□論 文□

# Network Equilibrium Models of Urban Location and Travel Choices : Formulations and Applications

# 通行起 終点 및 交通手段選擇과 結合된 交通網 平衡模型의 構成및 應用

全 京 秀

(美 버팔로 在 뉴욕州立大學校 敎授)

CONTENTS



要 約

本 論文은 通行 起·終點 및 交通手段選擇의 問題를 交通網 平衡模型과 結合하여 이를 하나의 最適化 問題로 構成하는 方法 및 그의 實際應用 方法을 提示, 檢討하였다. 두가지의 行態的 特性, 즉 交通費用을 最小化하려는 使用者 平衡과 通行 起·終點 및 交通手段 등의 選擇에 있어 어느 水準의 相互交流를 維持하려고 하는 엔트로피 極大化 現狀이 折衷되어 最適化 問題를 形成하게 된다.

本 硏究의 目的은 이의 體系的 構成과 아울러 이를 여러 都市에서 設置, 使用되고 있는 既存의 UTPS 프로그램과 結合시킴으로써 既存의 老大한 데이타베이스를 使用케 하고 實際應用을 單純化시키는 데 있다. 開發된 模型과 方法은 시카고地域의 資料를 利用하여 그 妥當性이 檢討되었다.

## 1. Introduction

The network equilibrium problem is to predict the travel and location choices on a transportation system when the resulting travel times and costs are one basis for the choices. Solving this problem requires one to find an equilibrium among choices and travel times and costs, as contrasted with the case where choices are based on fixed travel times and costs. The choices here include not only travel options such as mode and route, but also location options including place of residence, employment and shopping, which are hypothesized to depend in part on travel times and costs.

The principles of network equilibrium in the context of route choice were first stated by Wardrop (1952), and independently formulated and analyzed in a mathematical sense by Beckmann et al. (1956). Since the late 1960s, such models have been the focus of an active, ongoing research effort; great progress has been made in understanding various formulations of these models and solving them computationally.

In this paper a network equilibrium problem of urban location and travel choices is formulated and applied to a large scale transportation system. The major concern of the paper is focused on the problems with regard to the model implementation in conjunction with the Urban Transportation Planning System (UTPS) software package. UTPS, developed by the Urban Mass Transportation Administration and the Federal Highway Administration, has been widely used by many planning agencies. The procedure, however, is confined to the conventional sequential approaches. Procedures appropriate for incorporation of the network equilibrium models within the module "UROAD", the highway assignment program in UTPS, are discussed in this paper.

## 2. Network Equilibrium Models of Urban Location and Travel Choices

The principle of network equilibrium originated in the area of trip assignment or route choice problem. The basic assumption underlying this principle is that each individual tripmaker seeks to minimize his/her travel cost. Because this cost is a function of link flows, however, an individual's travel cost depends on the choices of others. Thus a concept of network equilibrium is required. User-optimal equilibrium in a network is achieved when no traveler can reduce his/her travel cost by changing travel choices.

Wardrop (1952) originally proposed this equilibrium concept in the context of route choice; in this formulation the notion is extended in principle to include hierarchical travel choice sets involving route, mode, destination and even location choices. The generalized version of Wardrop's principle is written as follows:

1. all travel choice alternatives which are selected have equal travel cost, and
2. no unselected travel choice alternative has a lower travel cost than a selected alternative.

Utilizing the concepts of network equilibrium stated above, Beckmann et al. (1956) formulated the network assignment model as a mathematical programming problem. A simplified version of a problem introduced by Beckmann et al. is the fixed demand network equilibrium problem.

$$\min_{v} \sum_{a} \int_{0}^{v_a} s_a(x)\,dx \tag{1}$$

$$\text{s. t. } v_a = \sum_{i}\sum_{j}\sum_{r} p_{ijr}\,\sigma_{ijr}^{a}\,T \quad a \in A \tag{2}$$

$$\sum_{r} p_{ijr} = \bar{p}_{ij} \quad i=1,\cdots,I;\ j=1,\cdots J \tag{3}$$

$$p_{ijr} \geqq 0 \tag{4}$$

where $s_a(x)$ = generalized cost of travel of link a at person trip flow x; a constant factor converting the number of person trips into vehicle equivalents is assumed to be included in the cost function.

$v_a$ = flow of person trips on link a.

$\sigma_{ijr}^{a}$ = 1, if route r from zone i to zone j include link a;
= 0, otherwise.

$P_{ijr}$ = proportion of person trips from zone i to zone j using route r.

T = total number of person trips.

given exogenously.

A = set of links in the network.

$\overline{P}_{ij}$ = fixed proportion of person trips from zone i to zone j.

The problem is to find $(p_{ijr})$ and $(v_a)$, given the cost functions $(s_a(x))$, the incidence matrix $(\sigma^a_{ijr})$, T, and the origin-destination proportions $(\bar{p}_{ij})$. To find the optimality conditions, define the Lagrangian equation.

$$L=\sum_a \int_0^{v_a} s_a(x)\,dx+\sum_i\sum_j \lambda_{ij}\,(p_{ij} - \sum_r p_{ijr})+$$

$$\sum_i\sum_j\sum_r \lambda_{ijr}(-p_{ijr})$$

where $v_a$ is defined by equation (2) and $\lambda_{ij}$ and $\lambda_{ijr}$ are the Lagrange multipliers for constraints (3) and (4). The Kuhn-Tucker optimality conditions are:

$$\frac{\partial L}{\partial p_{ijr}}=T\sum_a s_a(v_a)\,\sigma^a_{ijr}+\lambda_{ij}(-1)+\lambda_{ijr}(-1)=0$$

$$p_{ijr}\lambda_{ijr}=0$$

These conditions may be interpreted as follows:

