BAYESIAN SHRINKAGE ESTIMATION OF THE RELIABILITY FUNCTION FOR THE LEFT TRUNCATED EXPONENTIAL DISTRIBUTION

MAN GON PARK

1. Introduction.

Consider the two parameters exponential distribution with positivity constraint on the truncation parameter defined by the probability density function,

\[ f(x|\theta, \lambda) = \lambda^{-1} \exp[-\lambda^{-1}(x-\theta)], \quad x>\theta, \lambda>0, \]

where \( \theta>0 \) for the density to be left truncated.

The model (1) will be referred to as the left truncated exponential distribution.

It is well-known that the left truncated exponential distribution is really appropriate as a lifetime distribution model for reliability and life-testing.

Evans and Nigm (1980) investigated that the use of the two parameters exponential distribution with no positivity constraint on the truncation parameter as a lifetime distribution model is unrealistic and may lead to inefficient inferences and prediction.

Both classical and Bayesian estimation of the reliability function for the two parameters exponential distribution with or with no positivity constraint on the truncation
parameter have studied by many authors, including Basu (1964), Varde (1969), Grubbs (1971), Pierce (1973), Sinha and Guttman (1976), Sinha and Kale (1980), Martz and Waller (1982), Trader (1985) and so on.

Trader (1985) considered the truncated normal distribution as a prior distribution for the truncation parameter in the left truncated exponential distribution.

The shrinkage estimation techniques have been advocated by a number of authors as a procedure for lowering the mean squared error (MSE) of the minimum variance unbiased estimator (MVUE) or maximum likelihood estimator (MLE) [see Thompson (1968), Mehta and Srinivasan (1971), Pandey and Singh (1980), Pandey and Srivastava (1985) and so on].

In recent years, the use of Bayes shrinkage estimation of the parameters for binomial, Poisson and normal distributions were considered by Lemmer (1981) at first. But he has been little paid attention to lifetime distributions-exponential, Weibull etc.

Pandey and Upadhyay (1985) considered the Bayes shrinkage estimation of the parameters for Weibull distribution, and discussed the relative s-efficiencies of these Bayes shrinkage estimators with respect to the unbiased estimators of Engelhardt and Bain (1977) on the basis of a Monte Carlo study of 500 random samples.

In this study, we will consider some Bayes shrinkage estimators of the reliability function in the left truncated exponential distribution.

First, we will give the MVUE and Bayes estimators of the reliability function with the noninformative and conjugate prior distributions in this model.
Next, using the Bayes estimator instead of the guessed value which is close to the true unknown value, such as that given by Pandey (1985), we will propose some Bayes shrinkage estimators of the reliability function in this model.

Finally, we will compare the relative s-efficiencies of the Bayes shrinkage estimators with respect to the MVUE by the Monte Carlo simulation and numerical evaluation technique in the sense of MSE.

2. MVUE and Bayes estimators of the reliability function.

Let $x_{(1)} \leq x_{(2)} \leq \cdots \leq x_{(r)}$ be the first $r$ ordered observations of $n$ failure times form the left truncated exponential distribution (1) under test without replacement.

For a given time $t$, the reliability function, the probability that survival until time $t$, is given by

\begin{equation}
R_t = 1 - F(t) = \int_0^\infty f(x)dx
= \exp \left(-\frac{t - \theta}{\lambda}\right), \ t \geq \theta,
\end{equation}

where $F$ is the cumulative distribution function of the failure time $x$.

