ are the same and equal to u_{ij} and (b) those costs are less than or equal to the ones for any route with zero flows. The fact that the total flow among the different routes between any O-D pair (i, j) should be the same as the total demand $D_{ij}(u)$ which in turn depends upon the congestion in the network through u_{ij} is seen in (3). For the fixed demand case, D_{ij} is assumed to be constant. The remaining equations (4) and (5) show that both flow and cost should not be negative.

An equivalent optimization problem for equations (1) to (5) was first formulated by Beckman et al. (1956), based on their observation that the optimality conditions to this problem correspond to the user-equilibrium pattern. Although they considered the variable demand case in which the travel demand is endogenously determined by equilibrium travel costs, we shall only give attention to the route choice problem for fixed demand. Their optimization problem is then:

$$\text{[P2] Minimize } \frac{1}{T} \Sigma_a \int_0^{V_a} s_a(x)\,dx \tag{6}$$

subject to:

$$\Sigma_r P_{ijr} = P_{ij} \quad \text{for all } i,j \tag{7}$$

$$P_{ijr} \geqslant 0 \tag{8}$$

and definitional equations:

$$v_a = \Sigma_{ijr} P_{ijr}\, \delta_{ar}\, T \quad \text{for all } a \tag{9}$$

In spite of its simple structure, this formulation has been the genesis of network equilibrium models which can be solved by flexible and powerful optimization techniques. In order to examine whether the optimality conditions of the equivalent mathematical programming problem satisfy the Wardrop's user-equilibrium principle, the Lagrangian is constructed:

$$L = \frac{1}{T} \Sigma_a \int_0^{Va} s_a(x)\, dx + \Sigma_{ij} u_{ij} \left(P_{ij} - \Sigma_r P_{ijr} \right) - \Sigma_{ijr} \theta_{ijr} P_{ijr} \tag{10}$$

The necessary conditions for the equivalent optimization problem may be expressed by:

$$\frac{\partial L}{\partial P_{ijr}} = \frac{1}{T} \Sigma_a s_a(v_a)\, \delta_{ar} T - u_{ij} - \theta_{ijr} = 0 \tag{11}$$

and

$$P_{ijr} \theta_{ijr} = 0 \tag{12}$$

If $P_{ijr} > 0$, which implies $\theta_{ijr} = 0$

$$\Sigma_a s_a(v_a)\, \delta_{ar} - u_{ij} = 0 \tag{13}$$

If $P_{ijr} = 0$, which implies $\theta_{ijr} \geq 0$

$$\Sigma_a s_a(v_a)\, \delta_{ar} - u_{ij} \geq 0 \tag{14}$$

Therefore, equations (13) and (14) satisfy our definitions of user-equilibrium conditions.

III. Parameter Estimation of the Combined Mode and Route Choice Model

1. Combined Mode and route Choice Model

The period for which urban networks are typically analyzed in urban transportation is the morning or afternoon peak-demand period. During these times, most trips cannot be easily avoided because of the home-to-work purpose for which they are performed. As a first approximation, then, the number of trips between each origin and destination (O-D) pair may be regarded as fixed. Some of these trips, however, may not be conducted over the road network but on alternative modes of transportation, such as public transit. This section describes a network equilibrium model in which the network includes both automobile and transit modes. The solution, therefore, includes the flow of transit patrons between each O-D pair in addition to the equilibrium vehicular flow pattern over the road network. The problem is referred to as the combined mode and route choice problem (CMR).

In the present context, assume that some of the network O-D pairs are connected by transit. The level of service offered by the transit system is independent of either the automobile flow or the transit patronage on the assumption that the transit capacity is large enough so that no congestion effects occur on the transit routes and the schedule is determined to accomodate the worst delays. The level of service is defined by variables such as travel time, transit fare, and some other fixed costs. For this purpose, the transit level of service between O-D pairs is summarized by some fixed terms.

Most traditional approaches to this problem assume that only travel time is taken into account to measure the level of service of those modes. This is unrealistic in cases where other factors such as fares, out-of-pocket money cost, parking cost, walking time to a station, transit feeder service, and so on are important. Thus, define the generalized cost incurred in traveling as a weighted sum of the level of service variables of each mode. In addition, for consistency, both automobile and transit flows should be expressed in terms of persons per unit of time. Since the road network is analyzed in terms of automobile flows, a vehicle occupancy factor must be used to convert person flow to vehicular flow over this network.

Under the assumptions made in the previous statements, one may formulate a mathematically equivalent optimization problem in accordance with Beckmann et al. (1956) which is:

$$\text{[CMR]} \quad \text{Minimize} \quad \gamma_1 \left[\Sigma_a \frac{R}{T} \int_0^{V_a} s_a(x)\,dx + \Sigma_i \Sigma_j P_{ijt}\, \tau_{ijt}\right] + \gamma_2 \left[\Sigma_a \frac{1}{T} \int_0^{V_a} k_a(x)\,dx + \Sigma_i \Sigma_j P_{ijt}\, \zeta_{ijt}\right] + \gamma_3 \left[\Sigma_i \Sigma_j P_{ijh} w_{ijh} + \Sigma_i \Sigma_j P_{ijt}\, \omega_{ijt}\right] \quad (15)$$

subject to:

$$\sum_{m \varepsilon M} P_{ijm} = P_{ij} \quad \text{for all } i,j \quad (16)$$

$$\sum_{r \varepsilon R_{ij}} h_r = p_{ijh}\, T/R + F_{ij} \quad \text{for all } i,j \quad (17)$$

$$-\Sigma_{ijm}\, p_{ijm} \ln(p_{ijm}/q_{ijm}) \geqslant S \quad (18)$$

In addition, the definitional constraints:

$$v_a = \Sigma_{ijr}\, h_r\, \delta_{ar} \quad \text{for } a \varepsilon A$$

$$p_{ijm},\, h_r \geqslant 0 \quad \text{for all } i,j,m,r \quad (19)$$

where q_{ijm} = a priori probability of a trip from i to j by mode m

P_{ijh} = proportion of highway person trips from i to j

P_{ijt} = proportion of transit person trips from i to j

P_{ijr} = number of highway vehicle trips on route r

w_{ijh} = terminal time of auto driver and passengers

F_{ij} = truck trips in auto equivalents from i to j

τ_{ijt} = fixed transit travel time

ζ_{ijt} = fixed transit fare

ω_{ijt} = fixed out-of-vehicle time for transit (e.g., headway, waiting time, excess time)

δ_{ar} = 1 if link a lies on route r
0 Otherwise

R = (Auto drivers + auto passengers) / auto drivers

s_a = in-vehicle travel time on link a by highway

k_a = operating cost on link a by highway

P_{ij} = fixed proportion of person trips from i to j

P_{ijm} = proportion of total trips from i to j on mode m

P_{ijmr} = proportion of total trips from i to j by mode m on route r

S = observed entropy value

T = total number of person trips in peak hour

$\gamma_1, \gamma_2, \gamma_3$ = are parameters to be calibrated

The model hypotheses used to formulate the above problem are:

1. all mode and route combinations from origin to destination which are selected have equal travel costs
2. no unselected combination has a lower travel cost
3. deviation from the minimization of travel costs is characterized by the entropy function

This problem is the multimodal version of Wardrop's first principle. The basic assumptions underlying this formulation are:

