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# Optimal Control of Large Flexible Structures Via Partial Decoupling

# (部分分離法에 의한 柔軟性이 있는 大型構造物의 最適制御)

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要 約

이 연구에서는 선형2계시스템을 상태변수형으로 변환하여 부분적으로 모오드를 분리한 후 제동을 요하는 모오드에만 최적제어법을 적용, 제어하므로서 제어의 영향이 나머지 모오드에 미치지 않는 진동제어 방법을 제시하였다. 여기서는 제동을 요하는 모오드에 해당하는 고유벡타만으로 모오드를 분리하는 방법을 채택하였기 때문에 대형시스템의 경우 계산시간을 줄일 수 있어서 극히 적은 수의 모오드에만 제동을 요하는 유연성이 있는 우주용 대형구조물의 진동제어에 효과적으로 이용될 수 있을 것으로 기대된다.

**Abstract**

Linear second-order matrix systems representing dynamics of large flexible structures are recast in a state space form. By an efficient partial decoupling technique, a few of low frequency modes are decoupled from the rest of modes, and an optimal control procedure is developed in such a way that damping is added to the selected modes without control spillover to uncontrolled modes. Since the partial decoupling requires only eigenvectors of the sected modes, the computing time can be reduced significantly in large systems. Therefore, the method of partial decoupling and control developed in this work may be applicable to vibration control of large flexible space structures.

## I. Introduction

With the advent of a space shuttle transportation system, there is considerable interest in control of large flexible space structures (LFSS). See References 1 and 2 for surveys. One of control problems foı LFSS is vibration control which involves maintaining the shape of critical structures of LFSS, such as a phased array antenna or a solar energy collecting panel, against possible disturbances after deployment.

LFSS may be described as a continuum by a set of simultaneous partial differential equations. Because of difficulties in implementation of distributed parameter control systems, a usual approach to modeling is to convert the partial differential equations into an infinite number of ordinary differential equations by spatial discretization[3]. A finite number of modes are then retained in the model. To maintain reasonable model

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fidelity, however, the order of the system may be quite high (say $n \gg 100$), which makes analysis and controller synthesis particularly challenging[4].

Linear optimal control theory or pole allocation method may be applied to the approximated model to maintain the stability of LFSS [5,7]. However, since linear optimal control requires the solution of a matrix Riccati equation of an order equal to twice the number of modeled modes, the computational load for several hundred modes makes ordinary type of coupled control impractical. In addition, even though a small number of critical modes are chosen to be controlled, the state feedback control scheme is likely to induce so-called control and observation spillover[8]. Spillover refers to the phenomenon in which the energy intended to go into the controlled modes is pumped into the uncontrolled modes. To reduce these difficulties some papers concerned with the control of LFSS adopt the independent modal-space control[9] method. The method is based on the idea of coordinate transformations, whereby the system is completely decoupled into a set of independent second-order systems in terms of the modal coordinates. This independent modal-space control not only guarantees controllability but also guarantees that no control spillover into the modeled modes occurs, provided the number of actuators used is equal to the order of the discretized system[10]. But it has high hardware requirements.

The work in this paper will take a midway direction between the independent modal-space control and coupled control: Linear optimal regulator scheme will be applied to subsystems of partially decoupled modes, not of completely decoupled modes. As a result, the computational load for the solution of the Riccati equation will be reasonable and the control spillover problem will be eliminated by decoupling the selected modes for control from the other modes of the model. Seriousness of high hardware requirements can be reduced owing to the work [11] for controllability and observability criteria of linear second-order models.

## II. Problem Statement

Neglecting the low natural damping of LFSS the dynamics of the structure after spatial discretization of the partial differential equations are characterized by the linear second-order matrix differential equation[12];

$$\hat{M}\ddot{u}(t) + \hat{K}u(t) = \hat{B}_0 f(t), \qquad (1)$$

where the mass matrix $\hat{M} \epsilon R^{nxn}$ is symmetric positive definite and the stiffness matrix $\hat{K} \epsilon R^{nxn}$ is symmetric positive semi-definite. The vector $u(t) \epsilon R^n$ describes the node displacement and $f(t) \epsilon R^m$ is the force vector acting on the structure through the actuator matrix $B_0 \epsilon R^{nxm}$.

