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Optimum Parameter Determination of PLL Used in Timing Clock Recovery Circuit

(타이밍 클럭 복원 회로에 사용된 PLL의 최적 파라미터 결정)

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要 約

능동 지연 1차 루우프 여파기를 사용한 2차 PLL(phase-locked loop) 회로의 폐회로 전달함수를 구하였다. 회로의 과도 응답시간, 선형 분석에 의한 잡음대역, 그리고 근 궤적에 의한 안정도를 시스템 성능 기준으로 고려하였을 때 최적 댐핑 팩터의 값은 1.0이 되었고, 자연 주파수(natural frequency)의 값은 사용 신호 주파수에 따라서 응답 시간과 잡음대역의 상반 관계를 고려하여 결정할 수 있음을 확인하였다.

Abstract

The closed-loop transfer function of 2-nd order PLL(phase-looked loop) of which loop filter has active-lag 1-st order is found. Considering the three criteria of system performance: the transient response time of the circuit, noise bandwidth by the linear analysis and stability which uses root-locus method, the optimum value of damping factor is 1.0 and the natural frequency which depends upon the signal frequency can be determined after consideration of the trade-off relationship between the transient response time and the noise bandwidth.

I. Introduction

In digital/optical PCM data transmission, timing clock is recovered for the original data to be found from the input data stream in receivers or repeaters. Self-timing method is very useful for the clock recovery, in which there are two typical schemes: tuned-circuit scheme and PLL (phase-locked loop) scheme [1] [4] [6]. Without the loss of practical generality, PLL is well-accepted.

On the report of T. Shimamura and I. Eguchi [3], the proper value of damping factor is 5 to 8 when the number of repeater chain is 100, or 15 to 18 when the number of repeater chain is 1000 in the viewpoint of minimization of jitter accumulation. But in this paper the noise bandwidth is found to be minimum at the point of damping factor of about 0.5 which is the same with the result of F.M. Gardner's analysis [6].

The noise bandwidth in which damping factor is 5 to 18 is about 14 to 50 times than the minimum bandwidth. It is quite natural that the larger the noise bandwidth is, the poorer the system performance is. In addition to the noise bandwidth, the transient response time and the stability are considered for the performance enhancement of 2-nd order PLL which is generally used in timing recovery circuit.

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II. Second-order PLL

In Fig. 1. the active-lag typed loop filter is shown which is used in the second-order PLL.

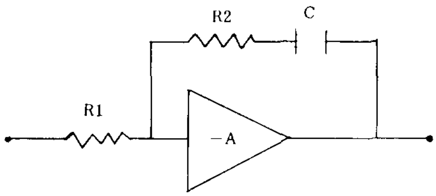


Fig. 1. 1-st Order Active Loop Filter.

For large A in Fig. 1, L(s) which is the transfer function of loop filter is represented in eq. (1).

$$L(s) = -\frac{sT_2 + 1}{sT_1} \qquad (1)$$

where $T_1 = R1*C$, $T_2 = R2*C$

The PLL block diagram which uses the loop filter in Fig. 1 is demonstrated in Fig. 2.

Eq. (2) is the expression of closed-loop transfer function [5] [6].

$$H(s) = \frac{\theta o\,(s)}{\theta i\,(s)} \qquad (2a)$$

$$H(s) = \frac{KvKpL\,(s)}{s + KvKpL\,(s)} \qquad (2b)$$

where Kv is the transfer gain of VCO,
Kp is the transfer gain of PD.

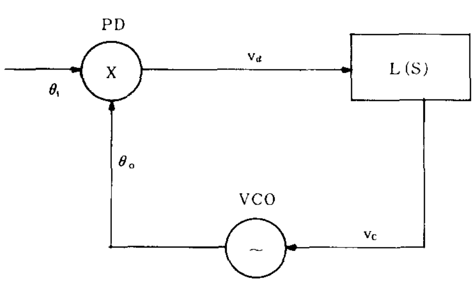


Fig. 2. 2-nd Order PLL.

Combining the eqs. (1) and (2), we can obtain the eq. (3).

$$H(s) = \frac{2\xi w_n s + w_n^2}{s^2 + 2\xi w_n s + w_n^2} \qquad (3)$$

where ω_n is $(KvKp/\,T_1)^{1/2}$ which is natural frequency, ξ is $(T_2\,\omega_n)/2$ which is damping factor.

