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NUMERICAL RANGES OF OPERATORS ON HILBERT C^*-MODULES

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Let B be a unital C^*-algebra over $\mathbf{C}$. Let X be a Hilbert B-module with a B-valued inner product $\langle , \rangle$, and let $L(X)$ be the Banach algebra of all bounded linear operators on X. We define for $T \in L(X)$ a spatial numerical range, $W_B(T) = \{f(\langle Tx, x\rangle) : (x, f) \in \Pi\}$, where $\Pi = \{(x, f) : x \in$ unit sphere of X, $f \in$ unit sphere of B', and $f(\langle x, x\rangle) = 1\}$, and obtain for X over $\mathbf{C}$: (1) the closure of the convex hull of $W_B(T)$ equals the algebra numerical range of T in $L(X)$; (2) $W_B(T)$ is connected, but not closed; (3) an extension of Williams' theorem on the Hilbert space and an application of our concept to a unital C^*-algebra.

Secondly, we extend results of J.K. Canavati concerning a definition and properties of a numerical range for the class of all numerically bounded operators on a Banach space. In particular, we introduce a new class of B^*-numerically bounded operators on X and define the numerical range $\Omega(T)$ for any operator T of this class. In the case that T is a bounded linear operator, $\Omega(T)$ coincides with the closure of $W_B(T)$. Among other properties, we show that $\Omega(T)$ is compact and connected and obtain analogue of Lumer's formula for the class of Lipschitz maps.

Finally, in terms of our spatial numerical range, we define a Hermitian operator, a positive operator and an operator "closed to Hermitian" in some sense, and obtain some results (as Vidav's theorem and Sinclair's theorem in a complex Banach algebra) of them.

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