Basu (1964) obtained the MVUE of the reliability function at time $t$ to be

\begin{equation}
\hat{R}_i = \begin{cases} 
\frac{n-1}{n} \left\{1 - \frac{t - x_{(i)}}{y_r}\right\}^{-2}, & x_{(i)} < t < x_{(i)} + y_r \\
1, & x_{(i)} > t \\
0, & t > x_{(i)} + y_r
\end{cases},
\end{equation}

where $y_r = \sum_{i=1}^r (x_{(i)} - x_{(1)}) + (n-r)(x_{(i)} - x_{(1)})$. 
The likelihood function is given by

\( L(\theta, \lambda|x_{(1)}, x_{(2)}, \ldots, x_{(r)}) \propto \exp\{-\lambda^{-1}[y + n(x_{(1)} - \theta)]\} \times \exp\{-\lambda^{-1}x\} \exp\{-\lambda^{-1}y\} \exp\{-\lambda^{-1}[x_{(1)} - \theta]\} \exp\{-\lambda^{-1}[x_{(2)} - \theta]\} \ldots \exp\{-\lambda^{-1}[x_{(r)} - \theta]\} \)

where \( 0 < \theta < x_{(1)}, \lambda > 0 \).

The noninformative joint prior distribution of \( \theta \) and \( \lambda \) is taken as [Sinha and Guttman(1976)]

\( g_1(\theta, \lambda) = \begin{cases} (1 - b), & \text{if } \theta = \theta_0, \lambda = \lambda_0 \\ b/\lambda^a, & \text{otherwise,} \end{cases} \)

where \( a > 0, 0 < b \leq 1, \lambda > 0, 0 < \theta < x_{(1)}, \theta_0, \lambda_0 \) are the prior values in the vicinities of the true values \( \theta \) and \( \lambda \), respectively, and the prior distribution has weight \((1 - b)\) in the prior values and weight \( b \) in the rest intervals.

We obtain the joint posterior distribution of \( \theta \) and \( \lambda \) as

\( g_1(\theta, \lambda|x_{(1)}, x_{(2)}, \ldots, x_{(r)}) = \frac{L(\theta, \lambda|x_{(1)}, x_{(2)}, \ldots, x_{(r)})g_1(\theta, \lambda)}{\int_{0}^{x_{(1)}} \int_{0}^{x_{(2)}} \ldots \int_{0}^{x_{(r)}} L(\theta, \lambda|x_{(1)}, x_{(2)}, \ldots, x_{(r)})g_1(\theta, \lambda) \, d\lambda \, d\theta} \).

Therefore, from (4), (5) and (6), the Bayes estimator of the reliability function with the noninformative prior distribution under the squared-error loss can be written as

\( R^{*}_{\lambda_1} = E[R_i|x_{(1)}, x_{(2)}, \ldots, x_{(r)}] = \int_{0}^{\infty} R_i g_1(\theta, \lambda|x_{(1)}, x_{(2)}, \ldots, x_{(r)}) \, d\lambda \, d\theta = \frac{P_{r+a-2}(p_1 + 1, p_2 + t - x_{(1)}, p_3 + t) + Q(\theta_1 + t - \theta_0)}{P_{r+a-2}(p_1, p_2, p_3) + Q(\theta_0)} \)

where

\( P_{r+a-2}(p_1, p_2, p_3) = b \frac{\Gamma(r + a - 2)}{p_1} \frac{1}{p_2^{r+a-2}} \left(1 - \left(\frac{p_2}{p_3}\right)^{r+a-2}\right), \)
\[ Q(g_i) = (1-b)\lambda_{0}^{-\tau} \exp\left(-\frac{g_i}{\lambda_{0}}\right), \]
\[ p_1 = n, \quad p_2 = y_r, \quad p_3 = y_r + nx_{(1)}, \quad q_1 = y_r + nx_{(1)} - n\theta_0. \]

Also, we can use the conjugate joint prior distribution of \( \theta \) and \( \lambda \) as [Evans and Nigm(1980)]

\[ g_2(\theta, \lambda) = \begin{cases} \frac{1}{\lambda^e} \exp\left(-\frac{d+h(m-\theta)}{\lambda}\right), & \text{if } \theta = \theta_0, \ \lambda = \lambda_0 \\ \frac{1}{\lambda^e} \exp\left(-\frac{d+h(m-\theta)}{\lambda}\right), & \text{otherwise,} \end{cases} \]

where to be a proper prior distribution we must have \( c>2, \ d>0, \ h>0, \) and \( 0<\theta<m, \ \lambda>0. \)

