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—Research Paper—

A Goal Programming Model for Reverse Resource Allocation

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ABSTRACT

This paper deals with the development and analysis of a quantitative model for reducing operating budgets of the academic units of a small private university, while reflecting the diverse goals of the university. A zero-one goal programming approach is used to design and implement the model for budgetary decision-making. The goal programming model can facilitate academic planning and decision-making by providing valuable information.

1. INTRODUCTION

The subject of reverse resource allocation (i.e., budget cut) continues to receive increasing emphasis in colleges and universities as the environment of resource limitation permeates many institutions of higher learning. Some major factors contributing to the financial predicament of the university are enrollment shifts from humanities to business, reassessment of the value of a college education, enrollment declines, and rapidly rising costs. In addition to these, both the federal and state governments have cut back their support to academia, thus swithching of priorities to more expedient social problems. Research grants have been reduced or eliminated by both foundations and private industry.

In recent years, there has been tendency to place much emphasis on "balancing the budget" at the expense of the academic goals of the institution. The crucial issue in the university administration is not just financial efficiency. The operational policy must be based on the combined philosophies of many diverse groups within the university. The very purpose, concept, and function of the administrators, faculty and students must be embodied in budgeting decisions.

The purpose of this paper is to develop and analyze a quantitative model for reducing the operating budgets of the academic units of a university while reflecting diverse goals of the university and allowing some degree of autonomy in decision-making with respect to budgetary matters. To achieve

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this objective, a goal programming (GP) model was developed based on the data obtained from a small private university in the State of New York to reflect the multiple competitive goals of the university. Specifically, the model employs a zero-one (0-1) goal programming process to reflect the indivisibility of the budget packages.

There are a number of goal programming studies which have appeared in the academic journals [2], [3], and [6]. However, this present study differs from the existing studies in that it addresses the problem of budget reductions rather than expansions. It is large in scope, encompassing every level of academic units within the university, incorporating input from administration at all levels of the academic hierarchy.

2. A GOAL PROGRAMMING MODEL

The GP model data is drived from a private co-educational university offering undergraduate programs in natural sciences, social sciences, humanities, business, and education, The undergraduate population is approximately 2,400 students. The GP model is primarily concerned with establishing optimal portfolios of budget-cut packages for each school in a university, and optimal packages of budget-cut items for each department within the schools.

Overview of the Model

The GP model in this paper involves three administrative levels : academic vice-president, deans of schools or divisions, and chairmen of departments. The model will allow decision maker at each level to select the portfolio of budget cut-backs which has the least negative impact on his multiple objectives.

The budget process is sequential in nature, using top-down goal decomposition approach. The academic vice-president initially determines goals of the university (i.e., the highest-level goals). These goals are generally quite broad in nature rather than very detailed. These goals are then decomposed into the goals of the deans of the schools. This decomposition results in the formulation of subgoals which contribute to the goals of the higher level. A dean may add one or more additional goals which are not in direct conflicts with those of the academic vice-president. Finally, the same process is repeated by chairmen at the department level.

After the goals are established at the department level, the information flow is reversed. Each department formulates budget-cut alternatives with its dollar savings amounts and establishes priority ranking of its goals. Department chairman assesses the adverse impact each alternative has on the development goals using paired-comparison and eigenvalue prioritization techniques [4], [5], [7]. Each department sets up its own GP model in which the zero-one (0-1) decision variables correspond to budget-cut alternatives. The model is solved for several possible budget level cuts, usually between three and five in number. Corresponding to the budget levels, optimal sets of budget-cut alternatives, called portfolios, are communicated upward to the dean of the school.

The same procedures are repeated by each dean based on the set of mutually exclusive portfloios received from his departments, along with any budget-cut options he may wish to add. Dean's budget-cut alternatives are then communicated upward to the academic vice-president. The academic vice-president uses the model to make a final selection of portfolios so as to best achieve the highest-level goals for the known total value of budget cuts and allocates the budget cuts.

The following assumptions are made for the model development.

1. Any goal within a hierarchical level can be represented by a linear combination of the budget-cut alternatives.
2. The goals of an academic unit can be identified.
3. It is possible to prioritize goals.
4. A budget item is either not cut at all, or is cut only at one of the levels specified in the model.
5. Administrators are able to formulate and rank budget-cut alternatives with respect to their expressed goals.

Mathematical Statement of the Model

The mathematical statement of the GP model is presented below.