By changing coordinates u(t) to v(t) by $v(t) = \hat{M}^{1/2} u(t)$ and pre-multiplying both sides of (1) by $\hat{M}^{-1/2}$ one obtains

$$I\ddot{v}(t) + Kv(t) = B_0 f(t), \qquad (2)$$

where I is the (nxn) identity matrix, $K = \hat{M}^{-1/2} \hat{K} \hat{M}^{-1/2}$, and $B_0 = \hat{M}^{-1/2} \hat{B}_0$. It is easy to show that the change of coordinates preserves the system eigen values as well as the symmetry of the system.

The linear second-order system (2) is recast in state space form as;

$$\dot{x}(t) = Ax(t) + Bf(t), \qquad (3)$$

where

$$x(t) = \begin{bmatrix} v(t) \\ v(t) \end{bmatrix}, \quad A = \begin{bmatrix} 0_n & I_n \\ -K & 0_n \end{bmatrix},$$

$$\text{and } B = \begin{bmatrix} 0_{nxm} \\ B_0 \end{bmatrix} \qquad (4)$$

It is well known[13] that if $\langle A,B \rangle$ is controllable then control f(t) of system equation (3), which minimizes a scalar cost functional,

$$J(x,f,t) = \tfrac{1}{2} \int_0^\infty [x^T(t)Qx(t) + f^T(t)Rf(t)]\, dt$$

is written as

$$f(t) = -R^{-1}B^{T}Px(t), \qquad (6)$$

where 2nx2n matrix P satisfies the algebraic Riccati equation,

$$Q + A^{T}P + PA - PBR^{-1}B^{T}P = 0. \qquad (7)$$

The matrices $Q \epsilon R^{2n\times 2n}$ and $R \epsilon R^{m\times m}$ are weighting matrices chosen to fix the cost penalty for displacements and control efforts, respectively. It is assumed that Q is symmetric positive semi-definite and R is symmetic positive definite. The control law (6) will give the closed-loop system matrix,

$$A = A - BR^{-1}B^{T}P \qquad (8)$$

Problems of dimensionality and spillover to unwanted modes make this type of control hardly acceptable for the control of LFSS. Therefore, a simple control law which not only minimizes a given cost function but adds some damping to the selected modes will be devised in the paper. The method will be based on an efficient partial decoupling technique to be presented next.

## III. Partial Decoupling of the State Matrix

In this section the state matrix A defined in (4) will be blockwise diagonalized in a specific form so that the optimal control strategy can be carried out on a lower-order system of selected modes.

The eigenvalues of the state matrix A are along the imaginary axis and occur in complex conjugate pairs: If $\omega^2$, i=1, ..., n are the eigenvalues of K then $\pm j\omega_i i$ are the eigenvalues of A. This suggests that the spectral decomposition of A can be obtained from considering K rather than A.

If interest is in assigning damping to a few of low frequency modes, it is not necessary to compute all of eigenvectors of K. Since most of the energy of the system is in low frequency modes, a reasonably controlled structure would require control of the first few modes with the exception of the rigid modes. Under these circumstances the following theorem is useful for the control of LFSS.

*Theorem.* Let $E=B_{diag}(I_q, -I_{n-q})$, $H=\frac{1}{2}\Phi E\Phi^{-1}$ and $T = H + \frac{1}{2}E$, where $\Phi \epsilon C^{nxn}$ is an eigenvector matrix of $K \epsilon R^{nxn}$. Then T will diagonalize K blockwise into the form $K_B = B_{diag}(K_{B1}, K_{B2})$ under a similarity transformation; $K_B = TKT^{-1}$, where $K_{B1} \epsilon R^{qxq}$ has eigenvalues corresponding to the first q eigenvectors of K and the rest of them belong to $K_{B2}$.

*Proof* [14] : Let $\Phi$ be partitioned into four blocks;

$$\phi = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix}$$

where $\phi_{11} \epsilon C^{qxq}$ and $\phi_{22} \epsilon B^{(n-q)x(n-q)}$. Then with E defined as before, $T = H + \frac{1}{2}E = \frac{1}{2}[\Phi E + E\Phi]\Phi^{-1} = B_{diag}(\phi_{11}, -\phi_{22})\Phi^{-1}$. Since $K = \Phi D\Phi^{-1}$, $TKT^{-1} = B_{diag}(\phi_{11}D_1\phi_{11}, \phi_{22}D_2\phi_{22}^{-1})$ provided that no eigenvalue $\lambda_i$ is common to both $D_1$ and $D_2$. When we denote $K_{Bi} = \phi_{ii}D_i\phi_{ii}^{-1}$, i=1,2, the proof is completed.