From (8), the joint posterior distribution of \( \theta \) and \( \lambda \) becomes

\[ g_2(\theta, \lambda|x_{(1)}, x_{(2)}, \ldots, x_{(r)}) = \frac{L(\theta, \lambda|x_{(1)}, x_{(2)}, \ldots, x_{(r)}) g_2(\theta, \lambda)}{\int_0^\lambda \int_0^\lambda L(\theta, \lambda|x_{(1)}, x_{(2)}, \ldots, x_{(r)}) g_2(\theta, \lambda) d\lambda d\theta}, \]

where \( A = \min(m, x_{(1)}). \)

Therefore, from (4), (8) and (9), the Bayes estimator of the reliability function with the conjugate prior distribution under the squared-error loss can be written as

\[ R_{*_{2}} = E(R_{i2}|x_{(1)}, x_{(2)}, \ldots, x_{(r)}) = \int_0^\lambda \int_0^\lambda R_{i2}(\theta, \lambda|x_{(1)}, x_{(2)}, \ldots, x_{(r)}) d\lambda d\theta \]

\[ = \frac{P_{r+i-2}(p_1+h+1, p_2+nx_{(1)}+d+hm+t-m\theta)}{P_{r+i-2}(p_1+h, p_2+nx_{(1)}+d+hm+(n+h)A, p_3+d+hm)} Q(g_i). \]

3. Bayes shrinkage estimators of the reliability function.

Let us consider a shrunken estimator of the form
\[ T = k(\hat{\theta} - \theta_0) + \theta_0, \]

where \(0 \leq k \leq 1\), \(\hat{\theta}\) is the MVUE of \(\theta\) and \(\theta_0\) is a prior value which is close to the true unknown value \(\theta\).

This shrinkage estimator for \(\theta\) was considered by Thompson (1968) at first, and showed that \(T\) is more efficient than MVUE for mean parameter in the sense of MSE when sample size is small and \(\theta_0\) is in the vicinity of true value \(\theta\) in the normal, Poisson, binomial and gamma population.

Now we propose two classes of the Bayes shrinkage estimator of the reliability function:

\[ T_{RT1} = k_1(\hat{R}_t - R^*_{i1}) + R^*_{i1} = k_1\hat{R}_t + (1 - k_1)R^*_{i1}, \]

\[ T_{RT2} = k_2(\hat{R}_t - R^*_{i2}) + R^*_{i2} = k_2\hat{R}_t + (1 - k_2)R^*_{i2}, \]

where \(0 \leq k_1, k_2 \leq 1\), \(\hat{R}_t\) is the MVUE of \(R_t\), \(R^*_{i1}\) is the Bayes estimator of \(R_t\) with the noninformative prior distribution and \(R^*_{i2}\) is the Bayes estimator of \(R_t\) with the conjugate prior distribution.

4. Comparisons of the relative s-efficiencies of the Bayes shrinkage estimators with respect to the MVUE.

The relative s-efficiencies of the Bayes shrinkage estimators with respect to the MVUE of the reliability function are given by

\[ \text{REF}_1(T_{RT1}, \hat{R}_t) = \frac{\text{Var}(\hat{R}_t)}{\text{MSE}(T_{RT1})}, \]

\[ \text{REF}_2(T_{RT2}, \hat{R}_t) = \frac{\text{Var}(\hat{R}_t)}{\text{MSE}(T_{RT2})}, \]

where \(\hat{R}_t\) is the MVUE of \(R_t\), \(T_{RT1}\) is the Bayes shrinkage
estimator of \( R \) with the noninformative prior distribution (5) and \( T_{R12} \) is the Bayes shrinkage estimator with the conjugate prior distribution (8).