1. there is more than one mode
2. travel costs are independent of flows by other transportation modes (i.e., no interaction between modes)
3. the link congestion functions by a mode m for individual links are separable. In other words, the cogested travel cost by a mode m, $s_{am}(v_{am})$, for each link depends only upon the total flow by that mode, v_{am}, on that link (i.e., no interaction between link flows)

The Kuhn-Tucker optimality conditions for the constrained nonlinear programming problem give:

$$(1/\mu)\ln(p_{ijh}/q_{ijh}) + 1 + \frac{T}{R}U_{ijh} + \gamma_3 w_{ijh} + \lambda_{ij} = 0 \quad \text{for } p_{ijh} > 0$$

$$(1/\mu)\ln(p_{ijt}/q_{ijt}) + 1 + \gamma_1\tau_{ijt} + \gamma_2\zeta_{ijt} + \gamma_3\omega_{ijt} + \lambda_{ij} = 0 \quad \text{for } p_{ijt} > 0 \qquad (20)$$

$$\gamma_1(R/T)\Sigma_a s_a(v_a)\delta_{ar} + \gamma_2(1/T)\Sigma_a k_a(v_a)\delta_{ar} - U_{ijh} = 0 \text{ for } h_r > 0 \tag{21}$$

and constraints (16) – (18)

where λ_{ij} = a Lagrange multiplier associated with (16)

C_{ijm} : the generalized cost function by modes, is defined as:

$$C_{ijt} = \gamma_1 t_{ijh} + \gamma_2 k_{ijh} + \gamma_3 W_{ijh} \quad \text{for all } i,j \tag{22}$$

$$C_{ijt} = \gamma_1 \tau_{ijt} + \gamma_2 \zeta_{ijt} + \gamma_3 \omega_{ijt} \quad \text{for all } i,j \tag{23}$$

where C_{ijh} = generalized travel cost by automobile

C_{ijt} = generalized travel cost by transit

$t_{ijh} = \Sigma_a s_a(v_a)\delta_{ar}$ for $r \in R_{ijm}$ with $h_r > 0$

$k_{ijh} = -\Sigma_a k_a(v_a)\delta_{ar}$ for $r \in R_{ijm}$ with $h_r > 0$

μ = a reciprocal of the Lagrange multimplier associated with the constraint (18)

2. Calibration of the Lagrange Multiplier

The Lagrange multiplier can be calibrated by using the monotonicities of F and G which are defined in (A-1) and (A-2) of Appendix respectively. In this section, we consider how to estimate directly from problem (CMR) in (A-3). There are two relevant cases:

1. The estimate of F, $\hat{F}$, is available
2. The estimate of G, $\hat{G}$, is available

Consider the case when the estimate of F, the sum of integral of link costs, is available from base year data. Wilson (1970) proposed a kind of balancing factor approach when the associated travel cost is fixed. In gravity-like models, that approach converges to a stable solution under mild conditions. In one of the earlier calibration methods in the gravity models, Evans (1971) considered a numerical method for finding the value of μ for given mean travel cost. He first considered the plausible range of the mean travel cost in which $F(\mu) = \hat{F}$ is satisfied. He used a Newton method after the cost function is expanded in a Taylor series about $\mu = 0$, to update the value of μ. In this method, we are required explicitly to evaluate the derivative of the cost function. Having the new μ, the gravity model is reestimated, resulting in the updated mean travel cost. The process is continued until the prespecified convergence is satisfied. Similar ideas can be found in Hyman (1969) and Wagon and Hawkins (1970).

Nguyen (1977) showed that, assuming the underlying system is governed by an equilibrium route choice model, the link flows, v_a, are uniquely determined by the equilibrium cost. However, the route flows are not unique. Using his model, one may compute F and calibrate the traditional gravity model to obtain μ. If all of the link flows are availalbe from a survey, hence F, one can determine the equilibrium travel cost and use the Nguyen (1977) model to obtain an estimate of F. The calibration of μ is followed by the balancing factor approach.

A second approach to obtain μ has been proposed in Erlander and Stewart (1978) when the estimate of G, $\widehat{G}$, is available. In their model, the observed travel demands are required. Then the link flows can be computed by the equilibrium solving algorithms such as Evans (1976), LeBlanc (1975), and so on. Using the link flows, one can easily compute F and G. The balancing factor method is again used to estimate the value of μ from an estimate G. The aforementioned methods have been studied to reproduce the trip O-D matrix.

Boyce et al. (1983) suggested using the Newton-Raphson method to calibrate μ. By obtaining approximate first-order derivatives from the optimality conditions, they attempted to update the iterative values of μ. This process consists of an iteration between the estimation of and the solution of the combined model. A convergence proof was not given. For this research, a similar idea has been adopted. However, rather simple line search algorithms are considered, as introduced in next section; Empirical results for the Hull City network with two cost components in defining the generalized cost function have been reported in Boyce et al. (1983).

The following section concerns the parameters, μ, γ in the generalized cost function defined in (22) and (23). The maximum likelihood estimation method is defined, and solution algorithms for finding the maximum likelihood estimates of both μ and γ are suggested.

3. Maximum Likelihood Estimation

In this section, we shall present procedures which can be used to estimate the unknown parameter, including the generalized cost coefficients and the dispersion factor in the combined mode and route choice models introduced in the Section 1. Two basic techniques are available:

(1) least squares estimation

(2) maximum likelihood estimation

The choice of an estimation method depends upon the characteristics of data, the structure of the sampling design which generated the date and prior knowledge of the system being considered. It is very difficult to make a direct comparison between these methods, because there is no general agreement on which method is more appropriate in a given situation.

One principal task of implementing the CMR is to estimate P_{ijm} and v_a which are the trip proportions from origin i to destination j by mode m and link flows, respectively, for given parameter values. In other words, the $\{p_{ijm}\}$ and $\{v_a\}$ are now functions of parameters which are assumed to be unknown, provided the link costs and hence route costs are characterized by given cost functions in the objective function. The parameters such as γ_1, γ_2, γ_3, and μ therefore should be calibrated in order to realize the CMR. 'Calibration' as used here denotes finding the best estimates of the parameters of the model. Calibration and testing require survey data which were available from a survey carried out by the CATS in 1975.

In the course of this study, solution methods appropriate for the bilevel programming problem (BLPP) have been applied. The BLPP is defined as solving a hierarchy of two optimization problems where the constraint region of the first level is determined implicitly by the solution of the second level. In other words, when this formulation is applied to the present research problem, the upper level problem represents a maximum likelihood estimation over unknown parameters, while solving the CMR defines the second level problem.

Under the assumption that the sample data are randomly and identically selected, the joint density probability function may be given by the multinomial distribution:

$$P\{n_{ijm}\} = \frac{N!}{\pi_{ijm}\, n_{ijm}!}\, \pi_{ijm}\, p_{ijm}(\theta)^{n_{ijm}} \tag{24}$$

subject to : $\Sigma_{ijm}\, n_{ijm} = N$, and $\Sigma_{ijm}\, p_{ijm}(\theta) = 1$

where n_{ijm} = observed trip frequencies from i to j by mode m

$P\{n_{ijm}\}$ = joint probability of $\{n_{ijm}\}$

$p_{ijm}(\theta)$ = probability to be estimated corresponding to the unknown parameters, θ

N = total sample size

Taking the logarithm of equation (24), we obtain a log likelihood function as defined in the previous section:

$$L(\theta) = \Sigma_{ijm}\, n_{ijm} \ln p_{ijm}(\theta) + \ln W$$

$$\text{where} \quad W = \frac{N!}{\pi_{ijm}\, n_{ijm}!} \tag{25}$$

Disregarding the constant term,

$$L(\theta) = \Sigma_{ijm}\, n_{ijm} \ln p_{ijm}(\theta) \tag{26}$$

Dividing by the total sample size N, we obtain

$$L(\theta) = \Sigma_{ijm}\, \bar{p}_{ijm} \ \ln\ p_{ijm}(\theta) \tag{27}$$

which has a similar form to the (negative) entropy function used in the constraints of the CMR.