1. If $p_{ijr}>0$, then $\lambda_{ijr}=0$, and $c_{ijr}=\sum_a s_a$ $s(v_a)\,\sigma^a_{ijr}=\lambda_{ij}/T$ for all chosen routes r, where $c_{ijr}$ is the travel cost from zone i to zone j on route r;
2. If $p_{ijr}=0$, then $c_{ijr}\geqq \lambda_{ij}/T$

These two conditions are equivalent to the Wardrop conditions stated above. Condition 1 states that if the proportion of trips from i to j on route r is positive, then the travel cost on the route is equal to the travel costs on all routes chosen from i to j. Condition 2 states if the proportion of trips is zero, then the travel cost is no less than the cost on any chosen route.

In the preceding discussion, it was assumed that each person chooses the least cost alternatives for route choice. This may be interpreted as user cost minimizing behavior. From the empirical observation of urban location and travel choice behavior, however, it is apparent that many people do not select the minimum cost alternative. In other words, some individuals choose higher cost alternatives. For example, individuals do not always choose the nearest destination nor the minimum cost mode. Among the several location and travel choices, cost minimizing behavior would appear to apply most aptly to route choice, and to a lesser extent to mode choice; it is certainly inappropriate for choice of destination and location. Deviation from cost minimizing behavior accordingly occurs principally in the location and destination choices and to a lesser extent in mode and route choices.

This behavior may be characterized as choices of more costly alternatives by some travelers. These choices may reflect a desire to obtain a benefit not represented in the travel costs, or may indicate that travelers assign different values to the components of generalized travel cost: travel time, fares, operating costs, etc. Since the objective here is to focus on aggregate travel behavior, a measure of the dispersion of choices across routes, modes, destinations or locations is needed. Consider for this purpose a general measure of the dispersion of choices, as defined in information theory and known as entropy:

$$s=-\sum_k p_k \ln(p_k)$$

where $P_k$ = proportion of travelers choosing alternative k, k = 1, . . . K;

S = measure of the dispersion of the choices across the K alternatives.

If there are K choice alternatives unconstrained by other factors, the range of entropy S is $0\leqslant S\leqslant \ln K$. if the proportion of travelers choosing each alternative is equal, i.e.

$$p_k=\frac{1}{K},$$

S becomes

$$S=-K\cdot\frac{1}{K}\ln\frac{1}{K}=\ln K$$

If, at the other extreme, all travelers choose one alternative, the entropy is zero.

Therefore, the magnitude of S is determined by the number of alternatives. The higher the value of entropy S, the more even is the choice among alternatives. Thus it seems natural to use the entropy as a broad measure of the "dispersion" of choices. Erlander (1977) has characterized S for origin-destination choices as a measure of interactivity or accessibility among zones. Boyce and Janson (1980) referred to the same concept as measuring the level of spatial interaction. The more generalized characterization of route, mode, destination or location dispersion is used here. As a basis for describing travel choices, define:

$P_{ijmr}$= proportion of all person trips, T, traveling from location choice i to destination choice j by mode choice m and route choice r.

Then for the choice proportion, $P_{ijmr}$, the following dispersion measures may be defined among the many other hierarchical structures.

$$-\sum_i\sum_j\sum_m\left(\sum_r p_{ijmr}\right)\ \ln\left(\sum_r p_{ijmr}\right)=S_{LDM} \tag{5}$$

(location-destination-mode)

$$-\sum_i\sum_j\left(\sum_n\sum_r p_{ijmr}\right)\ \ln\left(\sum_m\sum_r p_{ijmr}\right)=S_{LD} \tag{6}$$

(location-destination)

The above measures of choice dispersion implicitly assume the same weights or attractiveness among choice alternatives. Since each location or destination has different attractiveness, zone-specific variables may be included in the above measures. Such variables are conveniently introduced through the notion of an a priori probability in the entropy function. For example, equation (5) may be rewritten in the location choice model as follows:

$$-\sum_i\sum_j\sum_m\left(\sum_r p_{ijmr}\right)\ \ln\left(\sum_r p_{ijmr}/\bar{p}_j\right)\geq S_{LDM}^1 \tag{7}$$

where $\bar{P}_i$ is an a priori distribution of location choice, i.e., the given proportion of trips originating at zone i. (A bar above the variable p denotes that the value of p is given exogenously).

The same equation (5) may be expressed in the destination choice model as follows:

$$-\sum_i\sum_j\sum_m\left(\sum_r p_{ijmr}\right)\ \ln\left(\sum_r p_{ijmr}/\bar{p}_i\right)\geq S_{LDM}^2 \tag{8}$$

where $\bar{P}_j$ is the given proportion of trips ending at zone j. Of course, $S_{LDM}^1$ and $S_{LDM}^2$, which are the measured values of choice dispersion, have different values.