A Monte Carlo study has been performed on the relative s-efficiencies of the proposed Bayes shrinkage estimators with respect to the MVUE and the numerical values of the relative s-efficiencies of proposed Bayes shrinkage estimators with respect to the MVUE have been evaluated by use of the computer system.

The Monte Carlo simulation on the relative s-efficiencies of \( T_{R1} \) and \( T_{R2} \) with respect to \( \hat{R} \) has been performed as the following four parts.

Part 1: 500 random samples of the first \( r \) ordered failure times were generated from the left truncated exponential distribution (1) with the parameters \( \lambda \) and \( \theta \) such that \( \theta/\theta_0 \) is fixed at 1 and \( \lambda/\lambda_0 \): 0.50(0.25) 1.75 varies with \( (n,r) \), and \( \text{REF} \) (\( T_{R1}, \hat{R}_r \)) were evaluated for \( a=1, b=0.2 \) (0.2) 0.8 and \( k_1=0.2 \) (0.2) 0.8 to avoid complexity on the table 1.

Part 2: 500 random samples of the first \( r \) ordered failure times were generated from the left truncated exponential times (1) with the parameters \( \lambda \) and \( \theta \) such that \( \lambda/\lambda_0 \) is fixed at 1 and \( \theta/\theta_0 \): 0.50(0.25) 1.75 varies with \( (n,r) \), and \( \text{REF} \) (\( T_{R1}, \hat{R}_r \)) were evaluated for \( a=2, b=0.2 \) (0.2) 0.8 and \( k_1=0.2 \) (0.2) 0.8 on the table 2.

Part 3: 500 random samples of the first \( r \) ordered failure times were generated from the left truncated exponential distribution (1) with the parameters \( \lambda \)
and \( \theta \) such that \( \theta/\theta_0 \) is fixed at 1 and \( \lambda/\lambda_0: 0.50 (0.25) 1.75 \) varies with \((n, r)\), and \( \text{REF}_2 \) \( (T_{Rt2}, \hat{R}_t) \) were evaluated for \( c=4, \ d=2, \ h=1, \ m=2, \ b=0.2(0.2) 0.8 \) and \( k_2=0.2(0.2) 0.8 \) on the table 3.

**Part 4:** 500 random samples of the first \( r \) ordered failure times were generated from the left truncated exponential distribution (1) with the parameters \( \lambda \) and \( \theta \) such that \( \lambda/\lambda_0 \) is fixed at 1 and \( \theta/\theta_0: 0.50(0.25) 1.75 \) varies with \((n, r)\), and \( \text{REF}_2(T_{Rt2}, \hat{R}_t) \) were evaluated for \( c=5, \ d=2, \ h=1, \ m=2, \ b=0.2(0.2) 0.8 \) and \( k_2=0.2(0.2) 0.8 \) on the table 4.

Throughout the table 1-4, we obtain the following results:

(a) \( T_{Rt1} \) is more efficient than MVUE \( \hat{R}_t \) in the sense of MSE for all possible values of \( n, r, a, b \) and \( k_1 \) contained the effective interval which is in the vicinity of true value \( \lambda \) or \( \theta \).

(b) \( T_{Rt2} \) is also much more efficient than MVUE \( \hat{R}_t \) in the sense of MSE for all possible values of \( n, r, c, d, h, m, b \) and \( k_2 \) contained the effective interval which is in the vicinity of true value \( \lambda \) or \( \theta \).

(c) When the guessed value \( \lambda_0 \) is true, that is \( \lambda/\lambda_0 \) is 1, \( T_{Rt1} \) and \( T_{Rt2} \) are most efficient in the sense of MSE.