The unknown probabilities $P_{ijm}(\theta)$ are given by [CMR]. Thus, the equivalent mathematical formulation can be expressed as:

(MLE1):

$$\underset{\gamma}{\text{Max}}\ L(\gamma) = \Sigma_{ijm}\, \bar{P}_{ijm} \ \ln\ P_{ijm}(\gamma) \tag{28}$$

subject to: $\gamma \ \varepsilon\ \Omega$

where $P_{ijm}(\gamma)$ solves:

$$\underset{(p,v)\varepsilon X}{\text{Min}}\ F\{(p,v);\gamma\} \tag{29}$$

and

$$X = \{(p,v);\ \{p_{ijm}\} \text{ and } \{v_a\} \text{ are feasible to the CMR}\} \tag{30}$$

$$\Omega = \{\ \gamma\ ;\ \Sigma_k\, \gamma_k = 1 \quad \text{and} \quad \gamma_k \geqslant 0 \quad \text{for all}\ \ k\ \} \tag{31}$$

The vector θ is defined as an scalar product of the vectors, γ and μ. Since γ is a normalized vector, then θ sums to μ. This definition is convenient when we wish to examine the effects of each cost component separately from μ. It should be noted that the μ is a scaling factor determined in part by the entropy constraint.

As an alternative formulation, Boyce et al. (1983) have the equivalent formulation to the problem (MLE1):

(MLE2): $$\text{Min}_{\theta}\ -\Sigma_{ijm}\, \bar{p}_{ijm} \ \ln\ p_{ijm}(\theta) \tag{32}$$

subject to:

$$p_{ijm}(\theta) = p_{ij} \frac{q_{ijm} \exp(-U_{ijm}(\theta))}{\sum_k q_{ijk} \exp(-U_{ijk}(\theta))} \tag{33}$$

$$\{p_{ijm}\} \in X$$

where $U_{ijm} = \mu C_{ijm}$ and taking $\theta = \mu\gamma$

C_{ijm} was defined in (22) and (23)

Difficulties arise when one deals with the constraint set X which is non convex because of the nature of the logit-type demand function. The logit function is neither convex nor concave with regard to the parameter θ. Thus the set covering the logit function, may not be generally convex, even if the set itself is closed and bounded. For these reasons, it seems very difficult to find global solutions to the equivalent optimization problem. Of important interest may be the structure of cost components which tends to be monotonic corresponding to θ. In such cases, we may search for the location of local minima in the vicinity of the good initial starting point.

In developing a systematic procedure to find a (local) optimum value for θ which also maximizes the likelihood function, we should take into account the fact that some cost functions are not differentiable so that we may not incorporate the class of gradient methods. The best way in this cases may be to develop a systematic procedure which requires only function evaluations (i.e., likelihood function). Under such circumstances, a kind of line search algorithm is proposed. The next section deals with such algorithms in more detail.

4. Solution Algorithms

Many early methods which have been suggested for minimization were developed based on ad hoc ideas without much theoretical background. Most methods require only the evaluation of the objective function. If the problem is in few variables, however, it is likely that some sort of repeated bisection in each one of variables could be tried so as to establish a region in which the minimum point exists. Then an attempt might be made to reduce the area of this region systematically.

Among many variations of this idea, the *alternating variables method* has been applied to certain cases. In this method, on iteration n (n = 1, 2, ..., N), the variable x_k alone is changed in a trial to reduce the objectivve function value, and the other variables are kept fixed. After iteration k, when all the variables have been changed, then the whole cycle is repeated until convergence is obtained. One possible improvement that has been used in this research is to make a change in each coordinate direction which reduces the objective function value as much as possible. The method mentioned here may have several weaknesses such as (1) inefficient and sometimes unreliable performance; (2) oscillatory behavior; (3) no profound theoretical background. In additon, it ignores the possibility of interaction between the variables.

Some substantial progress has been made to the above-mentioned method in such a way that the points at the beginning and end of a cycle determine a line along which better progress might be made. Thus, if a provision is made for searching along this line, a more efficient method might result. This idea gives rise to Hooke and Jeeves (1961), the DSC method by Swann (1964), Powell's (1964) conjugate-direction algorithm, and the simplex methods of Spendley et al. (1962) and Nelder and Mead (1965).

The direct-search strategies for generating a sequence of improving approximations to the solution are based simply on comparisons of function values; generally, but not always, the methods are heuristic in nature. However, Powell's (1964) conjugate-direction method might be exceptional. When these methods are incorporated, there are some advantages that result in a very wide range of applications.

One advantage of these methods can be found when the objective function is not continuous. Thus, they are effective when differentiation is impossible. In such cases, gradient-based methods can prove to be inefficient or even ineffective. Another advantage of the use of these methods is that they require less preparation than the gradient methods. Furthermore, bacause of their lack of assumptions about the objective function, they are often more useful than gradient methods. This does not mean that they are superior to the gradient methods; however, one should not ignore them from a practical point of view. They usually aim at producing a good solution rather than an optimal one, with the hope that the good solution approximates the optimal.

Because there is lack of a proof of convergence, in contrast to the methods that have a mathematical basis for their convergence, one may not give rigorous convergence criteria. In that sense, they are less reliable than methods making use of derivatives.

An alternating variable method has been used for the calibration of the Chicago network. The value of the likelihood function defined in (27) is reduced on every iteration, which usually implies that the stationary point turns out to be a local minimum. Additional evidence of convergence is a decreasing residual that is defined as:

$$Q^n = x^n - x^{n-1} \tag{34}$$

If Q^n goes to zero, then it may be possible to make statements about the convergence.

IV. Computational Results

This chapter summarizes computational results of estimating parameters that are required in order to implement the combined mode and route choice model. The networks used for this research were the Chicago sketch planning network described below and the Hull City network that has been used by Nguyen (1974) for testing his network equilibrium model.

1. Network Representation

Sketch planning is a phase of the planning process that is designed to increase both the responsiveness and the efficiency of transportation planning (UMTA, 1979). It has several beneficial features such as: (1) the preparation of data is relatively simple, compared with conventional planning tools in which large efforts are made for more detailed analysis, with the hope that the evaluation results are proportional to the level of effort; (2) computer operation is relatively inexpensive; (3) the outputs are relevant and meaningful to decision-makers in the sense that conventional planning tools often require much interpretation of the results before they are useful.

Eash et al. (1984) describe an effort to construct a sketch planning network for the Chicago region. The network and zone system in that sketch planning network has been aggregated from CATS's 1,797 zone *regional* system. Each sketch planning zone corresponds approximately to nine regional zones, and range in size from nine to thirty-six square miles, resulting in a 317 zone system. The network consists of roughly 1,250 bidirectional arterial and freeway links. Each zone centroid is located at the center of a zone and is connected to adjacent zones by at least two, but no more than four arterial links. Freeway interchanges are coded at most points of intersection with arterial links and as close as possible to their locations. The zone system of the sketch planning network is given in Figure 1.

