論 文
35~8~4

행렬부호함수를 이용한 이산치 계통의 모델 저차화

Model-Reduction of Linear Discrete Large-Scale Systems by use of the Sign Function

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요 약

본 논문은 행렬부호함수를 사용하여 대단위계통에 대한 이산치 저차화 모델을 결정하는 방법을 제시한다. 행렬부호함수에 의해 投影연산자를 정의하였으며 이를 사용하여 모델 저차화에 대한 알고리즘을 구하였다. 시뮬레이션을 통해 제안된 알고리즘이 매우 유용함을 보였다.

Abstract

This paper presents an approach for determining the discrete reduced-order models for large-scale system by using matrix sign function. We define projection operators based on the matrix sign function and develop the algorithm for model-reduction by using them. Simulation studies show that the proposed algorithm is very useful.

1. Introduction

The analysis and design problem of high-dimensional physical systems is not an easy task because of computational difficulties and economic considerations. Thus model-reduction methods have recently received considerable attentions in the analysis and synthesis of large-scale systems.[1),2)]

In the frequency domain, the reduced-degree models can be determined via the continued fraction methods[3)], the dominant or equivalent dominant pole retention methods[4),5)], the moment or dominant data-matching methods, [6)] etc. On the other hand, the reduced-order models can be determined via the aggregation methods[1)], the singular perturbation method[7)], the state-space reduction techniques or time-scale analysis in the time domain.[1)] The time domain model-reduction methods mentioned above need either the major knowledge of overall characteristics, or the eigenvalues and eigenvectors of the system matrices.

To develop a model-reduction method for large-scale systems without preknowledge of the major characteristics of the systems, J.D.Roberts applied matrix sign function to Marshall's reduc-

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接受日字 : 1986年 2月 7日

tion technique[3]. This method is restricted only to the continuous systems. In his reduction procedure, he didn't present the generalized matrix factorization algorithm.

In this paper, since digital computers are increasingly being used to implement control systems, we extend the Roberts' continuous reduction algorithm to discrete systems by defining the projection operators. In this reduction algorithm, we propose the simple, generalized matrix factorization algorithm by using elementary operation of a matrix.

This paper is organised as follows: in chapter 2, we review the matrix sign function and its numerical computation. In chaper 3, we extend the Roberts' algorithm to discrete systems. In chapter 4, two examples(weakly coupled system and strongly coupled system) are presented to demonstrate the proposed model-reduction algorithm. In chapter 5, we show several conclusions.

2. The Matrix Sign Function and Its Numerical Computation

This chapter introduces briefly the definition of the matrix sign function and its numerical computation.[9]

The function of a matrix can be defined in terms of its Jordan canonical from. If A is an nxn real or complex matrix, it can be represneted by following equivalent form.

$$A=M\wedge M^{-1} \tag{1}$$

where M is the matrix of eigenvectors for the eigenvalues in $\wedge$ and $\wedge$ is the Jordan canonical from for the eigenvalues of A such that

$$\Lambda=\begin{bmatrix} J_1 & & & 0 \\ & J_2 & & \\ & 0 & \ddots & \\ & & & J_k \end{bmatrix} \tag{2}$$

k : Jordan block numbers

J_i : Jordan block for the eigenvalues λ_i of A

i=1, 2, ..., k

Let $f(\lambda)$ be a function defined on the spectrum of A, ie., for each eigenvalue λ_i of A, $f(\lambda_i)$ is defined. A function of the matrix A, f(A), is defined as

$$f(A)=Mf(\Lambda)M^{-1} \tag{3}$$

where

$$f(\Lambda)=\text{diag}[f(J_1),\ f(J_2),\ \ldots,\ f(J_k)] \tag{4}$$

and $f(J_i)$ for each Jordan block is given by

$$f(J_i)=\begin{bmatrix} f(\lambda_i) & f'(\lambda_i)/1! \cdots & f^{(n-1)}(\lambda_i)/(n-1)! \\ 0 & f(\lambda_i) \cdots & f^{(n-2)}(\lambda_i)/(n-2)! \\ \cdots & \cdots & \cdots \\ 0 & 0 & f(\lambda_i) \end{bmatrix} \tag{5}$$