Here we can see that the natural frequency and the damping factor change according to the variation of the transfer gain of devices and the passive elements.

III. Transient Response Time

In line coding for digital communications, bipolar NRZ(nonreturn-to-zero) data without DC component will be transmitted. BUU(bipolar-to-unipolar unit) in repeaters or receivers converts this signal to the signal with the discrete spectral components. The unipolar NRZ data stream will reach the timing recovery circuit through the BPF(bandpass filter).

From eq. (3) the output response will be eq. (4) when the input data of 1 is applied.

$$C(s) = \frac{2\xi w_n s + w_n^2}{s^2 + 2\xi w_n s + w_n^2}\;\frac{1}{s} \qquad (4)$$

In order to study the transient response time (rise time), eq. (4) must be inversely transformed to time-domain. The consequent equations (5), (6), (7) are classified in accordance with the variation of damping factor.

When damping factor ξ is greater than 1.0,

$$\begin{aligned} c(t) = 1 &+ \frac{1}{2(\xi^2 - \xi\sqrt{\xi^2-1}\ -1)}\, e^{-(\xi-\sqrt{\xi^2-1})w_n t} \\ &+ \frac{e^{-(\xi+\sqrt{\xi^2-1})\,w_n t}}{2(\xi^2 + \sqrt{\xi^2-1}\ \xi - 1)} \\ &+ \frac{\xi}{\sqrt{\xi^2-1}}\, e^{-(\xi-\sqrt{\xi^2-1}\,)\,w_n t} \\ &- \frac{\xi}{\sqrt{\xi^2-1}}\, e^{-(\xi+\sqrt{\xi^2-1}\,)w_n t} \end{aligned} \qquad (5)$$

When damping factor ξ is less than 1.0,

$$c(t) = 1 + \frac{e^{-\xi w_n t}}{\sqrt{1-\xi^2}} \sin(w_n\sqrt{1-\xi^2}\,t+\alpha) + \frac{2\xi \sin\sqrt{1-\xi^2}\,w_n t}{\sqrt{1-\xi^2}} e^{-\xi w_n t} \quad (6)$$

where α is $-\tan^{-1}(\sqrt{1-\xi^2}\ \xi)$.
When damping factor ξ is equal to 1.0,

$$c(t) = 1 - e^{-w_n t} + w_n t\, e^{-w_n t} \quad (7)$$

In practical digital/optical transmission system the natural frequency ω_n has at least 3000 [rad/sec] (especially, 3653 [rad/sec] in 1344 channel (2 * T3 level) 90 Mbps fiber optic system). For the mathematical expression of transient response time (rise time: 10% to 90% response), the approximation is accomplished to obtain the eqs. (8), (9).

$$t_R = \frac{2.197}{(\xi + \sqrt{\xi^2 - 1})\, w_n} \quad \text{for } \xi \gg 1.0 \quad (8)$$

$$t_R = \frac{0.6325}{\xi w_n} \quad \text{for } \xi \ll 1.0 \quad (9)$$

In Fig. 3 transient response time (rise time) versus damping factor is plotted.

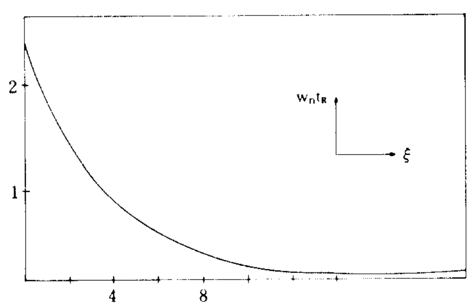


Fig. 3. Rise Time of PLL.

IV. Noise Bandwidth

Noise accompanying the signal causes the VCO (voltage-controlled oscillator) phase to fluctuate. Noise performance is studied in case for the stationary, bandpass, Gaussian noise. Noise bandwidth of the loop through the linear analysis is depicted as eq. (10) which is generally well-accepted [1] [2] [5] [6].

$$BW_n = \int_0^\infty |H(j2\pi f)|^2\, df \quad (10)$$

After eq. (3) is combined into eq. (10), complex integration is performed by use of residue threom according to the region of variation of damping factor. The resultant equations for the noise bandwidth are as follows.