Now following Evans' (1973) and Erlander's (1977) approaches on the combined trip distribution and assignment problem, Beckmann's network equilibrium problem (equations 1-4) is incorporated with the dispersion constraints on the choices of destination and mode. For the combined destination, mode and route choice model, the a priori probability of destination choice $\bar{P}_j$ is assumed to be known, and included in the dispersion constraints. The known probability $\bar{p}_j$ may be interpreted in many ways; one plausible interpretation is the benefit of choosing workplace or shopping center j for each trip purpose. The problem is stated as follows:

$$\min_{V,P}\sum_m\sum_a\int_0^{v_a^m} s_a^m(x)\,dx \tag{9}$$

$$\text{s. t. } v_a^m=\sum_i\sum_j\sum_r p_{ijmr}\,\sigma_{ijmr}^a T \quad a\varepsilon A_m \tag{10}$$

$$\sum\sum\sum p_{ijmr}=\bar{p}_i \qquad i=1,\ \cdots\cdots,\ I \tag{11}$$

$$-\sum_i\sum_j\left(\sum_m\sum_r p_{ijr}\right)\ \ln\left(\sum_m\sum_r p_{ijmr}/\bar{p}_j\right)\geq S_{LD} \tag{12}$$

$$-\sum_i\sum_j\sum_m\left(\sum_r p_{ijmr}\right)\ \ln\left(\sum_r p_{ijmr}/\bar{p}\right)\geq S_{LDM} \tag{13}$$

$$p_{ijmr} \geqq 0 \tag{14}$$

Two Lagrange multipliers, 1/n and $1/\mu$, are defined here for the constraints(12) and(23) respectively. Then the Lagrangian equation for the problem is

$$L=\sum_m\sum_a\int_0^{v_a^m} s_a^m(x)\,dx+\sum_i\sum_j\lambda_i\Big(\bar{p}_i-\sum_m\sum_r p_{ijmr}\Big)$$
$$+\frac{1}{\eta}\Big[S_{LD}+\sum_i\sum_j\Big(\sum_m\sum_r p_{ijmr}\Big)\ \mathrm{In}\ \Big(\sum_m\sum_r p_{ijmr}/\bar{p}_j\Big)\Big]$$
$$+\frac{1}{\mu}\Big[S_{LDM}+\sum_i\sum_j\sum_m\Big(\sum_r p_{ijmr}\Big)\ \mathrm{In}\ \Big(\sum_r p_{ijmr}/\bar{p}_j\Big)\Big]+$$
$$\sum_i\sum_j\sum_m\sum_r \lambda_{ijmr}(-p_{ijmr})$$

Differentiating with respect to $P_{ijmr}$ gives the following optimality conditions:

$$\frac{\partial L}{\partial p_{ijmr}}=T\sum_a s_a^m(v_a^m)\,\sigma_{ijmr}^a+\lambda_i(-1)+$$
$$\frac{1}{\eta}\Big[\mathrm{In}\Big(\sum_m\sum_r p_{ijmr}/\bar{p}_j\Big)+1\Big]$$
$$+\frac{1}{\mu}\Big[\mathrm{In}\Big(\sum_r p_{ijmr}/\bar{p}_j\Big)+1\Big]+\lambda_{ijmr}(-1)=0$$

and $\lambda_{ijmr}\,p_{ijmr}=0$

These may be interpreted as follows:

1. If $p_{ijmr}\rangle\ 0$, tken $\lambda_{ijmr}=0$ and

$$\mathrm{In}\ \Big(\sum_r p_{ijmr}/\bar{p}_j\Big)=-\frac{\mu}{\eta}\Big[\mathrm{In}\ \Big(\sum_m\sum_r p_{ijmr}/\bar{p}_j\Big)+1\Big]+$$
$$\mu\lambda_i-1-\mu T\sum_a s_a^m(v_a^m)\,\delta_{ijmr}^a \tag{15}$$

$$\text{or}\ -\frac{1}{\mu T}\Big[\mathrm{In}\Big(\sum_r p_{ijmr}/\bar{p}_j\Big)+\frac{\mu}{\eta}\ \Big(\mathrm{In}\Big(\sum_m\sum_r p_{ijmr}/\bar{p}_j\Big)$$
$$+1\Big)-\mu\lambda_i+1\Big]=\sum_a s_a^m(v_a^m)\,\delta_{ijmr}^a\equiv c_{ijmr} \tag{16}$$

Since the left hand side of equation (16) is independent of the route r, chosen routes must have equal travel cost for each ijm combination. These equal travel costs are designated $c_{ijm}$, with

$c_{ijm} = c_{ijmr}$ for all chosen routes r

2. If $P_{ijmr} = 0$, then $c_{ijmr} \geqslant c_{ijm}$

Next consider equation (15). Exponentiating (15) gives:

$$\sum_r p_{ijmr}=\Big(\sum_m\sum_r p_{ijmr}\Big)^{-\frac{\mu}{\eta}}\bar{p}_j^{\frac{\mu+\eta}{\eta}}\quad \exp(\lambda_i'-\mu T c_{ijm}),$$

where $\lambda_i'=\mu\lambda_i-1-\frac{\mu}{\eta}$

By summing this equation with respect to m and simplifying, one obtains

$$\sum_r p_{ijmr}=\bar{p}_j\exp\Big[\frac{\eta\lambda_i'}{\eta+\mu}\Big]\frac{\exp(-\lambda' c_{ijm})}{\Big(\sum_m\exp(-\lambda' c_{ijm})\Big)^{\frac{\mu}{\eta+\mu}}},$$

where $\lambda'=\mu T$

Using the composite cost definition proposed by Williams (1977), let

$$[\exp(-\lambda'\tilde{c}_{ij})]^{\frac{\eta}{\eta+\mu}}=\Big[\sum_m\exp(-\lambda' c_{ijm})\Big]^{\frac{\eta}{\eta+\mu}} \tag{17}$$

where $\tilde{c}_{ij}$ is the composite cost of travel from i to j by all modes as given by the above expression; equation (17) can be simplified by letting

$$\exp(-\beta'\tilde{c}_{ij})=\exp\Big(\frac{-\lambda'\eta}{\eta+\mu}\tilde{c}_{ij}\Big)=\exp(\ \lambda'\tilde{c}_{ij})^{\frac{\eta}{\eta+\mu}},$$

where $\beta'=\dfrac{\lambda'\eta}{\eta+\mu}$

Letting $p_{ijm*}=\sum_r p_{ijmr}$, and solving for $\lambda_i'$ in terms of constraint (11), one obtains the following result:

$$p_{ijm*}=p_i\frac{\bar{p}_j\exp(-\beta'\tilde{c}_{ij})}{\sum_j \bar{p}_j\exp(-\beta'\tilde{c}_{ij})}\frac{\exp(-\lambda' c_{ijm})}{\sum_m\exp(-\lambda' c_{ijm})} \tag{18}$$