A scalar sign function can be defined as

$$f(\lambda)=sign(\lambda)=\begin{cases} +1, & \text{if } Re(\lambda)>0 \\ -1, & \text{if } Re(\lambda)<0 \end{cases} \tag{6}$$

The corresponding matrix sign function is then

$$sign(A)=M\,sign(\wedge)M^{-1} \tag{7}$$

and

$$sign(\wedge)=\text{diag}\{sign(J_1),\ sign(J_2),\ \ldots,\ sign(J_k)\}$$

since (8)

$$\frac{d}{d\lambda_i}[sign(\lambda_i)]=0 \tag{9}$$

it follows according to Eq.(5) that $sign(J_i)$ is diagonal. It is important to note that the eigenvalues of sign(A) are ±1 and its eigenvectors are the same as the original matrix A.

An efficient iterative algorithm to compute the matrix sign function as defined from Eq.(6) to Eq.(8) was established by use of the Newton-Raphson's method. This algorithm is as follows.[8]

$$S(k+1)=\alpha_k S(k)+\beta_k S(k)^{-1},\ S(0)=A \tag{10}$$

where

$$\alpha_k=\|S(k)^{-1}\|/(\|S(k)\|+\|S(k)^{-1}\|)$$
$$\beta_k=\|S(k)\|/(\|S(k)\|+\|S(k)^{-1}\|)$$

For this iterative computation, there are some stopping conditions, some of which are

$$|S(k+1)-S(k)|\leq\mu$$

$$|S^2(k+1)-I_n| \leq \mu$$
$$|\text{trace } S^2(k+1)-n| \leq \mu$$

where μ is a small, preselected error bound.

3. Model-reduction of discrete linear Large-Scale Systems

Consider a discrete linear large-scale time-invariant system with (A, B, C, D) represented by

$$X(k+1)=AX(k)+Bu(k) \quad (11a)$$
$$y(k)=CX(k)+Du(k) \quad (11b)$$

where $X(k) \in R^n$, $u(k) \in R^m$, and $y(k) \in R^p$ are state, input and output vectors, respectively. A, B, C, and D are matrices of appropriate dimensions.

Taking Z-transform in Eq.(11a), we have

$$X(z)=(zI-A)^{-1}Bu(z) \quad (12)$$

For the matrix step input, $u(k)=u_s(k)u_1$

$$X(z)=(zI-A)^{-1}\frac{z}{z-1}Bu_1$$
$$=(I/z+A/z^2+\cdots)\frac{z}{z-1}Bu_1 \quad (13)$$

where

$u(z)=Z\{u(k)\}=\frac{z}{z-1}u_1$, $u_s(k)$: unit step sequence, u_1 : $(m \times 1)$ vector

Taking inverse Z-transform in Eq.(13), we have

$$X(k)=\sum_{n=1}^{\infty} u_s(k-n)A^{n-1}Bu_1$$
$$=\text{Sum}(A)Bu_1 \quad (14)$$

where

$$\text{Sum}(A) \triangleq \sum_{n=1}^{\infty} u_s(k-n)A^{n-1} \quad (15)$$

Thus the matrix step response of Eq.(11) can be expressed as

$$y_s(k)=\{C\text{Sum}(A)B+Du_s(k)\}u_1 \quad (16)$$

Assuming that

$$A=M\begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix}M^{-1} \quad (17)$$

where

Λ_1 : a class of eigenvalues of slow modes
Λ_2 : a class of eigenvalues of fast modes

From Eq.(3) and Eq.(15), Sum(A) becomes

$$\text{Sum}(A)=M\begin{bmatrix} \text{Sum}(\Lambda_1) & 0 \\ 0 & \text{Sum}(\Lambda_2) \end{bmatrix}M^{-1} \quad (18)$$

Now the matrix step response can be approximated by replacing fast modes by straight-through components.[1]

$$y_s(k) \cong \left\{CM\begin{bmatrix} \text{Sum}(\Lambda_1) & 0 \\ 0 & (I-\Lambda_2)^{-1}u_s(k) \end{bmatrix} M^{-1}B+Du_s(k)\right\}u_1 \quad (19)$$

To obtain the reduced-ordcr model by applying the matrix sign function to Eq.(19), we define the projection operators for the mode separation.

