균일한 확률 밀도를 갖는 위상 불규칙 신호의 전력 스펙트럼 밀도

Power Spectral Density of Jittered Signal with Uniform Probability Density Function

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요 약

위상이 불규칙적으로 변하는 RZ(Return-to-Zero)와 NRZ (Nonreturn-to-Zero) 신호에 대하여 전력 밀도 스펙트럼을 구하였고 신호의 펄스폭 점유율은 가변으로 하였다. 이때 불규칙 위상의 확률 분포는 구간 내에서 일정하다고 가정한다. 단극성 지터없는 신호는 입력된 신호의 기본 주파수의 정수배마다 스펙트럼의 이산성분이 존재하며 이것은 데이타를 찾기 위한 타이밍 신호로써 이용된다.

그러나 지터가 유입되는 경우에는 이 이산 신호성분이 점차 감소하게 되며, 균일한 확률 분포를 갖는 지터의 경우는 완전히 소멸하였음을 확인하였다.

ABSTRACT

We have derived the power spectral density of RZ (Return-to-Zero) and NRZ Nonreturn-to-Zero) signals having the variable duty ratio, where jitter probability density is assumed to be uniform in an interval. For the unipolar jitter-free signal, the discrete components are distinctly shown at the signal frequency, fs, which is used in tracking the timing clock. When uniform jitter is injected to the digital signal, however, this discrete components disappear in power spectral density function.

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I. INTRODUCTION

In digital transmission system, it is desired that the signal should be sampled at the accurate time instants. Such occasions, however, rarely happen in practice. The CCITT defines jitter as "short term variations of the significant instants of a digital signal from their ideal positions in time". In general the sources of jitter are the Mux/Demux process, the limited channel bandwidth, channel noise, and CRU ⟨Clock Recovery Unit⟩ of the repeater or receivers. And it is generated and accumulated along the chain of repeaters.[1-3] For the spectrally efficient signal transmission, the data signal is coded to have no dc component by the bipolar line coding method, and through the BUU (bipolar to unipolar unit) it is converted to the unipolar signal that has the discrete spectral components at the stage of repeater or receivers. The fundamental discrete component is used to be the timing clock which decides the data value.

In this paper, we show that if the jitter with uniform probability density is injectted to the digital signals, it is physically observed that the discrete components of signal disappear, while the continuous components increase.

II. SIGNAL ANALYSIS

We supposed that the signal has jitter with the uniform probability density, and has the variable duty ratio. But in computer simulation, the fixed duty ratio 0.5 is used.

The equations of the signal shown in Fig. 1 are represented as follows.

1) In the case of the bipolar jitter-free signal

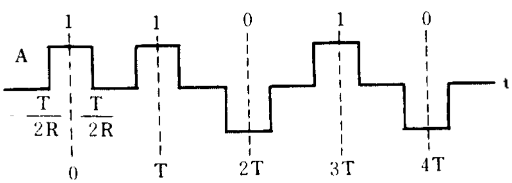

Fig. 1. Bipolar jitter-free signal

$$g_1(t) = \begin{cases} A, & -T/2R < t < T/2R \\ 0, & \text{elsewhere} \end{cases} \qquad (1)$$

$$g_2(t) = \begin{cases} -A, & -T/2R < t < T/2R \\ 0, & \text{elsewhere} \end{cases} \qquad (2)$$

, where $g_1(t)$, $g_2(t)$ stand for the transmitted data value, one and zero, respectively, and $1/R$ is duty ratio. By Fourier transforming the signal $g_1(t)$, $g_2(t)$, we obtain,

$$G_1(f) = \int_{-\infty}^{\infty} g_1(t)\, e^{-j2\pi ft}\, dt = \frac{A}{\pi f} \sin(\pi f T/R) \qquad (3)$$

$$G_2(f) = \int_{-\infty}^{\infty} g_2(t)\, e^{-j2\pi ft}\, dt = \frac{-A}{\pi f} \sin(\pi f T/R) \qquad (4)$$