Equation (18) is a nested logit function of destination and mode choice, where route choice is given by the user-optimal network equilibrium principle.

The combined location, mode and route choice problem given destination choice may be formulated in a similar approach to the previous problem. In this case constraint (11) is replaced by:

$$\sum_i\sum_m\sum_r p_{ijmr}=\bar{p}_j \qquad j=1,\cdots,J$$

and $\bar{p}_i$ replaces $\bar{p}_j$ in the dispersion constraints. By analogy to the combined destination, mode and route choice model, the combined location, mode and route choice model follows:

$$p_{ijm*}=\bar{p}_j\frac{\bar{p}_i\exp(-\beta' c_{ij})}{\sum_i(\bar{p}_i\exp(-\beta' c_{ij})}\frac{\exp(-\lambda' c_{ijm})}{\sum_m\exp(-\lambda' c_{ijm})} \quad (19)$$

## 3. Solution Algorithms—A Generalization of Evans' Approach

The above problems have a structure similar to Evans' combined distribution and assignment problem. This class of problems has the form, min $[f(v) + g(p)]$, as an objective function, where $f(v)$ is the sum of the integrals of the cost functions, and $g(p)$ consiss of one or more entropy functions weighted by their respective Lagrange multipliers. The set of constraints consists of linear functions, which insure the balancing of trips and the conservation of network flows.

Evans' algorithm is initiated with a feasible solution of the main problem, consisting of an initial solution of the distribution problem followed by an all-or-nothing assignment to the minimum cost paths. Her distribution problem is a doubly-constrained gravity models with a negative exponential cost function; it may be solved by use of balancing factors or Furness' iterative procedure (Murchland, 1966). A sequence of these distribution and assignment subproblems is solved. Following each pair of problems, a line search is performed to find the optimal combination of the main problem and subproblem distribution and assignment solutions, producing a new solution to the combined problem with a lower value of the objective function. The steps are repeated until a suitable level of convergence is obtained.

This algorithm has been generalized to apply to various combined choice problems. A generalized version of Evans' iterative algorithm for solving this class of model is stated below. The following variables are fist defined:

$v_a^m(n)$ = flow of persons on link a of mode m in the main problem solution at iteration n

$P_{ijmr}(n)$ = proportion of persons choosing choice set ijmr in the main problem solution at iteration n

$w_a^m(n)$ = flow of persons on link a of mode m in the subproblem solution at iteration n

$q_{ijmr}(n)$ = proportion choosing choice set ijmr in the subproblem at iteration n.

Then the algorithm consists of the following steps:

1. Choose an initial feasible solution, $v_a^m(1)$ and $P_{ijmr}(1)$; set n = 1.
2. Compute minimum cost paths: for every origin zone i, find the minimum cost path to every destination zone j on mode m; designate these costs, $c_{ijm}(n)$.
3. Compute choice proportions: calculate $q_{ijm}(n)$ according to the optimality criterion for the problem of interest based on the travel cost $c_{ijm}(n)$ corresponding to the main problem solution.
4. Assign trips: assign $Tq_{ijm}(n)$ to the minimum cost paths, resulting in $w_a^m(n)$.
5. Perform convergence check: if convergence criterion is satisfied, STOP; otherwise,
6. Perform line search: find the value λ

which minimize the following function:

$$\min_{0\leq\lambda\leq1} f[(1-\lambda)v(n)+\lambda w(n)]+$$

$$g[(1-\lambda)p(n)+\lambda q(n)] \quad (20)$$

$$\text{set}: v(n+1)=(1-\lambda)v(n)+\lambda w(n) \quad (21)$$

$$p(n+1)=(1-\lambda)p(n)-\lambda q(n) \quad (22)$$

update link cost, given v(n+1),
set n = n + 1,
go to step 2.

Evans (1973) proved the convergence of the above algorithm and provided convergence criteria. Frank (1978) and LeBlanc and Farhangian (1981) showed that the algorithm is more efficient compared with linearization algorithms.

The convergence criterion is obtained from an identification of a lower bound on the optimal value of the problem at each iteration as done by LeBlanc et al. (1973). A stopping criterion for the proposed models is derived as follows:

$$\nabla f[v(n)]\cdot[w(n)-v(n)]+\nabla g[p(n)]\cdot[q(n)-p(n)] \mid \leqq \epsilon$$

where $\epsilon$ is a quantity chosen according to the level of accuracy required. As solution approaches the equilibrium state, the ratio of the above criterion to the value of the objective function is expected to approach zero.

## 4. Description of Program "UROAD"

The model derived in the above sections were implemented on a large-scale transportation network serving Chicago region. The computer algorithms for solving the models were implemented using the Urban Transportation Planning System (UTPS) subprogram "UROAD". UROAD, a highway assignment and analysis program, was developed as part of the Urban Transportation Planning System (UTPS). UTPS was designed to provide practicing transportation planners with state-of-art methods for the analysis of multimodal transportation systems. Periodically, UTPS has been upgraded in response to the development of new planning techniques and advances in computer hardware.