3.1 Definition of projection operators

In discrete system, the class of eigenvalues located near the unit circle in the Z-plane are assigned to the slow mode and the class of eigenvalues near the origin are assigned to the fast mode. Thus, we need the reference circle with radius γ for mode separation in the Z-plane.

The positive real number γ is chosen as the geometric mean of the magnitudes of eigenvalues of A.

$$\gamma=\sqrt[n]{|\det(A)|} \quad (20)$$

where $\det(A)=\prod_{i=1}^{n} \lambda_i \quad (21)$

The reference circle with radius γ is shown in Figure 1.

In Figure 1, slow and fast mode of a discrete system are classified as follows.

$\lambda_i \in \{E_f | E_f \text{ is fast mode}\}$, if $|\lambda_i| < \gamma$
$\lambda_i \in \{E_s | E_s \text{ is slow mode}\}$, if $|\lambda_i| > \gamma$

Now projection operators for slow and fast modes can be defined by the use of Theorem 1.

Theorem 1.

The eigenvalues of $\text{sign}\{(A-\gamma I)(A+\gamma I)^{-1}\}$ are

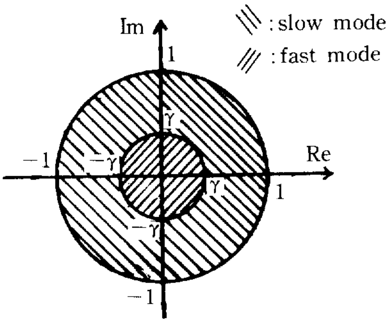

Fig. 1. Reference circle with radius γ on the Z-plane

± 1 according to the magnitude of the corresponding eigenvalues of matrix A. i. e.

$$\sigma_i[\mathrm{sign}\{(A-\gamma I)(A+\gamma I)^{-1}\}]=\begin{cases}+1, \text{ if } |\lambda_i|>\gamma \\ -1, \text{ if } |\lambda_i|<\gamma\end{cases}$$

where σi is the i-th eigenvalue of sign{$(A-\gamma I)(A-\gamma I)^{-1}$} corresponding to the i-th eigenvalue λ_i of A.

Proof)

Let λ_i be i-th eigenvalue of A and $\lambda_i=a_i+jb_i$ then from Eq.(17)

$$A-rI=M\begin{vmatrix}\ddots & & 0 \\ & \lambda_i-r & \\ 0 & & \ddots\end{vmatrix}M^{-1}$$

Thus

$$(A-rI)(A+rI)^{-1}=M\begin{vmatrix}\ddots & & 0 \\ & \frac{\lambda_i-r}{\lambda_i+r} & \\ 0 & & \ddots\end{vmatrix}M^{-1}$$

From Eq.(7)

$$\mathrm{sign}[(A-rI)(A+rI)^{-1}]$$
$$=M\begin{bmatrix}\mathrm{sign}(\cdot) & & & & 0 \\ & \ddots & & & \\ & & \mathrm{sign}\left(\frac{\lambda_i-r}{\lambda_i+r}\right) & & \\ & & & \ddots & \\ 0 & & & & \mathrm{sign}(\cdot)\end{bmatrix}M^{-1}$$

Thus

$$\sigma_i=\mathrm{sign}\left(\frac{\lambda_i-r}{\lambda_i+r}\right)=\begin{cases}+1, \text{ if } \mathrm{Re}\left(\frac{\lambda_i-r}{\lambda_i+r}\right)>0 \\ -1, \text{ if } \mathrm{Re}\left(\frac{\lambda_i-r}{\lambda_i+r}\right)<0\end{cases} \quad (22)$$

Since

$$\mathrm{Re}\left(\frac{\lambda_i-r}{\lambda_i+r}\right)=\frac{a_i^2+b_i^2-r^2}{(a_i+r)^2+b_i^2}=\frac{|\lambda_i|-r^2}{(a_i+r)^2+b_i^2}$$

Eq. (22) can be expressed equvalently as Eq. (23)

$$\sigma_i=\begin{cases}+1, \text{ if } |\lambda_i|>r \\ -1, \text{ if } |\lambda_i|<r\end{cases} \quad (23)$$