(d) \( T_{Rt2} \) is more efficient than \( T_{Rt1} \) in the sense of MSE.
Table 1. Relative s-efficiencies of $T_{k1}$ with respect to $\tilde{R}_1$

\[
\begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline
\lambda & 0.50 & 0.75 & 1.00 & 1.25 & 1.50 & 1.75 \\
\hline
\delta & & & & & & \\
\hline
0.2 & 0.4 & 0.6 & 0.8 & 0.2 & 0.4 & 0.6 & 0.8 & 0.2 & 0.4 & 0.6 & 0.8 & 0.2 & 0.4 & 0.6 & 0.8 \\
\hline
10 & 7 & 0.899 & 1.041 & 1.117 & 1.163 & 0.960 & 1.066 & 1.112 & 1.141 & 1.020 & 1.067 & 1.090 & 1.163 & 1.032 & 1.044 & 1.062 & 1.057 \\
20 & 15 & 0.783 & 0.950 & 1.036 & 1.039 & 0.856 & 0.979 & 1.040 & 1.077 & 0.920 & 0.988 & 1.035 & 1.057 & 0.970 & 1.008 & 1.021 & 1.031 \\
20 & 15 & 0.988 & 1.011 & 1.075 & 1.122 & 0.939 & 1.034 & 1.060 & 1.032 & 1.005 & 1.025 & 1.042 & 1.073 & 1.065 & 1.013 & 1.022 & 1.030 \\
20 & 15 & 0.830 & 0.975 & 1.031 & 1.053 & 0.919 & 0.931 & 1.033 & 1.033 & 0.947 & 0.992 & 1.021 & 1.030 & 0.974 & 0.937 & 1.013 & 1.027 \\
\hline
\end{array}
\]
Table 2. Relative efficiencies of $T_{N1}$ with respect to $R_1$

(a = 2)

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\frac{b}{\theta_0}$</th>
<th>$\eta$</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
<td>10 7</td>
<td>2.055</td>
<td>2.057</td>
<td>2.168</td>
<td>2.113</td>
<td>2.536</td>
<td>2.483</td>
<td>2.406</td>
<td>2.254</td>
<td>2.220</td>
<td>2.155</td>
<td>2.067</td>
<td>1.917</td>
<td>1.538</td>
<td>1.508</td>
<td>1.474</td>
<td>1.444</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20 15</td>
<td>0.930</td>
<td>0.930</td>
<td>0.930</td>
<td>0.932</td>
<td>1.466</td>
<td>1.466</td>
<td>1.467</td>
<td>1.468</td>
<td>1.857</td>
<td>1.857</td>
<td>1.857</td>
<td>1.856</td>
<td>1.551</td>
<td>1.551</td>
<td>1.551</td>
<td>1.550</td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>10 7</td>
<td>5.136</td>
<td>5.376</td>
<td>2.704</td>
<td>2.061</td>
<td>3.188</td>
<td>2.498</td>
<td>2.053</td>
<td>1.682</td>
<td>2.051</td>
<td>1.784</td>
<td>1.585</td>
<td>1.404</td>
<td>1.386</td>
<td>1.315</td>
<td>1.247</td>
<td>1.179</td>
<td></td>
</tr>
<tr>
<td>1.00</td>
<td>10 7</td>
<td>2.887</td>
<td>2.146</td>
<td>1.815</td>
<td>1.614</td>
<td>2.138</td>
<td>1.740</td>
<td>1.544</td>
<td>1.419</td>
<td>1.520</td>
<td>1.428</td>
<td>1.325</td>
<td>1.256</td>
<td>1.259</td>
<td>1.188</td>
<td>1.147</td>
<td>1.113</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20 15</td>
<td>3.017</td>
<td>2.029</td>
<td>1.623</td>
<td>1.384</td>
<td>2.160</td>
<td>1.672</td>
<td>1.426</td>
<td>1.271</td>
<td>1.640</td>
<td>1.393</td>
<td>1.260</td>
<td>1.170</td>
<td>1.264</td>
<td>1.174</td>
<td>1.120</td>
<td>1.080</td>
<td></td>
</tr>
<tr>
<td>1.25</td>
<td>10 7</td>
<td>2.126</td>
<td>1.763</td>
<td>1.613</td>
<td>1.528</td>
<td>1.732</td>
<td>1.513</td>
<td>1.419</td>
<td>1.364</td>
<td>1.426</td>
<td>1.309</td>
<td>1.256</td>
<td>1.224</td>
<td>1.187</td>
<td>1.140</td>
<td>1.118</td>
<td>1.104</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20 15</td>
<td>1.311</td>
<td>1.254</td>
<td>1.234</td>
<td>1.222</td>
<td>1.222</td>
<td>1.182</td>
<td>1.168</td>
<td>1.161</td>
<td>1.141</td>
<td>1.116</td>
<td>1.108</td>
<td>1.103</td>
<td>1.067</td>
<td>1.056</td>
<td>1.052</td>
<td>1.050</td>
<td></td>
</tr>
<tr>
<td>1.75</td>
<td>10 7</td>
<td>1.722</td>
<td>1.576</td>
<td>1.520</td>
<td>1.491</td>
<td>1.489</td>
<td>1.395</td>
<td>1.360</td>
<td>1.341</td>
<td>1.286</td>
<td>1.242</td>
<td>1.222</td>
<td>1.211</td>
<td>1.135</td>
<td>1.112</td>
<td>1.103</td>
<td>1.098</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20 15</td>
<td>1.254</td>
<td>1.231</td>
<td>1.223</td>
<td>1.219</td>
<td>1.183</td>
<td>1.167</td>
<td>1.161</td>
<td>1.158</td>
<td>1.117</td>
<td>1.107</td>
<td>1.103</td>
<td>1.102</td>
<td>1.056</td>
<td>1.051</td>
<td>1.050</td>
<td>1.049</td>
<td></td>
</tr>
</tbody>
</table>
Table 3. Relative s-efficiencies of $T_{SR}$ with respect to $\widehat{R}$.