Phase 1: Department Level

If there are s structural constraints, f functional constraints and r levels of g goal constraints, the model can be stated as follows:

$$\text{Minimize } V = P_1 \sum_{i=1}^{s} (d_i^+ + d_i^-) + P_2 \sum_{i=s+1}^{s+f} (d_i^+ + d_i^-) + P_3 (d_{s+f+1}^+ + d_{s+f+1}^-) + \sum_{j=1}^{r} \sum_{i=s+f+2}^{s+f+r} P_j \alpha_{ij} d_i^+ \quad (1)$$

Subject to

$$f_i(x) + d_i^- - d_i^+ = a_i \qquad i=1, \cdots, s \text{ (structural constraints)} \quad (2)$$

$$h_i(x) + d_i^- - d_i^+ = a_i \qquad i=s+1, \cdots, s+f \text{ (functional constraints)} \quad (3)$$

$$\sum_{j=1}^{n} F(X_j)X_j + d_i^- - d_i^+ = b \qquad i=s+f+1 \text{ (budget-cut constraint)} \quad (4)$$

$$\sum_{j=1}^{n} W(X_i)X_j - d_i^+ = 0 \qquad i=s+f+2, \cdots, s+f+r \text{ (other goal constraints)} \quad (5)$$

and

$$X_j = \begin{cases} 1 & \text{if alternative is chosen} \\ 0 & \text{otherwise} \end{cases}$$

$d_i^-,\ d_i^+ \geq 0.$

where α_i = weights within a priority level

$f_i(X),\ h_i(X)$ = linear combinations of decision variables corresponding to the structural and functional characteristics of the problem

$F(X_j)$ = the dollar savings of budget alternative X_j
X_j = a department budget-cut decision variable, $j=1,\cdots, n$
$W_i(X_j)$ = the relative impact of X_j on goal i
a_i = contsants (usually 0 or 1) determined uniquely by the specific situation being modeled
b = a budget-cut parameter. The model will be solved for three to five different values of b
d^+ = positive deviation from goal
d^- = negative deviation from goal

In the objective function (1), the highest priority, P_1, is placed on structural constraints (2) with equal weighting, because these constraints must be satisfied in order to assure a feasible solution. The functional constraints (3) are given the second priority level, P_2, with equal weights in the objective function. The decision variables in (3) correspond to interrelated activities and are mutually dependent upon one another. The budget-cut constraint (4) is placed at the third priority level, P_3. This is the first goal constraint specifying the dollar amount of budget-cut requirement to be satisfied as closely as possible. In other goal constraints (5), $W(X)$ is a linear function of decision variables with weights derived from the eigenvalue technique. The right-hand side value of 0 represents the goal's least negative impact when all variables are zero. These goal constraints are prioritized by the department chairmen with P_4, through P_n priorities in accordance with their importance.

Phase 2: Division Level
The model is similar to that of Phase 1, substituting Y for X and B_j for $F(X_j)$, since the Y_i will yield savings of B_j.

Phase 3: University Level
The model is similar to that of Phase 2, substituting Z for Y. This moded is solved only once, since the value of B here is the total amount of budget cuts.

3. A ZERO-ONE GP MODEL FOR DEPARTMENT A1

There are 26 GP models in this study: 20 departmental, 5 divisional, and 1 university models. For illustration, a 0-1 GP model for Department A1 is only presented below as an example.

$$\text{Minimize } V=P_1\left(\sum_{i=1}^{6} d^-_{i,1}+d^+_{i,1}\right)+P_2(d^-_{7,1}+d^+_{7,1})+P_3d^+_{8,1}+P_4d^+_{9,1}$$

$$\begin{aligned}\text{subject to } & X_1(1)+X_1(2)+W_1+d^-_{1,1}-d^+_{1,1}=1\\ & X_3(1)+X_3(2)+W_2+d^-_{2,1}-d^+_{2,1}=1\\ & X_4(1)+X_4(2)+W_3+d^-_{3,1}-d^+_{3,1}=1\\ & X_5(1)+X_5(2)+W_4+d^-_{4,1}-d^+_{4,1}=1\\ & X_6(1)+X_6(2)+W_5+d^-_{5,1}-d^+_{5,1}=1\end{aligned}$$

$$X_7(1)+X_7(2)+W_6+\bar{d}_{6,1}-\overset{+}{d}_{6,1}=1$$

$$300X_1(1)+1300X_1(2)+500X_2(1)+200X_3(1)+650X_3(2)+500X_6(1)+1000X_6(2)$$
$$+500X_7(1)+2000X_7(2)+\bar{d}_{7,1}-\overset{+}{d}_{7,1}=b$$

$$35X_1(1)+63X_1(2)+49X_2(1)+8X_3(1)+211X_3(2)+144X_4(1)+206X_4+127X_5(1)$$
$$+20X_5(2)+18X_6(1)+54X_6(2)+50X_7(1)+44X_7(2)-\overset{+}{d}_{8,1}=0$$

$$129X_1(1)+64X_1(2)+62X_2(1)+11X_3(1)+15X_3(2)+16X_4(1)+18X_4(2)+18X_5(1)$$
$$+214X_5(2)+169X_2(1)+52X_6(2)+47X_7(1)+185X_7(2)-\overset{+}{d}_{9,1}=0$$

and

$$X_j=\begin{cases}1 \text{ if alternative is chosen}\\ 0 \text{ otherwise}\end{cases}$$

$$\overset{+}{d}_1,\ \bar{d}_i \geq 0$$

The model was solved using a modified 0-1 GP package [8]. For each of the 20 departments at the university studied, a separate portfolio of budget reductions was established corresponding to a 10%, a 15%, or a 20% cut in that department's operating expense budget. The five division models were run corresponding to 5%, 10%, 15%, or 20% reduction in that division's budget. Finally, at the highest academic level, the model was run once corresponding to an overall reduction of 10%. Thus, a total of 81 runs were made on a Prime 550 Computer. A CPU time for these runs required almost nine minutes. All of the structural, functional, and budget-cut goals were completely satisfied on 81 runs. The academic unit goals were not completely satisfied on any of the runs.