(Q. E. P)

Now we define two projection operators $P_s(A)$, $P_f(A)$ which can separate slow and fast modes as follows

$$P_s(A)=\mathrm{sign}^+\{(A-\gamma I)(A+\gamma I)^{-1}\} \quad (24a)$$
$$P_f(A)=I-\mathrm{sign}^+\{A-\gamma I)(A+\gamma I)^{-1}\} \quad (24b)$$

where

$$\mathrm{sign}^+(E)=\frac{1}{2}[I+\mathrm{sign}(E)] \quad (25)$$

E : arbitrary square matrix

And these projection operators have properties as follows;

i) $P_s(A)+P_f(A)=I$

ii) $P_s(A)=MP_s(\Lambda)M^{-1}$

iii) $P_f(\Lambda)=MP_f(\Lambda)M^{-1}$

iv) $P_s(\Lambda)=\left[\begin{array}{c:c} I & 0 \\ \hdashline 0 & 0\end{array}\right]$, $P_f(\Lambda)=\left[\begin{array}{c:c} 0 & 0 \\ \hdashline 0 & I\end{array}\right]$

v) $P_s(A)^n=P_s(A)$, $n=2, 3, \cdots$

vi) $P_f(A)^n=P_f(A)$, $n=2, 3, \cdots$

There properties are obtained easily from Eq. (3), Eq.(24) and Eq.(25).

3.2 Model-reduction by the projection operators

The expression for $y_S(k)$ given Eq.(19) can be expressed by the use of properties of two projection operators $P_S(A)$, $P_f(A)$.

$$y_S(k)\cong\bar{y}_S(k)=\{CP_S\mathrm{Sum}[P_S(A)A]B+[CP_f(A)(I-A)^{-1}B+D]u_S(k)\}u_1 \quad (26)$$

The derivation of Eq.(26) is shown in Appendix

Let $\bar{A}=P_S(A)A$, $\bar{B}=B$, $\bar{C}=CP_S(A)$, and $\bar{D}=CP_f(A)(I-A)^{-1}B+D$, then

$$\bar{y}_S(k)=\{\bar{C}\mathrm{Sum}(\bar{A})\bar{B}+\bar{D}u_S(k)\}u_1 \quad (27)$$

Thus, $\bar{y}_S(k)$ is the matrix step response of the system expressed by ($\bar{A}$, B, $\bar{C}$, $\bar{D}$). These matrices are reduced in rank but are still of the same dimension as (A, B, C, D). It remains to transform this to a representation using a lower dimensional matrix of full rank. The crucial step is to find matrices H(n×r), $\bar{\bar{A}}$(r×r), K(r×n) such that

$$\bar{A}=H\bar{\bar{A}}K \quad (28b)$$

$$KH=I_r,\ \mathrm{Rank}(\bar{A})=\mathrm{Rank}(\bar{\bar{A}})=r \quad (28a)$$

From Eq.(15), Eq.(27), and Eq(28), $\bar{y}_S(k)$ becomes

$$y_S(k)=\{\bar{C}H\mathrm{Sum}(\bar{\bar{A}})KB+\bar{D}u_S(k)\}u_1 \quad (29)$$

To determine the required matrices H, K, we need following procedures.

i) determine J(n×r), K(r×n) such thbt $\bar{A}$=JK (i.e., J−K factorization)
ii) define $\bar{\bar{A}}$=KJ
iii) define H=J$\bar{\bar{A}}^{-1}$

Then, Eq.(28) satisfied as follows

$$H\bar{\bar{A}}K=J\bar{\bar{A}}^{-1}\bar{\bar{A}}K=JK=\bar{A} \quad (30a)$$

$$KH=KJ\bar{\bar{A}}^{-1}=\bar{\bar{A}}\bar{\bar{A}}^{-1}=I_r \quad (30b)$$

Therefore, the matrix step response of completely reduced-order system given by

$$y_S(k)=\{\bar{\bar{C}}\mathrm{Sum}(\bar{\bar{A}})\bar{\bar{B}}+\bar{\bar{D}}u_S(k)\}u_1 \quad (31)$$