2. Generalized Cost Function

The concept of a cost function in transportation planning has been perceived as the core of the analysis of travel demand. Accordingly, transportation cost functions have been estimated for various modes with the objective of describing the performance of the corresponding technology. In spite of evident changes in transportation technology, most cost functions that have been used in past years do not take account of such changes.

This section is concerned with formulating the cost function that underlies an individual's choice between automobile and public transit in a manner consistent with the hypothesis that the travel choices are described by network equilibrium model. The generalized cost that has widely been used in practice is a weighted linear sum of attributes such as travel time, money cost, access time, waiting time, parking cost, and so on. The generalized cost function used in the Chicago network is defined as:

$$C_{ijh} = \gamma_1 t_{ijh} + \gamma_2 k_{ijh} + \gamma_3 w_{ijh} \quad \text{for all} \quad i, j \tag{35}$$

$$C_{ijt} = \gamma_1 \tau_{ijt} + \gamma_2 \zeta_{ijt} + \gamma_3 \omega_{ijt} \quad \text{for all} \quad i, j \tag{36}$$

where

C_{ijh} = the generalized cost of travel from i to j by auto

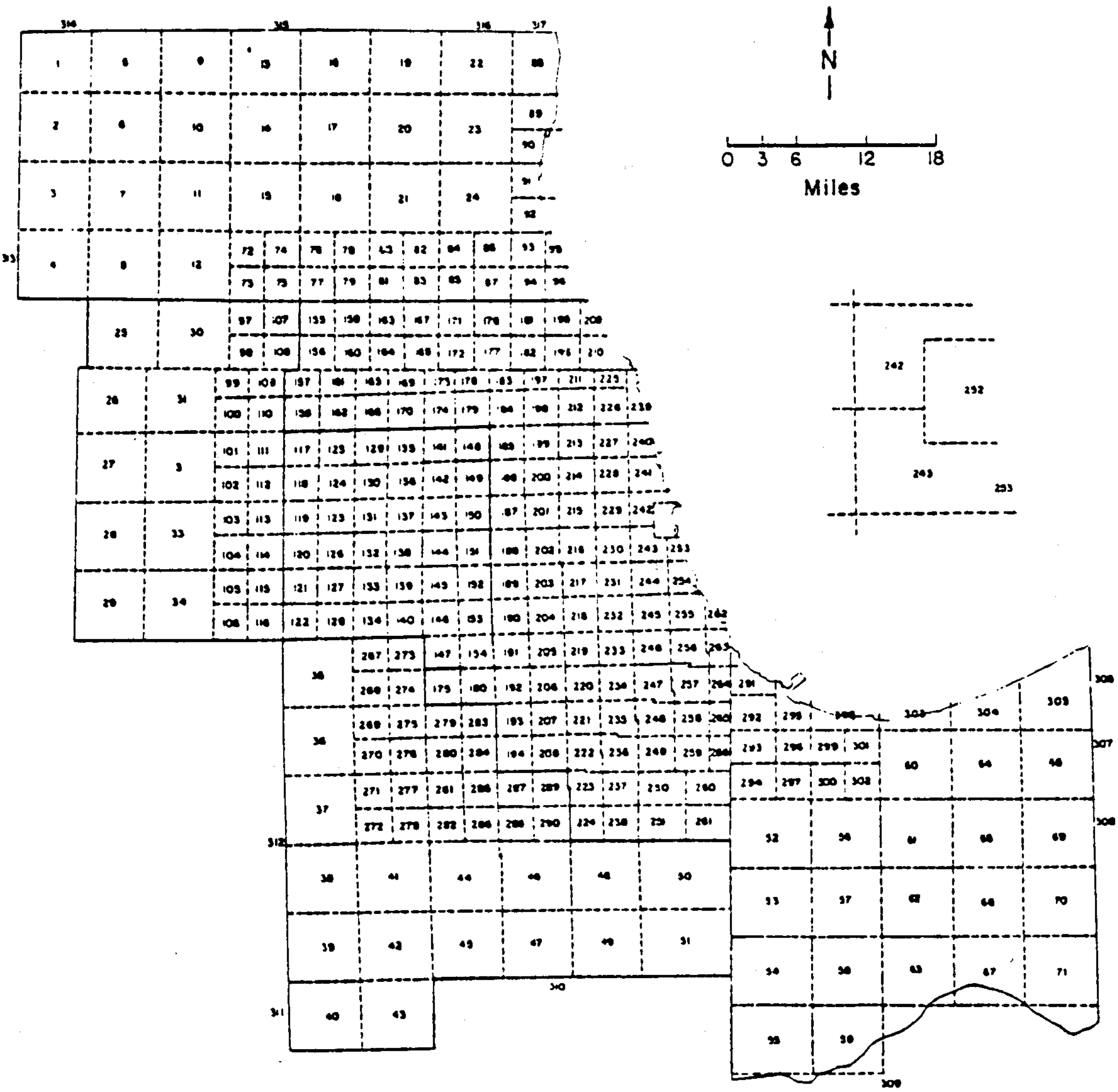

Figure 1. Sketch planning zone system for the Chicago Region

C_{ijt} = the generalized cost of travel from i to j by transit

t_{ijh} = auto in-vehicle travel time in minutes

k_{ijh} = auto operating cost, including parking fee, in cents

w_{ijh} = auto access/egress time in minutes

τ_{ijt} = transit in-vehicle travel time in minutes

ζ_{ijt} = transit operating cost in cents

ω_{ijt} = transit access/egress time in minutes

$\gamma_1, \gamma_2, \gamma_3$ are coefficients to be estimated

The transit cost terms are considered to be fixed, as given from CATS data base. The auto fixed cost includes terminal costs such as parking cost at each origin and destination. Auto in-vehicle travel time and operating cost are functions of link flows, and hense, route flows. For example, auto travel time is computed by the FHWA congestion function which is defined as:

$$s_a = t_{ao}[1.0 + 0.15(v_a/m_a)^4] \qquad (37)$$

where t_{ao} = travel time on link a with zero flow

v_a = flow of link a

m_a = capacity of link a

s_a = travel time on link a with flow v_a

Auto operating costs are estimated by the regression analysis, using the CATS (1978) data base as:

$$k_a = (8.07 - 4.9148z + 2.1515z^2 - 0.40686z^3 + 0.027493z^4)\ d_a \qquad (38)$$

where k_a = auto operating cost on link a in cents

$$z = \frac{u_a - 2.5}{15}$$

where u_a = travel speed on link a in miles per hour

d_a = travel distance in miles

The auto operating function k_a has been rederived so as to preserve an increasing function with respect to link flows (see Figure 2). Compared with values estimated by Chicago Area Transportation Study (CATS) in 1978, the values below 35 miles per hour are similar. However, the values for above 35 miles per hour decrease slightly, while the values of CATS function increase. One basis for the use of new auto operating function can be found in Sanders and Reynen (1979) who estimate auto fuel consumption by speeds. More detailed information related to the operating function is given in Boyce et al. (1981).