As it has been developed to date, UROAD is a comprehensive, flexible program for analysing highway networks, incorporating a number of "capacity restraint" assignment methods, including one approach based on network equilibrium concepts. The primary functions of the current version of UROAD are summarized as follows:

1. network construction and/or modification
   a. historical record file input
   b. test network generation
2. trip table construction and/or modification
   a. selected link assignment analysis
   b. time of day and directional loading
   c. test trip table generation
3. tree tracing (skim trees)
   a. link impedance calculation (time, distance, toll)
   b. minimum impedance paths (time, distance, and toll)
4. assignment
   a. all-or-nothing assignment (with optional turn penalties)
   b. all shortest paths assignment
   c. probabilistic multipath assignment (stochastic method)
   d. capacity restraint (equilibrium, iterative or CATS incremental)
5. highway travel and environmental impact summarization

In addition to the above functions, UROAD achieves much of its flexibility by allowing the user to alter, replace or insert certain functions through the inclusion of user-coded subroutines. The application of UROAD is illustrated in Figure 1, displaying its input and output files.

As part of UTPS, the program is integrated with the data structures of the other programs in the system. The program operates on an IBM 360/370 computer of a compatible system having both tape and disk input/output devices using the full Operating System/360. Core requirements vary from one UTPS program to another and, within programs, by problem size.

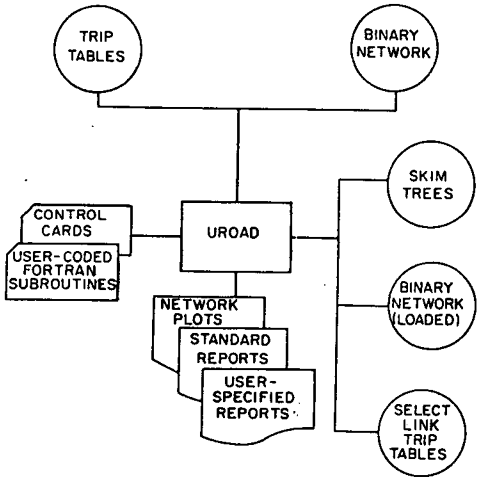


FIGURE 1. GENERAL FLOWCHART OF APPLICATION OF UROAD

Source: U. S. DOT (1979), UTPS Reference Manual

## 5. Implementation of Evans' Algorithm in UROAD

The generalized algorithm of Evans is incorporated with UROAD through the user-coded subroutine ROA23. A number of entry points are provided in subroutine ROA23 to furnish user-supplied routines. Each of these entry points is used to provide the following functions.

1. **Entry ROA23A** is called after UROAD control cards have been read. This entry is used to initialize user-supplied parameters which are specific to each problem. In addition, the information on auto access and egress walking time for each zone as well as auto parking costs are read through this entry point. Later in the process, these data are multiplied by the appropriate cost coefficients and added by the generalized link costs on the path to complete the highway path cost between origin-destination zones.
2. **Entry ROA23B** is called after each link record has been read. At this point the capacity and free-flow travel time from each input link record is saved for use during the network equilibrium process. Also a generalized link cost based on free flow is computed to obtain the initial solution.
3. **Entry ROA23C** is called after each minimum cost tree has been traced and the zone trip vector read. The main purpose of this entry point is to compute the trip probability from origin i to destination j by mode m, $P_{ijm}$, given trip vector, $\overline{P}_{ij}$, or trips by origin, $\overline{P}_i$, or trips by destination, $\overline{P}_j$, for each problem. For this purpose the generalized highway path cost is completed by combining the link-related cost to the origin/destination related costs, such as access/egress walking time and auto parking cost.

The transit cost vector also is read here. Then, the logit-type optimality condition derived in Section 2 is applied, producing $P_{ijm}$. At the end of this entry point, the proportion of auto trips is converted to vehicle trips by applying the vehicle occupancy ratio, and then adding the truck trips. This result, total highway vehicle trips, is assigned in the next step.

4. **Entry ROA23D** is called after each tree is assigned. No function, however, is processed at this point. An additional call to this entry point is made during each iteration, after all trees have been built and loaded. The functions of this entry points are as follows:
   a. perform the convergence test;
   b. given the main and subproblem sets of volumes on links and predicted trip tables, find the convex linear combination which minimizes the value of the objective function, equation (20);
   c. revise the link flows and trip tables as in equation (21) and (22);
   d. update the generalized link cost, given revised link flows.
5. **Entry ROA23E** is called after all iterations are completed. For the final link flows and trip tables, a parameter calibration routine is invoked at this entry point. Also some statistics on the results are reported.

Many of the above functions are not invoked in the implementation of the network equilibrium route choice model (Equation (1)-(4)), since this problem doesn't have a choice dispersion constraint.

For the network equilibrium model of equation (9)-(14), no trip table is input to the program; instead, trip origin and destination totals are read through entry ROA23A. The maximum number of iterations is set to nine because UROAD allows a maximum of 10 all-or-nothing assignments.

The functional relationship between the program UROAD and the user code is portrayed in Figure 2.

## 6. Model Application

The proposed procedures were applied to the aggregate network of the Chicago region prepared by the Chicago Area Transportation Study (CATS) for the sketch planning purpose. The network consists of 317 zones and 1,250 bidirectional arterial and freeway links. In this paper we do not attempt to provide details of applications; however, they may be referred to Chon (1982).