where

$$\bar{\bar{A}}=KJ \quad (32a)$$

$$\bar{\bar{B}}=K\bar{B}=KB \quad (32b)$$

$$\bar{\bar{C}}=CH=CP_S(A)J\bar{\bar{A}}^{-1} \quad (32c)$$

$$\bar{\bar{D}}=\bar{D}=C[I-P_S(A)](I-A)^{-1}B+D \quad (32d)$$

And the reduced-order model is expressed as follows

$$X_r(k+1)=\bar{\bar{A}}X_r(k)+\bar{\bar{B}}u(k),\ X_r(0)=\bar{\bar{C}}^T[\bar{\bar{C}}\bar{\bar{C}}^T]^{-1}CX(0) \quad (33a)$$

$$y_r(k)=\bar{\bar{C}}X_r(k)+\bar{\bar{D}}u(k) \quad (33b)$$

Thus the reduction problem of discrete system is concentrated on J−K factorization problem. This method discussed in next section.

3. 3 J-K factorization

A generalized factorization algorithm for a matrix A(n×n) with rank(A)=r≤n by using elementary operation is proposed as following steps.

Let

$$A=\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \quad (34)$$

Step 1

Taking the elementary row operation on (34), ve have

$$A'=\begin{bmatrix} a'_{11} & a'_{12}\cdots a'_{1r}\cdots a'_{1n} \\ 0 & a'_{22}\cdots a'_{2r}\cdots a'_{2n} \\ \vdots & \vdots \quad \vdots \\ 0 & \cdots \ a'_{rr}\cdots a'_{rn} \\ \hline \multicolumn{2}{c}{0_{(n-r)\times n}} \end{bmatrix} \quad (35)$$

Step 2

Taking the elementary column operation on (35), we have

$$A''=\left[\begin{array}{c|c} \begin{matrix} a''_{11} & & 0 \\ & \ddots & \\ 0 & & a''_{rr} \end{matrix} & 0_{r\times(n-r)} \\ \hline 0_{(n-r)\times r} & 0_{(n-r)\times(n-r)} \end{array}\right] \quad (36)$$

Next, diag$\{a''_{11}\cdots\cdots a''_{rr}\}$ can be factorized into itself and unit matrix.

$$[I_{r\cdot r}]\begin{bmatrix} a''_{11} & & 0 \\ & \ddots & \\ 0 & & a''_{rr} \end{bmatrix} \text{ or } \begin{bmatrix} a''_{11} & & 0 \\ & \ddots & \\ 0 & & a''_{rr} \end{bmatrix}[I_{r\cdot r}] \quad (37)$$

Thus (37) is reformed by the following form

$$\begin{bmatrix} I_{rr} \\ \hline 0_{(n-r)\times r} \end{bmatrix}\left[\begin{array}{c|c} \begin{matrix} a''_{11} & & 0 \\ & \ddots & \\ 0 & & a''_{rr} \end{matrix} & 0_{(n-r)\times r} \end{array}\right]=J'K' \quad (38)$$

Sine A″=J′K′, matrices J and K can be obtained by the taking inverse procedure in step 1, 2 to J′ and K′, respectively.

4. Numerical Examples

4. 1 Weakly Coupled System

Consider a fifth order system[10] with coupling ratio 0.25 as follows.

$$X(k+1)=\begin{bmatrix} 1.005 & -0.03 & 0.105 & -0.215 & 0.39 \\ -0.005 & 0.98 & -0.035 & 0.065 & -0.05 \\ -0.01 & 0.02 & 0.91 & 0.17 & -0.3 \\ -0.005 & 0.01 & 0.025 & 1.045 & -0.25 \\ 0. & 0. & 0. & 0. & 0.92 \end{bmatrix} X(k)+\begin{bmatrix} -0.02 & 0.01 & 0. \\ 0.01 & -0.02 & -0.03 \\ 0.01 & 0.01 & 0.03 \\ 0.02 & 0.01 & 0.04 \\ 0.01 & 0. & 0.01 \end{bmatrix} u(k),\ X(0)=0_{5\times1}$$

$$y(k)=\begin{bmatrix} 10. & -15. & 41. & -65. & 131. \\ 5. & -2. & 10. & -2. & 6. \end{bmatrix} X(k)$$