The similarity transformation in Theorem can be obtained from the knowledge of eigenvectors corresponding to those modes that are to be decoupled. Let $P_D$ be the eigenprojector[15] of the q modes that are to be decoupled;

$$P_D = \bar{\Phi}B_{diag}(I_q, 0_{n-q})\bar{\Phi}^{T} = \begin{bmatrix} \bar{\phi}_{11}\bar{\phi}_{11}^{T} & \bar{\phi}_{11}\bar{\phi}_{21}^{T} \\ \bar{\phi}_{21}\bar{\phi}_{11}^{T} & \bar{\phi}_{21}\bar{\phi}_{21}^{T} \end{bmatrix}$$

where it is assumed that $\bar{\Phi}$ is normalized such that $\bar{\Phi}^{-1} = \bar{\Phi}^{T}$. The matrix H defined earlier is then $H = P_D - \frac{1}{2}I_n$ and thus $T = P_D + B_{diag}(0_q, -I_{n-q})$. Since the matrix $P_D$ contains only the first q eigenvectors of K, $[\phi_{11}^{-T}\ \phi_{21}^{-T}]^{T}$, T has been shown to be constructed from those q eigenvectors.

Thus far, the spectral decomposition of K has been carried out but the system matrix A must be considered because this is the matrix of concern. Let $T_A$ be a new transformation matrix with $T_A = B_{diag}(T, T)$, where T is the similarity transformation that diagonalizes K blockwise into a symmetric matrix by Theorem and eigenprojector mentioned above. Then,

$$T_A A T_A^{-1} = \begin{bmatrix} 0_n & I_n \\ -TKT^{-1} & 0_n \end{bmatrix} \tag{9}$$

This gives a new system matrix with $K_B$ in the low left corner of the matrix, but the matrix (9) is not blockwise diagonalized. To diagonalize (9) blockwise, construct a row-column interchange matrix F,

$$F = \begin{bmatrix} I_q & & & \\ & & I_q & \\ & I_{n-q} & & \\ & & & I_{n-q} \end{bmatrix}$$

where zeroes are deleted in F for simplicity. The block diagonal form $A_B$ can then be found by

$$A_B = FT_A A T_A^{-1} F^{-1} \triangleq SAS^{-1} \tag{10}$$

where $S \triangleq FT_A$ and $A_B = B_{diag}(A_{B1}, A_{B2})$ with

$$A_{B1} = \begin{bmatrix} 0_q & I_q \\ -K_{B1} & 0_q \end{bmatrix}, \quad A_{B2} = \begin{bmatrix} 0_{n-q} & I_{n-q} \\ -K_{B2} & 0_{n-q} \end{bmatrix}$$

It can be seen from (10) that S is orthogonal and that S is constructed from T only. Therefore, all of the computations for the decomposition (10) can be carried out by considering K matrix which is nxn. As a result, it is not necessary to find eigenvectors of A since the necessary information is contained in K.

The spectral decomposition process will modify the state equation (3) and the state vector x(t) of (4) as follows: Let q(t) be a new state vector defined as

$$q(t) \triangleq Sx(t), \tag{11}$$

then the state equation (3) becomes

$$\dot{q}(t) = A_B q(t) + \bar{B}f(t) \tag{12}$$

where $A_B = SAS^{-1}$ and $\bar{B} = SB$.

## IV. Optimal Control of Undamped Decoupled Systems

It was shown in the previous section that the state matrix could be diagonalized blockwise with selected eigenvalues of $A \epsilon R^{2n \times 2n}$ placed in one of the selected block matrices. Let the block matrix for the undamped system have the general form;

$$A_B = SAS^{-1} = \begin{bmatrix} A_{B1} & 0 \\ 0 & A_{B2} \end{bmatrix} \tag{13}$$

where $A_{B1} \epsilon R^{2q \times 2q}$ has eigenvalues $|\omega_i| < \rho$ and $A_{B2} \epsilon R^{(2n-2q) \times (2n-2q)}$ has eigenvalues $|\omega_i| > \rho$ with $\rho$ a scalar variable and A is the undamped system matrix. The value of $\rho$ will be chosen to include desired modes in $A_{B1}$.