The performance of the models is evaluated in two ways: by examining the convergence rate to a usable solution, and by comparing the estimated link flows and trip tables to the observed ones.

The convergence of the algorithm may be judged in many ways: the change in the value of the objective function, the ratio of the stopping criterion to the value of the objective function, or the value of $\lambda$ which is used to form the linear combination of the main problem solution and the subproblem solution. Table 1 shows the results from the combined destination, mode and route choice model using the calibrated values of parameters (for more detail, see Chon, 1982). Values of the objective function and entropy are displayed in Figures 3 and 4 with respect to each iteration. The objective function value reached a stable point around iteration 6 and shows small increases at iteration 8 and 9. These increases resulting from round-off error appear in the later iterations when the solution approaches the optimal point.

Vehicle-miles of travel were tabulated by the zone in which the links are located for the model described above and for a fixed demand user-optimal network equilibrium route choice model computed on the same generalized cost function. The result of this comparison is shown by freeway and arterial links in Figures 5a and 5b. The

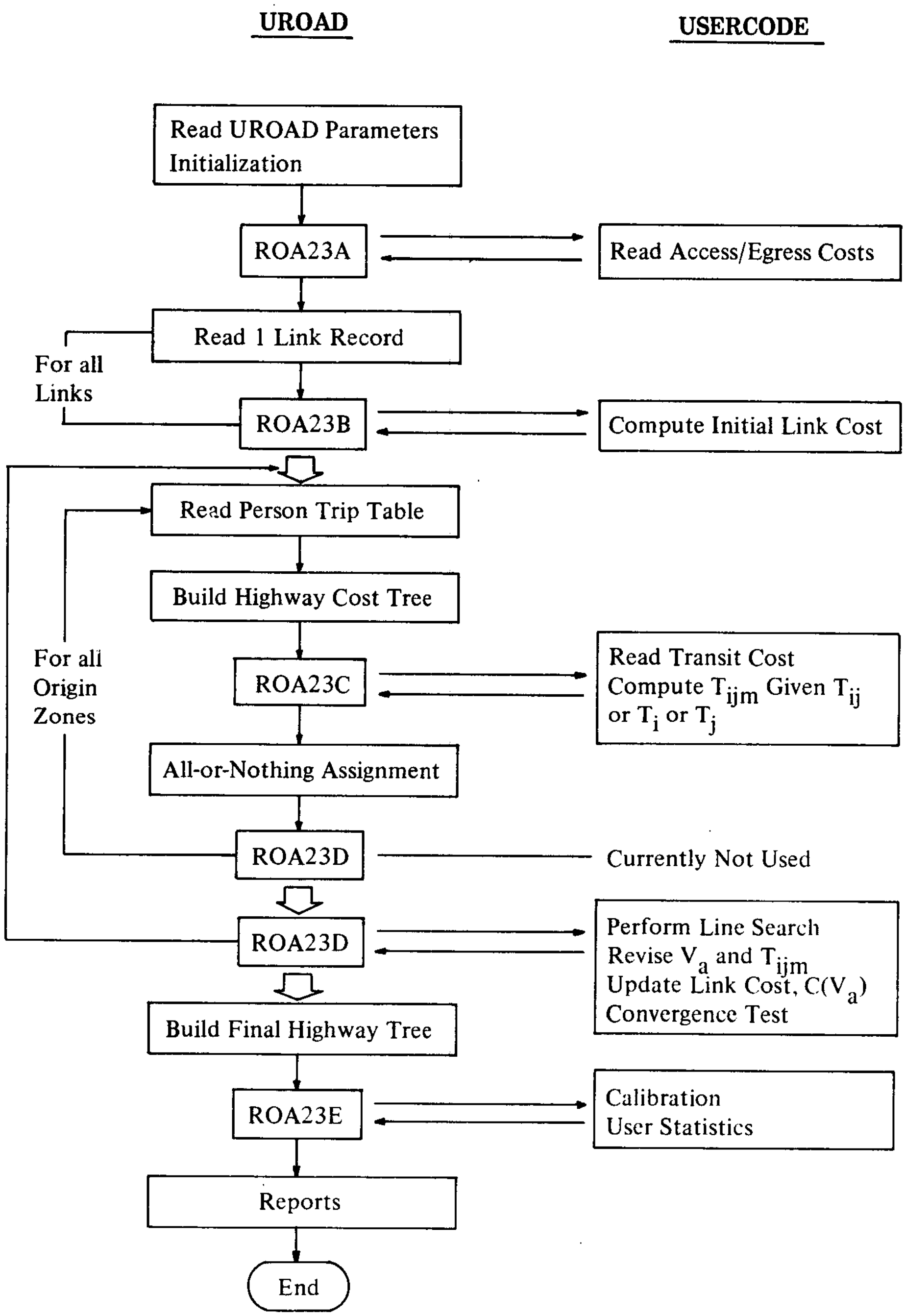


FIGURE 2. EXTENDED UROAD FLOWCHART

**TABLE 1. CONVERGENCE OF THE COMBINED DESTINATION, MODE AND ROUTE CHOICE MODEL**

$\mu = 0.171 \times 10^{-7}; \eta = 0.1 \times 10^{-4}$

Observed $S_{LDM}$ = 3.83959; Observed $S_{LD}$ = 3.57535; Observed Transit Share = 0.160