Another computational issue can be raised in defining the auto operating cost term in the objective function as introduced in (15). Since we should take the integral of k_a with regard to link flow v_a, some derivational effort is needed. The explicit function form is given as:

$$\Sigma_a \int k_a\, dx = \Sigma_a d_a \{8.9249 v_a - 5.4777 (1/\beta) A + 2.1449 (1/\beta)^2 B - 0.3522 (1/\beta)^3 C + 0.02 (1/\beta)^4 D\} \quad (39)$$

where
$$\alpha = \frac{t_{ao}}{4 d_a} \quad (40)$$

$$\beta = \frac{0.15\, t_{ao}}{4 d_a m^4} \quad (41)$$

$$r = (\alpha/\beta)^{(1/4)} \quad (42)$$

$$A = \frac{1}{r^3\sqrt{8}} \{ \frac{1}{2} \ln (\frac{r^2 + rx\sqrt{2} + x^2}{r^2 - rx\sqrt{2} + x^2}) + \tan^{-1} \frac{rx\sqrt{2}}{r^2 - x^2} \} \quad (43)$$

$$B = \{ \frac{x}{4r^4 (r^4 + x^4)} + \frac{3}{4r^4} A \} \quad (44)$$

$$C = \{ \frac{x}{8r^4 (r^4 + x^4)^2} + \frac{7}{8r^4} B \} \quad (45)$$

$$D = \{ \frac{x}{12x^4 (r^4 + x^4)^2} + \frac{11}{12 r^4} C \} \quad (46)$$

and $x = v_a$

3. Computational results for the Chicago Region

As mentioned before, we have adopted the alternating variable method as a method of solving the

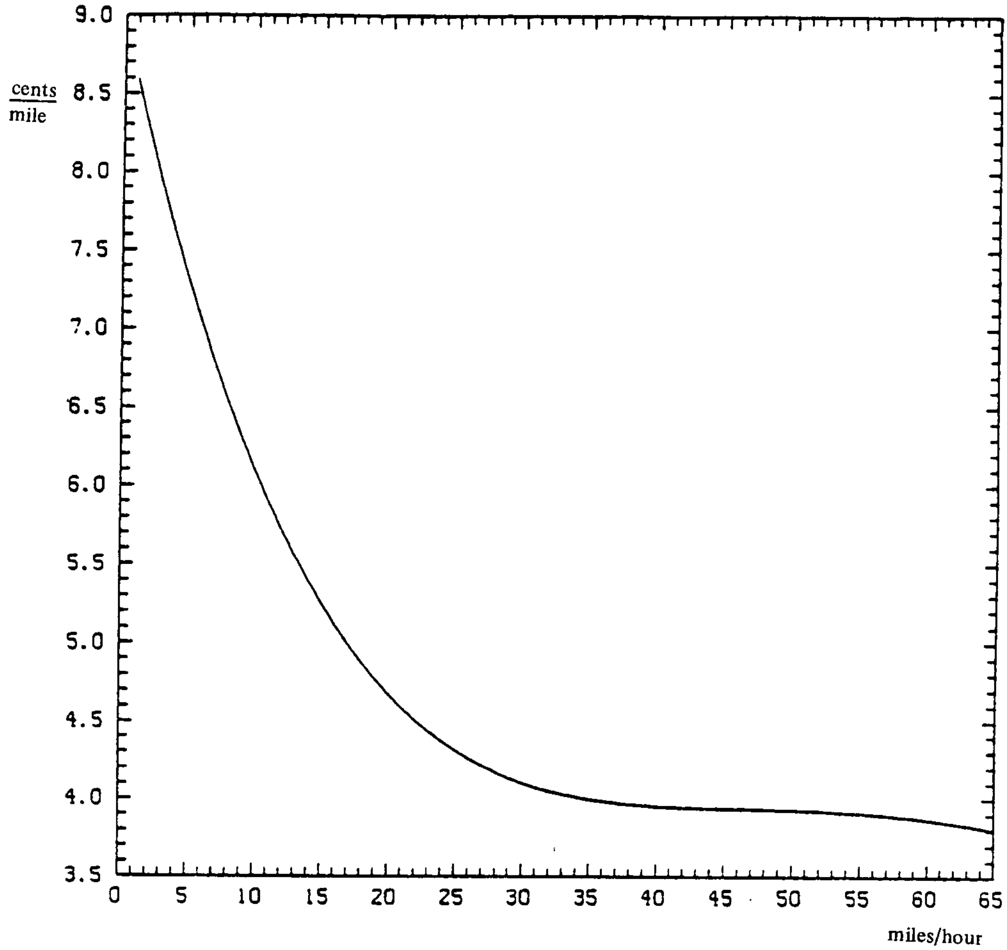

Figure 2. Automobile operating cost function

equivalent BLPP defined in the section III. About five cycles were completed, where a cycle is defined as the complete evaluation of one variable while other variables are fixed. As shown in Table 1, the initial starting point was obtained from the previous values estimated in Chon (1982). the value of the dispersion parameter was 1.1527×10^{-2}. Corresponding likelihood and estimated entropy values were 8.7997 and 8.7750, respectively. The observed entropy value is 8.7551.

In the second trial, the value of θ_1 was reduced about 20%. Then, the resulting entropy value is increased because the θ value was decreased. The resulting likelihood value also increased at this point. Thus, the next trial point was chosen in the opposite direction, resulting in a better solution in terms of value of the likelihood function. In order to improve the likelihood function value as much as possible, another point that was increased as much as 50% of the initial starting point was

Table 1. Estimation results for the Chicago network

	θ_1 ($\times 10^{-3}$)	θ_2 ($\times 10^{-3}$)	θ_3 ($\times 10^{-3}$)	MLE	S
Cycle 1					
	0.3227	0.0703	0.9060	8.7997	8.7750
	0.2582	0.0703	0.9060	8.8009	8.7829
	0.3873	0.0703	0.9060	8.7993	8.7669
	0.4841	0.0703	0.9060	8.8000	8.7553
Cycle 2					
	0.3873	0.0703	0.9060	8.7993	8.7669
	0.3873	0.1406	0.9060	8.7946	8.7576
	0.3873	0.1561	0.9060	8.7942	8.7551
	0.3873	0.2109	0.9060	8.7937	8.7468
	0.3873	0.2812	0.9060	8.7958	8.7355
Cycle 3					
	0.3873	0.2109	0.9060	8.7937	8.7468
	0.3873	0.2109	1.1780	8.8067	8.7141
	0.3873	0.2109	0.6342	8.7911	8.7939
	0.3873	0.2109	0.3624	0.8076	8.8605
Cycle 4					
	0.3873	0.2109	0.6342	8.7911	8.7939
	0.4841	0.2109	0.6342	8.7912	8.7801
	0.2743	0.2109	0.6342	8.7936	8.8095
	0.6455	0.2109	0.6342	8.7931	8.7600
Cycle 5					
	0.3873	0.2109	0.6342	8.7911	8.7939
	0.3873	0.2285	0.6342	8.7908	8.7902
	0.3873	0.2812	0.6342	8.7925	8.7777
	0.3873	0.1406	0.6342	8.7935	8.8107

where $\theta_1 = \mu\gamma_1$

$\theta_2 = \mu\gamma_2$

$\theta_3 = \mu\gamma_3$

$MLE = -\Sigma_{ijm}\bar{p}_{ijm}\ln(p_{ijm})$

$S = -\Sigma_{ijm}p_{ijm}\ln(p_{ijm})$

tried. However, the point failed to find the smaller value of the likelihood function. This terminates the first cycle by going back to the best point that has been obtained.

The second cycle was performed by varying θ_2 for fixed values of θ_1 and θ_3. The first trial point was chosen by increasing θ_2 by 200%. Since this point improved the likelihood function, the next trial point was also selected in the same direction this subsequent point also produced a better value of the likelihood function. Again, a further search effort was made in that direction until no improvement in that function was obtained. The second cycle stopped at that point.