Because $f_s = 1/T$, T is the period of the data symbol,

$$G_1(0) = G_2(0) = AT/R,\ G_1(mf_s) = G_2(mf_s) = 0, \quad m = \text{integer} \qquad (5)$$

According to the reference [4] the complete spectral density of random data signal which is composed of element signal, $g_1(t)$, $g_2(t)$, is found to have the following representation.

$$S(f) = 2 f_s p(1-p)\,|G_1(f) - G_2(f)|^2 + f_s^2\,[pG_1(0) + (1-p)G_2(0)]^2\,\delta(f) + 2f_s^2 \sum_{m=1}^{\infty} |pG_1(mf_s) + (1-p)G_2(mf_s)|^2\,\delta(f - mf_s) \qquad (6)$$

A summary of the terms and notations in eq. (6) follows:

S(f) = the complete power spectral density,

f_s = signaling frequency = symbol rate (=1/T) (in binary systems the bit rate equals the symbol rate, i. e., $f_b = f_s$),

$G_1(f)$ = the Fourier transform of the symbol $g_1(t)$,

$G_2(f)$ = the Fourier transform of the symbol $g_2(t)$,

p = probability of occurrence of $g_1(t)$ symbol,

1-p = probability of occurrence of $g_2(t)$ symbol,

m = an integer number (m=1, 2, 3,......).

From equations (3), (4), (5) and assumption (p = 1-p = 1/2), it follows

$$S(f)=\frac{2A^2T}{R^2}\sin c^2 Z+\frac{A^2}{R^2}\delta(f) \quad (7)$$

where $Z=\pi fT/R$, $\sin c\ x=\frac{\sin x}{x}$

(2) In the case of bipolar jittered signal

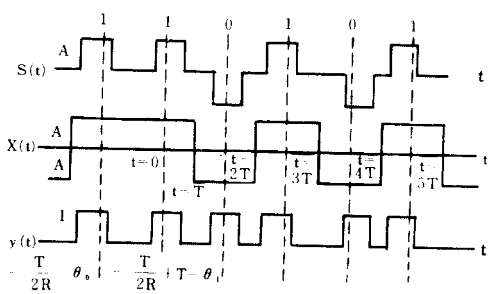

Fig. 2. Bipolar jittered signal

Fig. 2 depicts a sample of the bipolar jittered signal s(t) whose equation can be made up of x(t) and y(t). Each equation of such signals s(t), x(t), and y(t) is respectively siven by

$$s(t)=x(t)y(t)$$

$$=A\sum_m a_m \Pi\left(\frac{t-mT}{T}\right)\Pi\left(\frac{t-mT-\theta_m}{T/R}\right) \quad (8)$$

$$x(t)=A\sum_m a_m \Pi\left(\frac{t-mT}{T}\right) \quad (9)$$

$$y(t)=\sum_m \Pi\left(\frac{t-mT-\theta_m}{T/R}\right) \quad (10)$$

where $\Pi\left(\frac{t}{T}\right)=\begin{cases}1 & -T/2<t<T/2\\ 0 & \text{elsewhere}\end{cases}$

In Fig. 2, the time origin is specified for convenient evolution, but in fact no specific time origin should be chosen, and

where a_m = random variable having a value +1 or -1 with the equal probability

θ_m = random variable with a probability density function $p(\theta)$ uniform in interval [O, T]

Hence we can suppose that the above signal is defined as a wide-sense stationary and ergodic process, that is, an ensemble average of random variable at certain time point of the process equals to a time average over the entire time intervals.