Department A1's model has 13 0-1 decision variables, six 0-1 artificial variables, nine constraints, and four priority levels. The decision variables and their respective dollar savings are presented in Table 1.

TABLE 1

Variable	Description	Savings($)
$X_1(1)$	Student work program	300
$X_1(2)$		1,300
$X_2(1)$	Postage	500
$X_3(1)$	Memberships	200
$X_3(2)$		650
$X_4(1)$	Travel	500
$X_4(2)$		1,000
$X_5(1)$	Lectures	700
$X_5(2)$		1,100
$X_6(1)$	Office supplies	500
$X_6(2)$		1,000
$X_7(1)$	Library, books	500
$X_7(2)$		2,000

As the table indicates, seven budget categories are included in this model. Two reductions are made in each of six of these categories, while one category (postage) has only one reduction. An artificial variable is necessary each time when two or more reduction, in the same category occur. Consequently, six artificial variables are required.

Six of the nine constraints are of a structural nature, and hence appear at the first priority level. These constraints arise due to the mutually exclusiveness of variables $X_i(1)$ and $X_i(2)$, $i=1, 3, 4, 5, 6, 7$.

In order to use a standard 0-1 goal programming computer code, these structural constraints must be converted to goal format. This can be done by placing all structural constraints on the first priority level and rewriting the constraints in the form

$$\sum_{i=l}^{c} X_i(j) + W_i + \bar{d_i} - \overset{+}{d_i} = 1.$$

W_i is an artificial zero-one variable which is necessary to allow at most one of the $X_i(J)$ to be chosen at a particular i-level.

The seventh constraint is the budget constraint. The coefficient of each variable indicates the exact amount of the proposed reduction in that category. The right-hand-side constant b will vary for each of three runs of the program, corresponding to a 10%, 15%, or 20% reduction in the department's budget. For this department, the three values of b, corresponding to three separate runs, are $1,750, $2,625, and $3,500. The budget constraint appears at the second priority level.

Constraints 8 and 9 correspond to the two goals of the department. These constraints appear at the third and fourth priority levels, respectively. The coefficients of the variables in these equations are the weights calculated by the eigenvalue prioritization technique.

A comparison is made between the actual percentage allocation of the university's academic budget under existing budgetary practice and the percentage allocations resulting from the model when a 10% reduction in the operating budget is required. It is presented in Table 2. While there does not appear to be great differences in the distribution of funds among the various categories in this department, a word of caution should be made here. In the current budget process, chairmen indicated that it was difficult to increase any budget item substantially even if compensating decreases were made in other categories. If this tactic were taken, the results were that the items reduced were left at those lower levels and that category which was increased was cut back to its original budgeted vlaue. Chairmen thus found it advantageous to try to increase all budget items, even when not necessary. A redistribution of resources was difficult. Under the model, the chairman is able to decide the best (in terms of satisfying departmental goals) distribution of alloted funds among all budget line-items.

TABLE 2: Comparison of Actual and Model Budget Allocations for Department A1

Line-item	Actual Allocation (%)	Model Allocation(%)
Student Work	23.99	23.79
Postage	2.49	2.73
Memberships	3.13	2.38
Travel	10.42	13.32
Lectures	3.74	3.57
Repair of Equipment	1.25	0.00
Research	1.87	1.19
Supplies	28.15	26.85
Library	24.95	26.17

4. USER EVALUATION OF THE MODEL

The 0-1 GP model has been tested and evaluated by various administrators. Twelve chairmen favored using the model to determine operationg budgets, with only two votes against it. The majority of chairmen were uncertain as to the model's use for developing alternative budget allocations (7 yes, 10 unceratin and 3 no). Eleven chairmen would use the model to defend their budget recommendations, with 6 uncertain and 3 voting no. Most chairmen did not complain of excessive work or time involvement with the model (13 no, 4 yes, 3 uncertain). Most chairmen also provided favorable comments on the model use.

The deans of the five divisions were asked for their reactions to the results of the model. In all cases, favorable comments were made. These administrators realized that the fund should be allocated to those areas where most students were registered or where most of the research was being done. They all believed the model allocation did that.

The Academic Vice-President was very enthusiastic about the model results. He is presently using the results as a planning tool in next year's budget allocation.

5. SUMMARY

This paper presented a 0-1 GP model for optimal reverse allocation of budgetary resources at a private university. The model was developed and tested for budgetary decision-making. Tests of the model to an actual decision situation show that the model is indeed promising, thus bridging the gap between theory and practice.

A survey of users revealed a definite preference for the model results over the existing budgetary procedure. While it was developed specifically for the opearting budgets of academic units within a university, the model can be easily extended to and used by any organization experiencing a similar problem. It can be a viable decision-making tool for short-term financial planning.

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