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_0$</td>
<td>n</td>
<td>r</td>
<td>$\lambda$</td>
<td>n</td>
<td>r</td>
<td>$\lambda$</td>
<td>n</td>
<td>r</td>
<td>$\lambda$</td>
<td>n</td>
<td>r</td>
<td>$\lambda$</td>
<td>n</td>
<td>r</td>
<td>$\lambda$</td>
<td>n</td>
</tr>
<tr>
<td>0.50</td>
<td>10</td>
<td>7</td>
<td>0.459</td>
<td>0.563</td>
<td>0.609</td>
<td>0.946</td>
<td>0.651</td>
<td>0.731</td>
<td>0.832</td>
<td>1.002</td>
<td>0.890</td>
<td>0.908</td>
<td>0.949</td>
<td>1.032</td>
<td>1.058</td>
<td>1.021</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>15</td>
<td>0.272</td>
<td>0.358</td>
<td>0.472</td>
<td>0.670</td>
<td>0.391</td>
<td>0.478</td>
<td>0.585</td>
<td>0.753</td>
<td>0.576</td>
<td>0.416</td>
<td>0.721</td>
<td>0.939</td>
<td>0.821</td>
<td>0.869</td>
</tr>
<tr>
<td>0.75</td>
<td>10</td>
<td>7</td>
<td>2.234</td>
<td>2.238</td>
<td>2.252</td>
<td>2.261</td>
<td>2.321</td>
<td>2.146</td>
<td>1.998</td>
<td>1.943</td>
<td>1.933</td>
<td>1.785</td>
<td>1.633</td>
<td>1.509</td>
<td>1.414</td>
<td>1.343</td>
</tr>
<tr>
<td>1.00</td>
<td>10</td>
<td>7</td>
<td>11.442</td>
<td>7.537</td>
<td>5.311</td>
<td>2.675</td>
<td>4.653</td>
<td>3.819</td>
<td>3.139</td>
<td>2.485</td>
<td>2.442</td>
<td>2.221</td>
<td>2.011</td>
<td>1.766</td>
<td>1.490</td>
<td>1.436</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>15</td>
<td>2.023</td>
<td>1.907</td>
<td>1.799</td>
<td>1.655</td>
<td>1.992</td>
<td>1.822</td>
<td>1.686</td>
<td>1.543</td>
<td>1.698</td>
<td>1.572</td>
<td>1.471</td>
<td>1.367</td>
<td>1.325</td>
<td>1.271</td>
</tr>
<tr>
<td>1.50</td>
<td>10</td>
<td>7</td>
<td>1.872</td>
<td>1.864</td>
<td>1.862</td>
<td>1.852</td>
<td>1.869</td>
<td>1.779</td>
<td>1.714</td>
<td>1.651</td>
<td>1.631</td>
<td>1.544</td>
<td>1.479</td>
<td>1.417</td>
<td>1.302</td>
<td>1.266</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>15</td>
<td>0.909</td>
<td>0.968</td>
<td>1.017</td>
<td>1.110</td>
<td>1.055</td>
<td>1.062</td>
<td>1.034</td>
<td>1.126</td>
<td>1.133</td>
<td>1.142</td>
<td>1.106</td>
<td>1.110</td>
<td>1.083</td>
<td>1.075</td>
</tr>
<tr>
<td>1.75</td>
<td>10</td>
<td>7</td>
<td>1.366</td>
<td>1.421</td>
<td>1.486</td>
<td>1.582</td>
<td>1.431</td>
<td>1.429</td>
<td>1.426</td>
<td>1.452</td>
<td>1.366</td>
<td>1.326</td>
<td>1.307</td>
<td>1.301</td>
<td>1.202</td>
<td>1.173</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>15</td>
<td>0.723</td>
<td>0.802</td>
<td>0.981</td>
<td>1.061</td>
<td>0.940</td>
<td>0.890</td>
<td>0.944</td>
<td>1.022</td>
<td>0.942</td>
<td>0.961</td>
<td>0.987</td>
<td>1.022</td>
<td>1.002</td>
<td>0.600</td>
</tr>
</tbody>
</table>
Table 4. Relative s-efficiencies of $T_{R12}$ with respect to $R_1$