This process was continued after the second cycle until no better points were found. Since each point represents at least four iterations of Evans' algorithm, the computational efforts are quite intensive when one implements the procedure in a large-scale network like the Chicago sketch planning network. Further fine searching is documented in Table 4.

Table 2, summarizes the results associated with the best point obtained so far. The values are 120%, 300%, 85% of the initial values of θ_1, θ_2, and θ_3, respectively in CASE 2 denoted as the (local) minimum without consideration of the entropy constraint. Corresponding's are changed as much as 120%, 370%, and 86% in the (almost) active CASE 3. In these last two cases, the coefficients of modal operating cost are greatly increased, evidently because of the rederivation of the auto operating cost function so as to retain the convexity with regard to link flows. Thus, the coefficients of modal fixed terms, such as access/egress times for both modes, are relatively reduced.

Since the model implicitly determines the travel costs according to the concept of the network equilibrium, we may see the changes in the cost structrues associated with each parameter set. Table

Table 2. Final estimates for parameters in Chicago network

	θ_1 (x 10^{-3})	θ_2 (x 10^{-3})	θ_3 (x 10^{-3})	μ (x 10^{-3})	MLE	S
CASE 1:	.3227	.0703	.9060	11.527	8.7997	8.7750
	(.2485)	(.0541)	(.6974)			
CASE 2:	.3873	.2109	.7701	1.3684	8.7910	8.7676
	(.2830)	(.1542)	(.5628)			
CASE 3:	.3873	.2600	.7750	1.4223	8.7916	8.7579
	(.2723)	(.1828)	(.5449)			

where CASE 1 is represented by the initial starting point

CASE 2 is represented by the local minimum

CASE 3 is represented by the local minimum of the optimization problem constrained by an entropy function

Note that (.) denotes the estimated coefficient (i.e., γ) of the generalized cost function.

3 summarizes those values. In the results of CASE 2 and CASE 3, the values of time are \$1.10/hour for in-vehicle travel time and \$2.19/hour for access/egress in CASE 2 and \$0.89/hour and \$1.78/hour, respectively, for the CASE 3. Each value of time has been computed as:

$$\text{value of in-vehicle time (1)} = (\gamma_1/\gamma_2)\ 60. \tag{47}$$

$$\text{value of excess time (2)} = (\gamma_3/\gamma_2)\ 60. \tag{48}$$

Table 3. Travel cost structure of the Chicago network

(CASE 2)

Criteria	AUTO	TRANSIT	TOTAL
Estimated entropy	7.4228	1.3449	8.7677
Likelihood function	7.4388	1.3522	8.7910
Modal Choice (%)	82.890	17.110	100.
Mean travel cost by components			
Travel time (min)	31.659	31.671	
Money cost (cent)	32.725	56.752	
Access/eggress time (min)	4.954	17.659	
Generlized cost	17.717	26.480	

(CASE 3)

Criteria	AUTO	TRANSIT	TOTAL
Estimated entropy	7.3888	1.3692	8.7579
Likelihood function	7.4447	1.3469	8.7916
Modal choice (%)	82.556	17.444	100.
Mean travel cost by components			
Travel time (min)	34.920	31.967	
Money cost (cent)	32.263	56.229	
Access/egress time (min)	4.909	17.419	
Generalized cost	18.411	26.015	

Table 4. Data base used for contour Map for Chicago network

θ_1 $(\times 10^{-3})$	θ_2 $(\times 10^{-3})$	θ_3 $(\times 10^{-3})$	μ $(\times 10^{-2})$	S	MLE
(.3873)	.2109	.7701	.1368	8.767651	8.790976
(.28305)	(.15413)	(.56282)			
.3873	.2109	.6342	.1232	8.793935	8.791107
(.31426)	(.17113)	(.51461)			
.3873	.2300	.8050	.1422	8.758727	8.791394
(.27231)	(.15171)	(.56598)			
.3873	.2600	.7750	.1422	8.757982	8.791605
(.27231)	(.18280)	(.54489)			
.3873	.2109	.8498	.1448	8.754479	8.792587
(.26747)	(.14565)	(.58688)			
3873	.2900	.7450	.1422	8.757261	8.792648
(.27231)	(.20390)	(.52380)			
.3873	.1406	.6342	.1162	8.810712	8.793504
(.33328)	(.12099)	(.54574)			
.3873	.1758	.9060	.1469	8.752403	8.793646
(.26363)	(.11967)	(.61670)			
.3873	.2109	.9060	.1504	8.746806	8.793773
(.25748)	(.14021)	(.60231)			
.3873	.1561	.9060	.1449	8.755152	8.794241
(.26721)	(.10770)	(.62509)			
.3873	.1406	.9060	.1434	8.757617	8.794631
(.27010)	(.09805)	(.63184)			
.3873	.1406	.9241	.1452	8.754573	8.795420
(.26674)	(.09683)	(.63643)			
.3873	.2813	.9060	.1575	8.735561	8.795805
(.24597)	(.17865)	(.57538)			
.3873	.3516	.6342	.1373	8.760868	8.798197
(.28206)	(.25606)	(.46187)			
.3873	.0703	.9060	.1364	8.766917	8.799377
(.28403)	(.05155)	(.66442)			
.3873	.3735	.6342	.1395	8.755810	8.800740
(.27763)	(.26774)	(.45462)			
.3873	.2109	1.178	.1776	8.714157	8.806763
(.21805)	(.11874)	(.66321)			
.3873	.2109	.3624	.0961	8.860574	8.807629
(.40319)	(.21955)	(.37726)			
.3873	.1406	1.178	.1706	8.720402	8.808445
(.22704)	(.08242)	(.69054)			
.3873	.1406	2.084	.2612	8.660092	8.887303
(.14828)	(.05383)	(.79789)			

Note that (.) denotes the estimated coefficients (i.e., γ) of the generalized cost function.

In general, the value of time poses some problems. It represents the traveler's perception of a suitable trade-off between time and money. In other words, how much the traveler is willing to pay to save traveling time on a journey. The rate may vary with the socio-economic status of the traveler and also on the circumstances of the trip.

It is of interest that the estimated value of time from the results of our analysis is comparable with past experience. For example, Wilson et al. (1969) reported the following generalized cost function:

$$U_{ijm} = 0.66\, t_{ijm} + 1.32\, e_{ijm} + w d_{ijm} \tag{49}$$

where U_{ijm} = generalized cost from i to j by mode m

t_{ijm} = in-vehicle travel time in minutes by mode m between i and j

e_{ijm} = excess travel time in minutes by mode m between in and j

d_{ijm} = distance in miles by mode m between i and j

w = 2.00 for car travel
3.18 for train travel
3.06 for bus travel

It should be noted from Eq. (49) that excess travel time is valued at twice the value of in-vehicle travel time. A number of studies of the generalized cost of travel have noted this higher valuation of excess travel time by tripmaker (see Hutchinson, 1974). The generalized cost functions for automobile and transit given in Hutchinson (1974) are:

Auto cost = 2 (in-vehicle minutes) + 5 (excess time in minutes) + 0.5 (parking charges)

Transit cost = 2 (in-vehicle minutes) + 5 (excess time in minutes) + 1 (fare) + 15

After normalizing the weighting factors above, one may find it very similar to our results. Using the values shown in Table 4 , which consists of twenty points for a fixed value of $\theta_1 = 0.3873\text{x}10^{-3}$, a partial contour has been constructed in Figure 3. The X-axis represents the various values of θ_2, and θ_3 is shown on the Y-axis. The values depicted on the contour map were obtained by the difference between the likelihood values and the minimum likelihood value, and factoring up 10,000 times. Twenty points are uniformly distributed, and then the spline curve fitting technique developed at NCAR (1981) was applied to obtain each contour. The parallel lines donote the corresponding entropy values. A value of zero means the constraint equals the observed entropy value, 8.7551. The region above the zero line represents the violation of the entropy contraint; the inactive solutions to the entropy contraints correspond to the lower region. One interesting point has been observed from the relationship between the likelihood values and the estimated entropy value. In Figure ..3, the current best point is located in a region where the entropy function is active.