If x(t) and y(t) are statistically independent and stationary process, then the autocorrelation function of equation (8), s(t), is

$$R_s(\tau)=E[s(t)s(t-\tau)]$$

$$=E[x(t)y(t)x(t-\tau)y(t-\tau)]$$

$$=E[x(t)x(t-\tau)]E[y(t)y(t-\tau)]$$

$$=R_x(\tau)R_y(\tau) \quad (11)$$

Since the events are independent of others in different intervals, we only consider an arbitrary interval [O, T]. And from an ergodic property, using a time average over the interval [O, T], individual processes x(t), y(t) are related in the autocorrelation function of s(t) as follows.

$$R_x(\tau) = E[x(t)x(t-\tau)] = \frac{A^2}{T}\int_0^T \Pi\left(\frac{t}{T}\right)\Pi\left(\frac{t-\tau}{T}\right)dt = \begin{cases} \frac{A^2}{T}(T-|\tau|), & |\tau|<T \\ 0, & \text{elsewhere} \end{cases} \quad (12)$$

$$R_Y(\tau) = E[y(t)y(t-\tau)] = \frac{1}{T}\int_0^T \Pi\left(\frac{t}{T/R}\right)\Pi\left(\frac{t-\tau}{T/R}\right)dt = \begin{cases} \frac{1}{T}\left(\frac{T}{R}-|\tau|\right), & |\tau|<T \\ 0, & \text{elsewhere} \end{cases} \quad (13)$$

Now that the power spectral density and the autocorrelation function are Fourier transform pairs, the spectral density of jittered signal s(t) is derived as eq. (14) after combining the eqs. (11), (12), (13).

$$S(f) = \int_{-\infty}^{\infty} R_s(\tau)e^{-j2\pi f\tau}d\tau = 2A^2\int_0^{\frac{T}{R}}\left[\frac{1}{R}-\frac{1}{T}\left(\frac{1}{R}+1\right)\tau+\frac{1}{T}\tau^2\right]\cdot \cos(2\pi f\tau)d\tau = \frac{4A^2}{(2\pi f^2)RT}\left[(R-1)\sin^2 Z - \mathrm{sinc}\, 2Z + 1\right] \quad (14)$$

Like the analysis of the bipolar signal, element signal $g_1(t)$, $g_2(t)$ are as follows.

(3) **In the case of unipolar jitter-free signal**

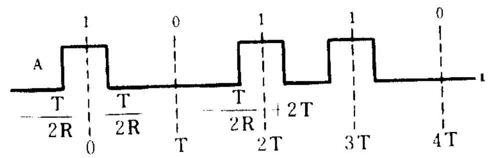

Fig. 3. Unipolar jitter-free signal

$$g_1(t) = \begin{cases} A, & -T/2R<t<T/2R \\ 0, & \text{elsewhere} \end{cases} \quad (15)$$

$$g_2(t) = 0, \quad \text{everywhere} \quad (16)$$

Fourier transforms of these signals are

$$G_1(f) = \int_{-\infty}^{\infty} g(t)e^{-j2\pi ft}dt = \frac{A}{\pi f}\sin\frac{\pi fT}{R} \quad (17)$$

$$G_2(f) = 0 \quad (18)$$

Assuming that the probability of occurrence of element signal $g_1(t)$ is equal to that of $g_2(t)$, $p=1-p=1/2$, then from equations (6), (17), (18) and $f_s=1/T$, the power spectral density function of unipolar jitter-free signal is

$$S(f) = \frac{A^2T}{4R^2}\left[2\,\mathrm{sinc}^2 Z + \delta(f) + \frac{2}{T}\sum_m \mathrm{sinc}\frac{m\pi}{R}\right] \quad (19)$$

(4) **In the case of unipolar jittered signal**

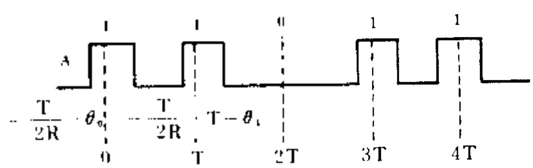

Fig. 4. Unipolar jittered signal

As shown in eq. (19), the unipolar jitter-free signal has both the discrete and the continjous components. This discrete component is available in extracting the timing information for the synchronization in the digital transmission system.