\begin{tabular}{ll|llll|llll|llll|llll|llll}
\hline
\hline
\hline
k & 0.2 & 0.4 & 0.6 & 0.8 & 0.2 & 0.4 & 0.6 & 0.8 & 0.2 & 0.4 & 0.6 & 0.8 & 0.2 & 0.4 & 0.6 & 0.8 \\
\hline
& 20 & 15 & 0.930 & 0.930 & 0.930 & 0.930 & 1.466 & 1.466 & 1.466 & 1.466 & 1.857 & 1.857 & 1.857 & 1.857 & 1.552 & 1.552 & 1.552 & 1.552 \\
\hline
\hline
\end{tabular}
5. Conclusions.

In the comparisons of the Monte Carlo relative s-efficiencies of the proposed Bayes shrinkage estimators for the reliability function with respect to the MVUE in the left truncated exponential distribution based on type II censoring, the proposed estimators are more efficient than MVUE in the sense of MSE for all possible values of $n, r, a, b, c, d, h, m, k_1$ and $k_2$ if $\lambda/\lambda_0$ and/or $\theta/\theta_0$ approach 1. Also, the Bayes shrinkage estimator with the conjugate prior distribution is more efficient than the Bayes shrinkage estimator with the noninformative prior distribution.

References


[18] Thompson, J.S. (1968), Some shrinkage techniques for estimating the mean, JASA, 63, 113-123.


Department of Applied Mathematics
National Fisheries University of Pusan
Pusan 608
Korea

Received August 18, 1986