Although some small suspicious points have been observed, the Figure 3 shows no general evidence of nonconvexity of the likelihood function with regard to the parameter θ.

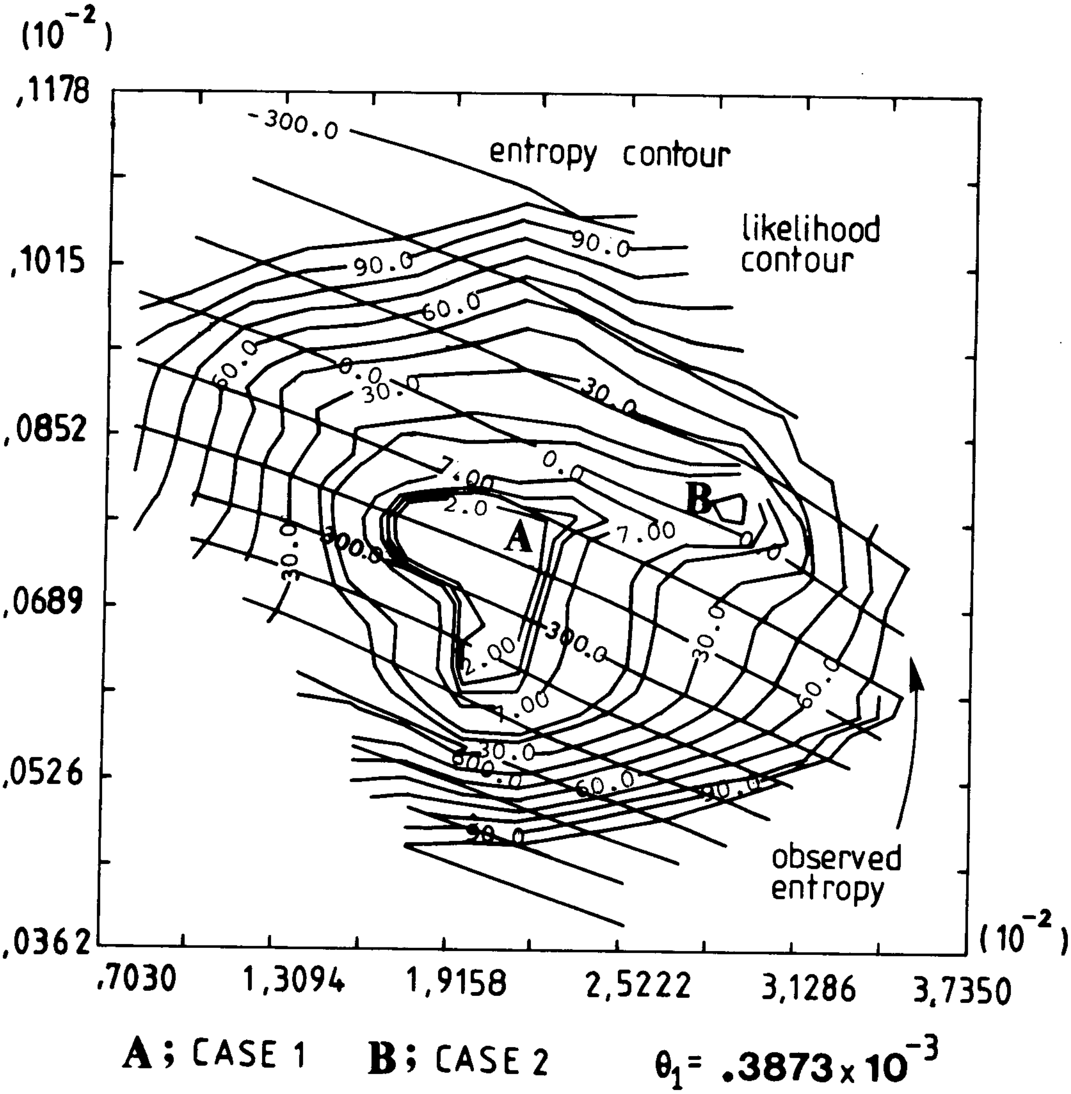

Figure 3. Contours of likelihood and entropy functions for the Chicago Region

4. Test of Model Performance

Tests of model performance have been made for the Chicago case in order to evaluate further validity of this approach. Eight evaluations are made against the past results reported in Chon's dissertation. As mentioned in the previous section, we considered two cases; (1) local minimum with an inactive constraint of the entropy function (2) local minimum with an (almost) active entropy constraint. For convenience, we denote CASE 1 for the former case and CASE 2 for the latter case. Table 5 summarizes the R^2 goodness-of-fit statistics corresponding to each case.

Table 5. Goodness-of-fit test for the Chicago Network (R^2)

	PAST RESULTS	CASE 1	CASE 2
TEST (A)	0.995	0.997	0.996
TEST (B)	0.980	0.987	0.986
TEST (C)	0.996	0.995	0.998
TEST (D)	0.998	0.998	0.999

Note:

TEST (A) = comparison of observed and estimated auto trips by origin

TEST (B) = comparison of observed and estimated transit trips by origin

TEST (C) = comparison of observed and estimated auto trips by destination

TEST (D) = comparison of observed and estimated transit trips by destination

As shown in Table 5, the results of this calibration, in general, give rise to a better fit against the observed data. Especially, the result of the evaluation on TEST (B) that compares the estimated transit trips with the observed transit trips by origin is notable.

5. Summary

In this study the estimates of the generalized cost coefficients are derived by the maximum likelihood estimation method. The results are compatiable with past research in this field. The method pursued in this study is so robust that one can estimate such coefficients without using the observed travel cost data. The only requirement in this method is the origin-destination trip matrix that is readily available in transportation planning from census data. In the analysis of the Chicago network, we conclude that the access/egress costs for both highway and transit modes are very important even in the congested network, while the travel time and operating cost are almost same. The model performance is fairly good in terms of R^2. The inherent structure of the likelihood function was analyzed over the parameter space. The contour maps obtained from the analysis empirically support the hypothesized unimodal shape of the likelihood function.