Consider now the algebraic Riccati equation for $\bar{P} \epsilon R^{2n \times 2n}$ and let $A_B$ be the decoupled matrix, thus $\bar{P}$ must satisfy

$$\bar{Q} + A_B \bar{P} + \bar{P} A_B - \bar{P}\bar{B}R^{-1}\bar{B}^T\bar{P} = 0 \tag{14}$$

where $\bar{Q} \epsilon R^{2n \times 2n}$ and $R \epsilon R^{m \times m}$ are weighting matrices for the state q(t) and the input f(t). The matrix $\bar{B} \epsilon R^{2n \times m}$ represents the control input matrix where $\dot{q}(t) = A_B q(t) + \bar{B}f(t)$ with $\bar{B} = SB$. If it is assumed that the algebraic Riccati equation (14) is decoupled so that $\bar{P}_{12} = 0$ and $\bar{P}_{21} = 0$ then (14) can be written as following set of matrix equations;

$$\bar{Q}_{11} + A_{B1}^T \bar{P}_{11} + \bar{P}_{11} A_{B1} - \bar{P}_{11}\bar{B}_1 R^{-1} \bar{B}_1^T \bar{P}_{11} = 0 \tag{15a}$$

$$\bar{Q}_{22}+A_{B2}^T\bar{P}_{22}+\bar{P}_{22}A_{B2}-\bar{P}_{22}\bar{B}_2R^{-1}\bar{B}_2^T\bar{P}_{22}=0 \tag{15b}$$

$$\bar{Q}_{12}+\bar{P}_{11}\bar{B}_1R^{-1}\bar{B}_2^T\bar{P}_{22}=0 \tag{15c}$$

$$\bar{Q}_{21}+\bar{P}_{22}\bar{B}_2R^{-1}\bar{B}_1^T\bar{P}_{11}=0 \tag{15d}$$

where $\bar{Q}_{ij}$ and $\bar{P}_{ij}$, i,j=1,2 are partitioned matrices of $\bar{Q}$ and $\bar{P}$ in usual way. Also, note that $\bar{B}=[\bar{B}^T, \bar{B}^T]^T$ with $\bar{B}_1\epsilon R^{2qxm}$ and $\bar{B}_2\epsilon R^{(2n-2q)xm}$.

Denoting equation (15a) as the Riccati equation associated with system 1 and (15b) with system 2, it follows that system 1 has the state equation

$$\dot{q}_1(t)=A_{B1}q_1(t)+\bar{B}_1f(t) \tag{16}$$

where $q(t)=[q^T(t), q^T(t)]^T$ and the cost functional,

$$J_1(q_1,f,t)=\int_0^\infty [q^T(t)\bar{Q}_{11}q_1(t)+f^T(t)Rf(t)]\,dt. \tag{17}$$

The other system has the state equation and the cost functional as follows;

$$\dot{q}_2(t)=A_{B2}q_2(t)+\bar{B}_2f(t) \tag{18}$$

$$J_2(q_2,f,t)=\int_0^\infty [q_2^T(t)\bar{Q}_{22}q_2(t)+f^T(t)Rf(t)]dt. \tag{19}$$

On the other hand, substituting (13) for $A_B$ in (14)

$$\bar{Q}+(S^{-1})^T A^T S^T \bar{P}+\bar{P}SAS^{-1}-\bar{P}\bar{B}R^{-1}\bar{B}^T\bar{P}=0 \tag{20}$$

and rearranging (20) gives

$$S^T\bar{Q}S+A^TS^T\bar{P}S+S^T\bar{P}SA-S^T\bar{P}SBR^{-1}B^TS^T\bar{P}S=0 \tag{21}$$

Defining $P=S^T\bar{P}S$ and $Q=S^T\bar{Q}S$ (21) is identical to the algebraic Riccati equation (7) for the original control problem (3) through (5).