The unipolar jittered signal is expressed in eq. (20).

$$s(t) = A\sum_{m} a_m \Pi\left(\frac{t-mT-\theta_m}{T/R}\right) \tag{20}$$

Here random variable θ_m is stationary and intervally independent, and has the probability density function $p(\theta)$ which is uniform over the interval [O, T]. Hence from the similar procedure as in the preceeding results, the autocorrelation of unipolar jittered signal s(t) is

$$R_s(\tau) = E[s(t)s(t-\tau)]$$
$$= \frac{A^2}{T}\left(\frac{T}{R} - |\tau|\right), \quad |\tau| < \frac{T}{2} \tag{21}$$

Therefore, the spectral density is obtained as follows.

$$S(f) = \int_{-\infty}^{\infty} R_s(\tau)\, e^{-j2\pi f\tau}\, d\tau$$
$$= 2A^2 \int_0^{\frac{T}{R}} \frac{1}{T}\left(\frac{T}{R} - \tau\right)\cos(2\pi f\tau)\, d\tau$$
$$= \frac{A^2T}{R^2}\delta(f) + A^2\,\mathrm{sinc}^2 Z \tag{22}$$

Thus no discrete component at the multiple frequencies of the signal frequency f_s shows itself in power spectrum.

III. COMPUTER SIMULATION

This section will visualize the above theoretical results through a plotting program. Eq. (19) and (22) are used in this simulation, and unspecified parameters are fixed as below values.

$\frac{1}{R}$ = pulse duty ratio = $\frac{1}{2}$, $(R>1)$

$p(\theta)$ = probability density function of θ_m

$= \frac{1}{T} = 1, \quad \left[-\frac{T}{2}, \frac{T}{2}\right]$

A = Amplitude of transmitted signal = 1.

$f_s = \frac{1}{T}$ = signal frequency

τ = variable of autocorrelation

The results of simulation are figuratively shown in Fig. 5, 6. For unipolar jitter-free signals, the power spectral density is shown in Fig. 5, and for unipolar jittered signal, it is shown in Fig. 6.

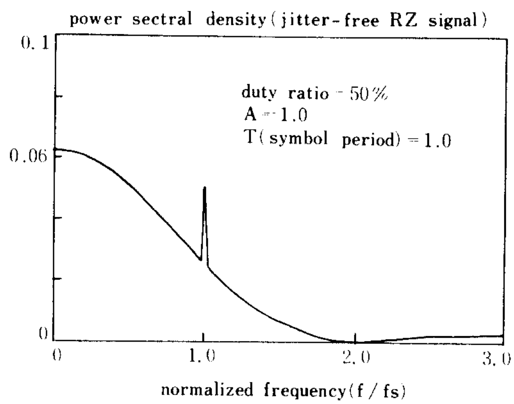

Fig. 5. The power spectrum of jitter-free signal

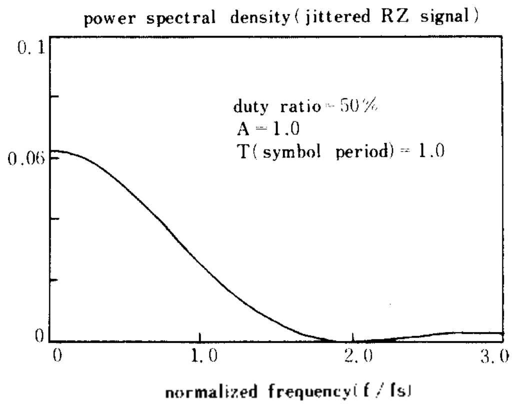

Fig. 6. The power spectrum of jittered signal

IV. CONCLUSION

We have derived the power spectral density functions for jitter -free signal and jittered signal respectively in section II, also observed that if no random phase variations are injected to the signal, the discrete components of the power spectral density exist at the frequency integer times of the signal frequency $f_s = 1/T$, as shown in Fig. 5.

But since jitter is generated during signal transmission, these discrete components would be reduced in accordance with the amount of phase variation.

In this paper, we choose the worst condition that jitter probability density $p(\Theta)$ is uniform in [O, T] to qualitatively study the effect of jitter on the signal power spectral density, where discrete components remarkably disappear as shown in Fig. 6. Considering the more suitable jitter probability density in our calculation of spectral density, we may preconceive the discrete components are diminished less than in the uniform case.

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