APPENDIX

In order to introduce the calibration problem of μ that is the reciprocal of the Lagrange multiplier associated with the entropy constraint, a slightly modified formulation from CMR in the previous section is considered: Define, ignoring q_{ijm} in (18) let

$$F(v) := \text{the objective function in (15)} \qquad \text{(A-1)}$$

$$G(p) := \sum_{ijm} P_{ijm} \ln (P_{ijm}) \qquad \text{(A-2)}$$

we have the equivalent formulation,

$$\text{(CMR)} \quad \underset{(v,p)}{\text{Minimize}} \quad F(v) + \frac{1}{\mu} G(p) \qquad \text{(A-3)}$$

Subject to: (15) – (16) and (18)

The problem (CMR) is a convex programming problem if the parameters to be calibrated, such as μ and γ are given. The convexity of the problem (CMR) has been examined by Evans (1976). Because s_a has been assumed to be continuous and strictly increasing, it is integrable and convex. In addition, k_a which is defined as highway operating cost is also an increasing function of the link flow. The sum of two convex functions is convex. The second derivative of G(p) always is non-negative, thus, it is convex. Therefore, the overall objective function defined in (A-3) is a strictly convex problem. If there exists a local minimum it is a unique and global one. In other words, once the parameters have been specified, the solution of (CMR) is unique. Thus, the calibration of the parameters in a model must be resolved before solving the (CMR). As stated earlier, quantifies the extent to which people take into account the equilibrium traveling cost when making decision. When goes to zero, they do not take account of traveling cost, and the travel demand will be distributed evenly among modes. On the other hand, when μ increases, the model produces modal demands that correspond to the least-cost mode.

Based on definitions of several fixed cost function, Evans (1970, 1971) explored the properties of μ in his trip distribution model, a typical gravity model which can also be derived by the entropy maximizing formulation of Wilson, (1967). In that case, the Lagrange multiplier, μ, which corresponds to a total system cost constraint, is reciprocally related to the multiplier associated with the entropy function in the (CMR). The role of μ is extensively investigated. The relationship between mean travel cost and μ is given in greater detail. If the mean cost is not equal to either its minimum or maximum value, μ can be determined from the given cost, because the mean cost is a decreasing function with respect to μ.

Erlander et al. (1978) considered another calibration problem in which μ was defined as the Lagrange multimplier associated with the entropy constraint as shown in (18). They investigated the properties of μ in the CDR model proposed by Florian et al. (1975). We shall prove some properties of μ in (CMR) in accordance with Erlander et al. (1978).

Suppose that we have obtained the optimal equilibrium solutions, v^* and p^* for given value of μ. As stated earlier, the problem to be solved is a convex programming problem in which the objective function is strictly convex with linear constraints. Thus, its necessary conditions are also sufficient. Since the v^* and p^* are now functions of μ, $v^*(\mu)$ and $p^*(\mu)$ are used explicitly. The equilibrium flows, v^*, and the equilibrium demands, p^* are well-defined functions for $\mu > 0$. If $\mu = 0$, the problem of the (CMR) is not properly defined. Hence, we confine ourselves to $\mu > 0$.

Lemma 1

The problem CMR has a unique optimal solution $v^*(\mu)$, $p^*(\mu)$.

Proof

It is obvious from the convexity of the objective function with regard to v and p and the convex domain with linear constraints.

Lemma 2

For $\mu > 0$, $F(\mu)$ is a decreasing function with respect to μ, and $G(\mu)$ is increasing.

Proof

For given μ_1, μ_2, we have

$$F(\mu_1) + \frac{1}{\mu_1} G(\mu_1) \leq F(\mu_2) + \frac{1}{\mu_1} G(\mu_2) \tag{A-4}$$

$$F(\mu_2) + \frac{1}{\mu_2} G(\mu_2) \leq F(\mu_1) + \frac{1}{\mu_2} G(\mu_1) \tag{A-5}$$

The inequalities in (A-4) and (A-5) are derived from the definition of the optimum solution. Thus, we have:

$$F(\mu_1) - F(\mu_2) \leqq \frac{1}{\mu_1} [G(\mu_2) - G(\mu_1)] \tag{A-6}$$

$$F(\mu_2) - F(\mu_1) \leqq \frac{1}{\mu_2} [G(\mu_1) - G(\mu_2)] \tag{A-7}$$

Thus,

$$\mu_2 [F(\mu_1) - F(\mu_2)] \leqq G(\mu_2) - G(\mu_1) \leqq \mu_2 [F(\mu_1) - F(\mu_2)] \tag{A-8}$$

Therefore,

$$(\mu_1 - \mu_2)[F(\mu_1) - F(\mu_2)] \leqq 0 \tag{A-9}$$

Equation (A-9) means that F is montone (decreasing) with respect to . In the same manner, we have

$$(\mu_2 - \mu_1)[G(\mu_2) - G(\mu_1)] \geqq 0 \tag{A-10}$$

Hence, G is also montone (but, increasing) with respect to μ. It follows that for $\mu_2 > \mu_1$, we have.

$$F(\mu_2) - F(\mu_1) \leqq 0 \tag{A-11}$$

$$G(\mu_2) - G(\mu_1) \geqq 0 \tag{A-12}$$

It should be noted that if either F or G is constant for $\mu_2 > \mu_1 > 0$, the other F, G, so that v, p all are constant. The following two theorems can be used for the sensitivity analysis of the equilibrium solutions.

Theorem

For $\mu > 0$, suppose that μ is not stable. Then, link flows are decreasing as μ increases. Furthermore, the corresponding total travel cost is also decreasing.

Proof

By the assumption that F is a well-defined function, F is differentiable. If one takes derivatives of F with respect to μ,

$$\frac{\partial F}{\partial \mu} = s_a(v_a) \frac{\partial v_a}{\partial \mu} < 0 \tag{A-13}$$

Since $s_a(v_a)$ is positive, we have

$$\frac{\partial v_a}{\partial \mu} < 0 \qquad \text{(A-14)}$$

Therefore, v_a is decreasing as μ increases. Suppose that (A-14) is satisfied for all a. Then we have:

$$\Sigma_a \; s_a(v_a) \varDelta v_a < 0 \qquad \text{(A-15)}$$

where $\varDelta v_a$ is partial derivative of v_a with respect to μ. Therefore, (total) travel cost is decreasing with μ.

Theorem

Having the same assumptions as in the previous theorem, that is, μ is not stable for $\mu > 0$, it is true to have either: if P_{ijm} is greater than (l/e), where e is a base of natural logarithm, then the travel demand is increasing with respect to μ; or, if P_{ijm} is less than (l/e), then the travel demand is decreasing with respect to μ. Furthermore, the corresponding entropy value is also increasing for $P_{ijm} >$ (l/e), and decreasing for $P_{ijm} <$ (l/e).

Proof

By the same assumption that G is well-defined, and hence, diffenentiable, we have:

$$\frac{\partial G}{\partial \mu} = (\ln p_{ijm} + 1) \frac{\partial p_{ijm}}{\partial \mu} > 0 \qquad \text{(A-16)}$$

Thus, we have either

$$\frac{\partial P_{ijm}}{\partial \mu} > 0 \quad \text{if} \;\; p_{ijm} > (1/e) \qquad \text{(A-17)}$$

or,

$$\frac{\partial P_{ijm}}{\partial \mu} < 0 \quad \text{if} \;\; 0 < Pij_m < (1/e) \qquad \text{(A-18)}$$

Furthermore, suppose that equations (A-17) and (A-18) are satisfied for all i, j, m; then we have:

$$\Sigma_{ijm}(\ln P_{ijm}) \varDelta P_{ijm} < 0 \quad \text{if} \;\; P_{ijm} > (1/e) \qquad \text{(A-19)}$$

or,

$$\Sigma_{ijm}(\ln P_{ijm}) \varDelta P_{ijm} > 0 \quad \text{if} \;\; 0 < P_{ijm} < (1/e) \qquad \text{(A-20)}$$

where ΔP_{ijm} is partial derivative of P_{ijm} with respect to μ. The corresponding entropy value $-G$ is decreasing or increasing, depending upon the value of P_{ijm}

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