From Eq.(20), radius of reference circle becomes

$$\gamma=0.97156$$

In Eq.(24), two projection operators $P_S(A)$, $P_f(A)$ obtained as follows

$$P_s(A)=\begin{bmatrix} 1. & 0. & 1. & -2. & 4. \\ 0. & 1. & 4. & 1. & -8. \\ 0. & 0. & 0. & 2. & -4. \\ 0. & 0. & 0. & 1. & -2. \\ 0. & 0. & 0. & 0. & 0. \end{bmatrix}$$

and

$$P_f(A)=\begin{bmatrix} 0. & 0. & -1. & 2. & -4. \\ 0. & 0. & -4. & -1. & 8. \\ 0. & 0. & 1. & -2. & 4. \\ 0. & 0. & 0. & 0. & 2. \\ 0. & 0. & 0. & 0. & 1. \end{bmatrix}$$

Since $\overline{A}=P_S(A)A$, the matrix $\overline{A}$ with reduced rank becomes

$$\overline{A}=\begin{bmatrix} 1.005 & -0.03 & 1.065 & -2.135 & 4.27 \\ 0.005 & 0.98 & -1.955 & 3.905 & -7.81 \\ -0.01 & 0.02 & 2.09 & -0.05 & -4.18 \\ -0.005 & 0.01 & -0.025 & 1.045 & -2.09 \\ 0. & 0. & 0. & 0. & 0. \end{bmatrix}$$

Since $\text{rank}(\overline{A})=3$, the matrix $\overline{A}$ can be factorized into $J(5\times3)$ and $K(3\times5)$ by section 3-3.

$$J=\begin{bmatrix} 1. & 0. & 0. \\ -0.00498 & 1. & 0. \\ -0.00995 & 0.02 & 1. \\ -0.00498 & 0.01 & 0.5 \\ 0. & 0. & 0. \end{bmatrix}$$

$$K=\begin{bmatrix} 1.005 & -0.03 & 1.065 & -2.135 & 4.27 \\ 0. & 0.98 & -1.96 & 3.9156 & -7.83 \\ 0. & 0. & 0. & 1.99 & -3.98 \end{bmatrix}$$

Thus reduced model is obtained by Eq.(32)

$$X_r(k+1)=\begin{bmatrix} 1.0049 & -3.005 & -0.0025 \\ 0.0049 & 0.98 & -0.0025 \\ -0.0099 & 0.02 & 0.995 \end{bmatrix} X_r(k)+\begin{bmatrix} -0.00975 & -0.00005 & -0.00985 \\ -0.0098 & -0.00005 & -0.0099 \\ 0. & 0.0199 & 0.0398 \end{bmatrix} u(k),\ X_r(0)=0_{3\times1}$$