| Iteration | Value of Objective Function | Percent Change | Value of Stopping Criterion | Ratio to Objective Function | $S_{LDM}$ | $S_{LD}$ | Transit Share | $\lambda$ |
|---|---|---|---|---|---|---|---|---|
| 1 | $-0.53553 \times 10^8$ | – | $0.90616 \times 10^8$ | 1.692 | 3.87783 | 3.76353 | 0.1762 | 0.4336 |
| 2 | $-0.54938 \times 10^8$ | –2.6 | $0.59700 \times 10^7$ | 0.109 | 3.86112 | 3.74905 | 0.1784 | 0.2877 |
| 3 | $-0.55363 \times 10^8$ | –0.8 | $0.14167 \times 10^7$ | 0.026 | 3.85267 | 3.74109 | 0.1795 | 0.2664 |
| 4 | $-0.55497 \times 10^8$ | –0.2 | $0.80325 \times 10^6$ | 0.014 | 3.84859 | 3.73734 | 0.1799 | 0.1712 |
| 5 | $-0.55548 \times 10^8$ | –0.1 | $0.51396 \times 10^6$ | 0.009 | 3.84187 | 3.73096 | 0.1806 | 0.3303 |
| 6 | $-0.55617 \times 10^8$ | –0.1 | $0.59897 \times 10^6$ | 0.011 | 3.83648 | 3.72624 | 0.1811 | 0.3485 |
| 7 | $-0.55798 \times 10^8$ | –0.3 | $0.69383 \times 10^6$ | 0.012 | 3.83672 | 3.72632 | 0.1811 | 0.1368 |
| 8 | $-0.55782 \times 10^8$ | 0.0 | $0.37325 \times 10^6$ | 0.007 | 3.83636 | 3.72593 | 0.1811 | 0.0942 |
| 9 | $-0.55588 \times 10^8$ | 0.3 | $0.29302 \times 10^6$ | 0.005 | 3.83440 | 3.72395 | 0.1812 | 0.3941 |

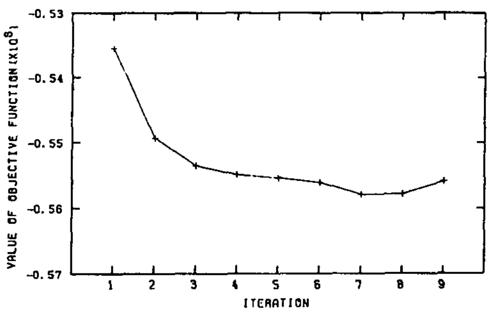


FIGURE 3 VALUE OF OBJECTIVE FUNCTION VS. ITERATION

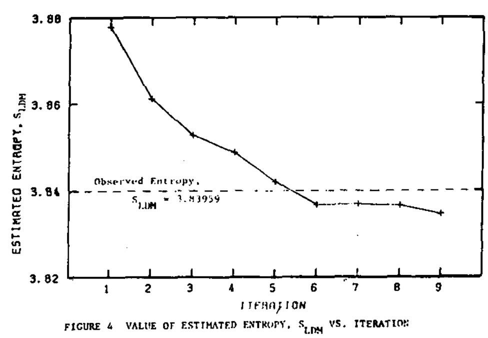


FIGURE 4 VALUE OF ESTIMATED ENTROPY, $S_{LDM}$ VS. ITERATION

correspondence may be seen quite close. The route choice model is considered to be an appropriate representation of "observed" link flows on the highly aggregated and therefore unobservable sketch planning highway network.

The estimated trips from the combined destination, mode and route choice model are compared to the observed trips in Figures 6a to 6d. Each of the scattergrams deals with auto trips by origin, transit trips by origin, auto trips by destination, and transit trips by destination, respectively. Many points that are exactly on 45-degree line in the figures represent the zones which do not have transit access or have extremely high transit costs. Since this model is bound by the number of trips by origin, the reciprocal relationship experienced in the combined mode and route choice model exists between the auto trips by origin and the transit trips by origin (Figures 6a and 6b). The overall overestimation of the transit share by the model (0.18122)is clearly shown in Figures 6a and 6b. Also the outlying points in Figures 6c and 6d were produced by the CBD zone.

The number of trips choosing the CBD zone as a destination is approximately 60 percent higher than the observed trips, mostly comprised of transit trips. This happened because the transit bias term in the generalized cost function was adjusted upwards for the CBD bound trips to improve the overall goodness of fit. Because of its extraordinarily higher potential as a trip attractor, it was quite difficult for the combined models to satisfy the CBD zone observations as well as many other small zones.

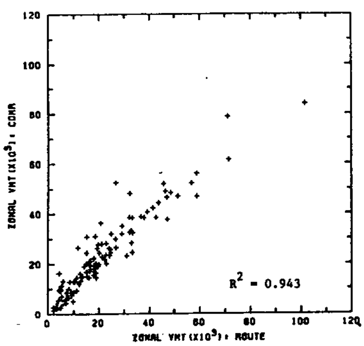


FIGURE 5a COMPARISON OF FREEWAY VEHICLE-MILES OF TRAVEL BY ZONE: COMBINED DESTINATION, MODE AND ROUTE CHOICE MODEL VS. ROUTE CHOICE MODEL

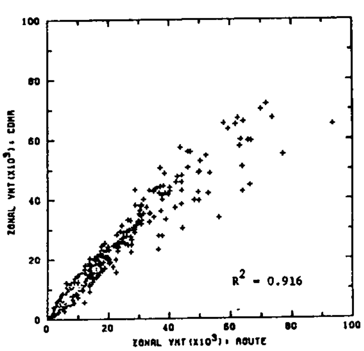


FIGURE 5b COMPARISON OF ARTERIAL VEHICLE-MILES OF TRAVEL BY ZONE: COMBINED DESTINATION, MODE AND ROUTE CHOICE MODEL VS. ROUTE CHOICE MODEL