Assume now that the first system is the desired system for damping. Then, $\bar{B}_2=0$ will leave system 2 undamped and $\bar{P}_{22}=0$.($P_{22}$ is not necessarily identical to zero, but certainly it is a solution to (15b) when $Q_{22}=0$. This means there is no cost penalty associated with $q_2(t)$. Also note that $\bar{P}_{22}=0$ satisfies (15c) and (15d) with $\bar{Q}_{12}=0$ and $\bar{Q}_{21}=0$). It then follows that the Riccati equation for the uncoupled system is

$$P=S^T\bar{P}S=\begin{bmatrix} S_{11}^T\bar{P}_{11}S_{11} & S_{11}^T\bar{P}_{11}S_{12} \\ S_{12}^T\bar{P}_{11}S_{11} & S_{12}^T\bar{P}_{11}S_{12}\end{bmatrix} \tag{22}$$

where $S_{ij}$, i,j=1,2 is partitioned matrices of S with appropriate dimensions. Since $\bar{P}_{11}$ is symmetric, P is also symmetric as desired. By the previous assumption the control input matrix $\bar{B}$ will have the form,

$$\bar{B}=\begin{bmatrix}\bar{B}_1 \\ 0_{(2n-2q)xm}\end{bmatrix} \tag{23}$$

Therefore, since S is orthogonal

$$B=\begin{bmatrix}B_1 \\ B_2\end{bmatrix}=S^{-1}\bar{B}=S^T\begin{bmatrix}\bar{B}_1 \\ 0_{nxm}\end{bmatrix} \tag{24}$$

where $B_1$, $B_2\epsilon R^{nxm}$. The numerical value of $\bar{B}_1$ can be chosen so that system 1 is controllable and that $B_1=0$ but $B_2\neq 0$ if the form of the state matrix (4) is to be maintained. This also implies that the hardware for actuators is simple.

The closed-loop system matrix of system 1 after optimal control is then written as

$$\hat{A}_{B1}=A_{B1}-\bar{B}_1R^{-1}\bar{B}_1^T\bar{P}_{11}, \tag{25}$$

and, from the equations (4),(8),(22), and (24) with $B_1=0$, the closed-loop system matrix of system (3) is given by

$$A=\begin{bmatrix} 0_n & I_n \\ -K+B_2R^{-1}B_2^TS_{12}^T\bar{P}_{11}S_{11} & B_2R^{-1}B_2^TS_{12}^T\bar{P}_{11}S_{12} \end{bmatrix} \qquad (26)$$

The required feedback control vector f(t) is obtained from (6), (22), and (24);

$$f(t)=-R^{-1}\bar{B}^T\bar{P}Sx(t), \qquad (27)$$

which can now be determined as R, $\bar{B}$ and $\bar{P}$ are known.

It may be possible to make $B_2R^{-1}B_2^TS_{12}^TS_{11}=0$ in (26) by properly selecting $B_2$ and the weighting matrices. In general, it will not be zero and the stiffness of the structure will be changed. It should also be pointed out that the matrix $C(=B_2R^{-1}B^TS^T\bar{P}_{11}S_{12})$ does not represent a model with passive damping as C does not have the proper structure. If the closed-loop system has the feedback defined as (27), however, there are no restrictions since this control law is not for a passive system.

## V. An Illustrative Example

The example chosen for computational purpose has a mass matrix $I_6$ and a stiffness matrix K,

$$K=\begin{bmatrix} 9 & -5 & & & & \\ -5 & 11 & -6 & & & \\ & -6 & 13 & -7 & & \\ & & 7 & 15 & -8 & \\ & & & 8 & 17 & -9 \\ & & & & -9 & 19 \end{bmatrix}$$

The system is defined in state space form with state vector $x(t)=[v^T(t), \dot{v}^T(T)]^T$;

$$x(t)=\begin{bmatrix} 0_6 & I_6 \\ -K & 0_6 \end{bmatrix} x(t)+\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} f(t). \qquad (28)$$

After an equivalence transformation S which decouples the two lowest modes was applied to (28), the system was changed to

$$\dot{q}(t)=\left[\begin{array}{cc|cc} 0_2 & I_2 & \multicolumn{2}{c}{0_{4\times 8}} \\ -K_{B1} & 0_2 & & \\ \hline \multicolumn{2}{c|}{0_{8\times 4}} & 0_4 & I_4 \\ & & -K_{B2} & 0_4 \end{array}\right] q(t)+\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} f(t),$$

where $q(t)=Sx(t)$ with

$$K_{B1}=\begin{bmatrix} 3.83448 & -1.65133 \\ -1.65133 & 2.35772 \end{bmatrix}$$

$$K_{B2}=\begin{bmatrix} 16.81055 & 0.23648 & 6.93450 & 2.07957 \\ 0.23648 & 17.86965 & -1.02621 & 3.41672 \\ 6.93450 & 1.02621 & 24.26231 & -5.16800 \\ 2.07957 & 3.41672 & -5.16800 & 18.86528 \end{bmatrix}$$