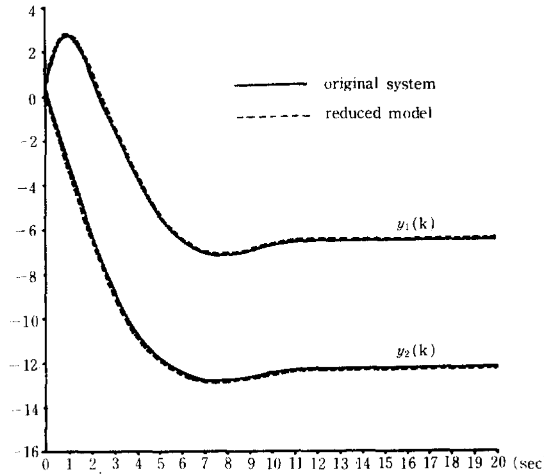

Fig. 2. Responses of $y_1(K), y_2(K)$ for input$[100]^T$

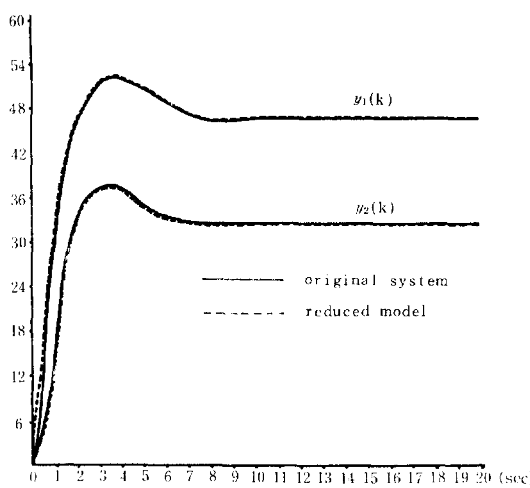

Fig. 3. Response of $y_1(K), y_2(K)$ for input$[001]^T$

$$y_r(k) = \begin{bmatrix} 9.9502 & -14.999 & 8.53 \\ 4.975 & -1.888 & 9.0528 \end{bmatrix} X_r(k) + \begin{bmatrix} 0.5 & -0.25 & 0. \\ 0.625 & -0.25 & 0.125 \end{bmatrix} u(k)$$

The resulting step response of both the original and reduced order model are plotted in Figure2, 3.

4.2 Strongly Coupled System

Consider a voltage regulator problem[1] whose fifth-order model with coupling ratio 0.7 is given by

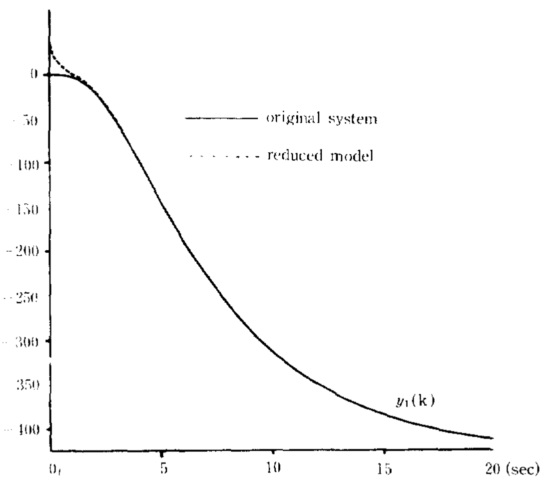

Fig. 4. Unit Step response of $y_1(K)$

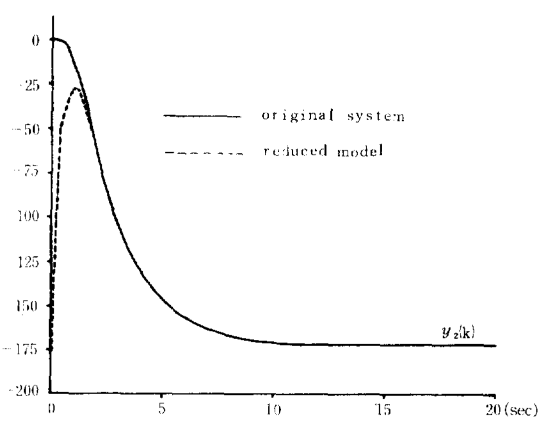

Fig. 5. Unit Step response of $y_2(k)$

$$X(k+1) = \begin{bmatrix} 0.998 & 0.005 & 0. & 0. & 0. \\ 0. & 0.995 & -0.016 & 0. & 0. \\ 0. & 0. & 0.971 & 0.171 & 0. \\ 0. & 0. & 0. & 0.95 & 0.15 \\ 0. & 0. & 0. & 0. & 0.98 \end{bmatrix} X(k) + \begin{bmatrix} 0. \\ 0. \\ 0. \\ 0.005 \\ 0.058 \end{bmatrix} u(k), \quad X(0) = 0_{5\times 1}$$

$$y(k) = \begin{bmatrix} 1. & 0. & 0. & 0. & 0. \\ 0. & 1. & 0. & 0. & 0. \end{bmatrix} X(k)$$

Reduced order model is obtained by the same procedure in example 4.1.ie

$$X_r(k+1) = \begin{bmatrix} 0.998 & 0.005 & 0. \\ 0. & 0.995 & -0.016 \\ 0. & 0. & 0.98 \end{bmatrix} X_r(k) + \begin{bmatrix} -0.85706 \\ 4.2852 \\ -0.00008 \end{bmatrix} u(k), \quad X_r(0) = 0_{3\times 1}$$

$$y_r(k) = \begin{bmatrix} 1.002 & -0.005 & -0.00008 \\ 0. & 1.005 & 0.0164 \end{bmatrix} X_r(k) + \begin{bmatrix} 31.617 \\ -163.5 \end{bmatrix} u(k)$$

The resulting response of both the original and reduced order model are plotted in Figure 4, 5.