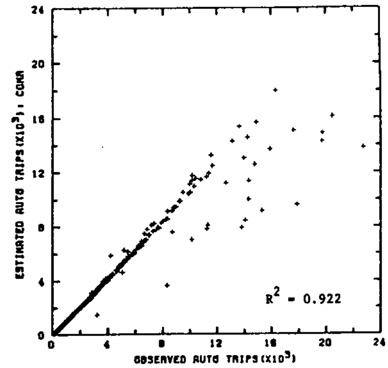


FIGURE 6a COMPARISON OF OBSERVED AND ESTIMATED AUTO TRIPS BY ORIGIN

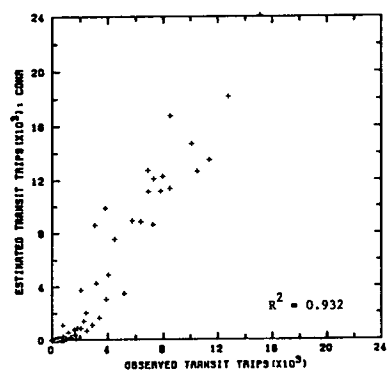


FIGURE 6b COMPARISON OF OBSERVED AND ESTIMATED TRANSIT TRIPS BY ORIGIN

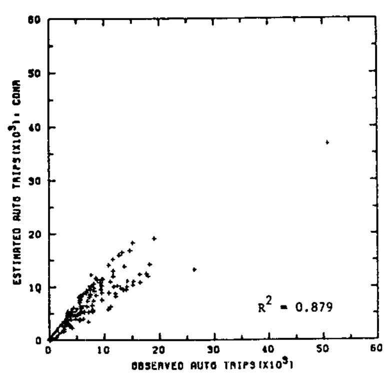


FIGURE 6c COMPARISON OF OBSERVED AND ESTIMATED AUTO TRIPS BY DESTINATION

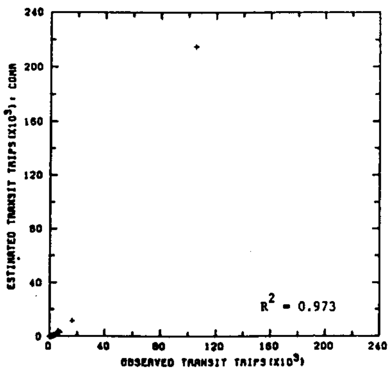


FIGURE 6d COMPARISON OF OBSERVED AND ESTIMATED TRANSIT TRIPS BY DESTINATION

## 7. Conclusions

This paper was more concerned on the problems of model formulations and their possibility of practical applications. Concurrently developed was a paper more devoted to the retrospective survey in this field (Boyce et. al. 1987). Readers are recommended to refer to the paper for more solid background.

Adopting UTPS as a basic tool for the implementation of the proposed models, greatly facilitated the applications of the models to a large-scale transportation system. This approach should allow many planning agencies to employ this class of models with ease as a convenient tool for sketch planning or TSM analysis.

## 8. References

Beckmann, M., McGuire, C.B. and Winsten, C.B. (1956) Studies in the Economics of Transportation, Yale University Press, New Haven, Conn.

Boyce, D.E. and Janson B.N. (1980) A Discrete Transportation Network Design Problem with Combined Trip Distribution and Assignment, Transportation Research, 14B, 147-154.

Boyce, D.E., LeBlanc, L.J. and Chon, K.S. (1987) Network Equilibrium Models of Urban Location and Travel Choices: A Retrospective Survey, forthcoming in the Journal of Regional Science.

Chon, K.S. (1982) Testing of Combined Urban Location and Travel Choice Models, Ph.D. Thesis, Department of Civil Engineering, University of Illinois, Urbana, Illinois.

Chon, K.S. and Boyce, D.E. (1985) Computational Results of Network Equilibrium Models of Urban Location and Travel Choices: Chicago and Stockholm, paper presented at the 32nd North American Meetings of the Regional Science Association, Philadelphia, Pensylvania, Nov. 15-17, 1985.

Erlander, S. (1977) Accessibility, Entropy and the Distribution and Assignment of Traffic, Transportation Research, 11, 149-153.

Evans, S.P. (1973) Some Applications of Mathematical Optimisation Theory in Transport Planning, Ph.D. Thesis University of London, London.

Frank, C. (1978) A Study of Alternative Approaches to Combined Trip Distribution-Assignment Modeling, Ph.D. Thesis, Department of Regional Science, University of Pennsylvania, Philadelphia, Pennsylvania.

LeBlanc, L.J. (1973) Mathematical Programming Algorithms for Large Scale Network Equilibrium and Network Design Problems, Ph.D. Thesis, Department of Industrial Engineering and Management Sciences, Northwestern University, Evanston, Illinois.

LeBlanc, L.J. and Farhangian, K. (1981) Efficient Algorithms for Solving Elastic Demand Traffic Assignment Problems and Mode Split-Assignment Problems, Transportation Science, 15, 306-317.

Murchland, J.D. (1966) Some Remarks on the Gravity Model of Traffic Distribution and an Equivalent Maximization Formulation, LSE-TNT-38, Transport Network Theory Unit, London Graduate School of Business Studies, London.

U.S. Department of Transportation (1979) UTPS Reference Manual, Urban Mass Transportation Administration, Washington, D.C.

Wardrop, J.G. (1952) Some Theoretical Aspects of Road Traffic Research, Proceedings, Institute of Civil Engineers, 1, Part II, 325-378.