Thus, system 1 was decoupled and given by (16) with

$$A_{B1}=\begin{bmatrix} 0_2 & I_2 \\ -K_{B1} & 0_2 \end{bmatrix} \qquad (29)$$

and the system was to be controlled to minimize the cost functional (17) with given $\bar{Q}_{11}=I_4$, R=1 and $\bar{B}_1=[0\ 0\ -1\ 1]^T$. The input matrix $\bar{B}_1$ was chosen so that system 1 was controllable. (For the state equation of the form (16) with $A_{B1}$ specified in (29), there exists a single vector $B_1$ such that $<A_{B1}, B_1>$ controllable unless subvector $[-1\ 1]^T$ of $B_1$ is in the null space of $(\lambda^2 I-K_{B1})$, where $\lambda\epsilon C$ satisfies $\det(\lambda^2 I-K_{B1})=0$. Since $\bar{B}_1$ is a single vector, it reduces the hardware requirements significantly. This fact can be deduced from Laub and Arnold's work [11]

The Riccati equation (15a) associated with (16) and (17) was solved by the eigenvector method [16] and solution was computed as

$$\bar{P}_1 = \begin{bmatrix} 6.31043 & 1.42106 & -0.47965 & 1.24103 \\ 1.42106 & 5.51307 & 0.90973 & 0.84844 \\ 0.47965 & 0.90973 & 2.66756 & 2.46582 \\ -1.24103 & 0.84844 & 2.46582 & 4.10804 \end{bmatrix}$$

With the assumption, $\bar{B}_2=0$, the solution P of (7) was computed by (22) and from (25) the closed-loop system matrix of system 1 was obtained as

$$\hat{A}_{B1} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -4.5959 & 1.5900 & -0.2017 & 1.6422 \\ 2.4127 & -2.2964 & 0.2017 & -1.6422 \end{bmatrix}$$

Finally the feedback control vector f(t) was computed from (27);

$$f(t) = \Gamma x(t)$$

where $\Gamma$ = [−0.4845 −0.5051 −0.2486 0.0167 0.1396 0.1097 −0.1796 0.1480 0.6793 1.0094 0.9489 0.5501].
The eigenvalues of the system before and after control were listed in Table 1 below.

Table 1. Eigenvalues $\lambda_i$ of the System Matrix.

| Mode i | $\lambda_i$ before control | $\lambda_i$ after control |
|---|---|---|
| 1 | $\pm j1.135$ | $-0.1713\pm j1.141$ |
| 2 | $\pm j2.215$ | $-0.7506\pm j2.117$ |
| 3 | $\pm j3.175$ | $\pm j3.175$ |
| 4 | $\pm j3.980$ | $\pm j3.980$ |
| 5 | $\pm j4.687$ | $\pm j4.687$ |
| 6 | $\pm j5.470$ | $\pm j5.470$ |

The input matrix B of the original system was computed by (24) and $B_2$ is given below with $B_1=0$.

$$B_2 = [-0.6696\ \ -0.4787\ \ 0.1589\ \ 0.6767\ \ 0.7775\ \ 0.4835]^T.$$

## VI. Conclusion

A precedure for optimal control of selected modes was investigated and it was shown that the control vector could be determined in such a way that damping was added to the low frequency modes with the other modes remaining undisturbed. The elimination of this control spillover was made possible by the partial decoupling technique for the specific system matrix. In addition, because of controllability criteria of linear second-order systems the hardware requirements on actuators part could be reduced significantly. However, since information on full states is required to construct the feedback control law, the hardware requirements on sensors part are still unsolved.

Because the partial decoupling precedure requires only eigenvectors of the selected modes the computing time is reduced considerably when the number of modes involved in vibration control is less than one fourth of the total modes. Therefore, in LFSS only a few of the low frequency modes need to be controlled and the method of decoupling and control developed in this paper may be applicable to vibration control of the structures.

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