5. Conclusions

In this paper, we present a model-reduction algorithm of linear discrete large-scale systems by projection operators based on the matrix sign function. As shown in examples, this newly proposed algorithm is quite useful for the model-reduction of discrete large-scale time-invariant systems. The main results of this paper are ;

a) we develop a model-reduction method for discrete large-scale systems by defining projection operators without preknowledge of the major characteristics of the system and arrangement of the eigenvalues of A.

b) We propose the more generalized, simpler J-K factorization algorithm by elementary operatition of a matrix.

An extension of this model-reduction algorithm to the case that eigenvalues exist on the reference circle is under research.

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Appendix

From Eq.(19) and properties of projection operators, Eq.(26) becomes

$$\bar{y}_s(k)=\{C[P_s(A)+P_r(A)]M\begin{bmatrix}\mathrm{Sun}(\Lambda_1) & 0\\ 0 & (\mathrm{I}-\Lambda_2)^{-1}u_s(k)\end{bmatrix}M^{-1}B+Du_s(k)\}u_1$$
$$=\{CP_s(A)M\begin{bmatrix}\mathrm{Sum}(\Lambda_1) & 0\\ 0 & (\mathrm{I}-\Lambda_2)^{-1}u(k)\end{bmatrix}M^{-1}B+CP_r(A)M\begin{bmatrix}\mathrm{Sum}(\Lambda_1) & 0\\ 0 & (\mathrm{I}-\Lambda_2)^{-1}u_s(k)\end{bmatrix}M^{-1}B+Du_s(k)\}u_1 \quad \text{(A1)}$$

The first term of Eq. (A1) derived as follows

$$CP_s(A)M\begin{bmatrix}\mathrm{Sum}(\Lambda_1) & 0\\ 0 & (\mathrm{I}-\Lambda_2)^{-1}u_s(k)\end{bmatrix}M^{-1}B$$
$$=CP_s(A)P_s(A)M\begin{bmatrix}\mathrm{Sum}(\Lambda_1) & 0\\ 0 & (\mathrm{I}-\Lambda_2)^{-1}u_s(k)\end{bmatrix}M^{-1}B$$
$$=CP_s(A)MP_s(\Lambda)\begin{bmatrix}\mathrm{Sum}(\Lambda_1) & 0\\ 0 & (\mathrm{I}-\Lambda_2)^{-1}u_s(k)\end{bmatrix}M^{-1}B \quad \text{(A2)}$$

In Eq.(A2), since $P_s(\Lambda)$

$$\begin{bmatrix}\mathrm{Sum}(\Lambda_1) & 0\\ 0 & (\mathrm{I}-\Lambda_2)^{-1}u_s(k)\end{bmatrix}=\begin{bmatrix}\mathrm{Sum}(\Lambda_1) & 0\\ 0 & 0\end{bmatrix},$$

from property iv), component $(\mathrm{I}-\Lambda_2)^{-1}u_s(k)$ can be replaced into any components.

Thus Eq. (A2) can be written as follows

$$CP_s(A)MP_s(\Lambda)\begin{bmatrix}\mathrm{Sum}(\Lambda_1) & 0\\ 0 & \mathrm{Sum}(\Lambda_2)\end{bmatrix}M^{-1}B$$
$$=CP_s(A)MP_s(\Lambda)\,\mathrm{Sum}(\Lambda)M^{-1}B$$
$$=CP_s(A)M\ \mathrm{Sum}[P_s(\Lambda)\Lambda]M^{-1}B$$
$$=CP_s(A)\,\mathrm{Sum}[MP_s(\Lambda)M^{-1}M\Lambda M^{-1}]B$$
$$=CP_s(A)\,\mathrm{Sum}[P_s(A)A]B$$

Next, the second term of Eq.(A2) can be derived by similar way in the first term.