When $0. < \xi < 1/\sqrt{2}$.

$$BWn = \frac{w_n(4\xi^2+1)}{8\sin[0.5*\arctan(2\xi\sqrt{1-\xi^2}/(1-2\xi^2))]} \quad (11)$$

When $\xi = 1\ \sqrt{2}$,

$$BWn = 3\sqrt{2}\, w_n/8 \quad (12)$$

When $1\ \sqrt{2} < \xi < \mathbf{1.0}$,

$$BWn = \frac{w_n(4\xi^2+1)}{8\cos[0.5*\arctan(2\xi\sqrt{1-\xi^2}\,/\,(2\xi^2-1))]} \quad (13)$$

When ξ is equal to 1.0,

$$BWn = w_n*(5/8) \quad (14)$$

When $\xi > 1.0$,

$$BWn = \frac{w_n[p-q](4\xi^2+1)}{16\xi\sqrt{\xi^2-1}} \quad (15)$$

where $p = \sqrt{2\xi^2 - 1 + 2\xi\sqrt{\xi^2-1}}$

$q = \sqrt{2\xi^2 - 1 - 2\xi\sqrt{\xi^2-1}}$

The noise bandwidth normalized by the natural frequency is graphically shown in Fig. 4 from which we can see that the noise bandwidth is minimum at damping factor of 0.49 and

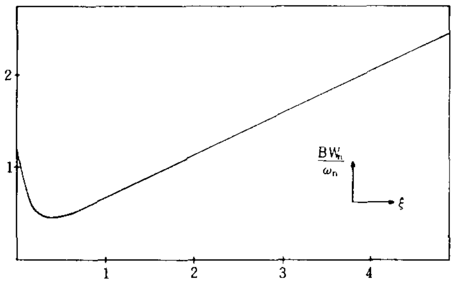


Fig. 4. Noise Bandwidth.

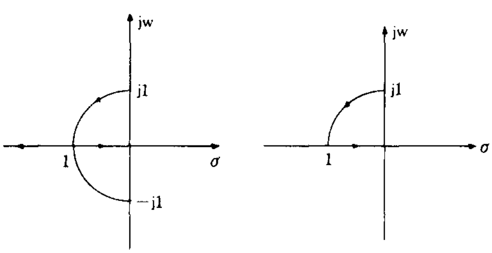
hereafter linearly increases in parallel with the growth of damping factor.

V. Stability

Of the system stability criteria, the root-locus method is applicated to this 2-nd order PLL circuit.

The root-locus is the trace of poles of the closed-loop transfer function (the movement of root of denominator) in s-domain as the parameters such as damping factor, natural frequency vary from zero to infinity.

The farther the poles go to the left hand side from the ordinate, the more stable the circit is. The root-locus for the 2-nd order PLL is shown in Fig. 5. The dominant root-locus which determines the stability is also drawn.



(a) total root-loci (b) dominant root-locus

Fig. 5. Root-Locus of 2-nd Order PLL.

From Fig. 5, we can plot the magnitude of stability to the variation of damping factor because poles of eq. (3) are $wn[-\xi\pm\sqrt{\xi^2-1}]$, which is demonstrated in Fig. 6.

In the region where damping factor has 1.0, the PLL circuit is most stable.

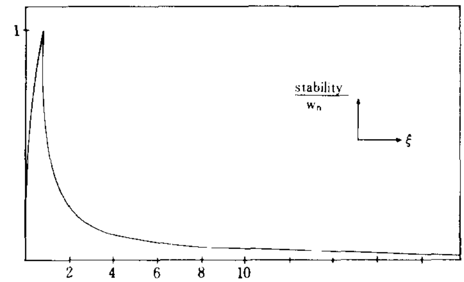


Fig. 6. Stability Variation.

VI. Discussion

Fig.3,4,6 are graphical results based on the analysis of circuit parameter for the performance enhancement of 2-nd order PLL. The smaller the noise bandwidth is, the smaller the transient response time is and the farther the poles exist from the imaginary axis in s-domain, the richer and the better the 2-nd order PLL circuit is. The global property which depends on the three criteria is shown in Fig. 7.

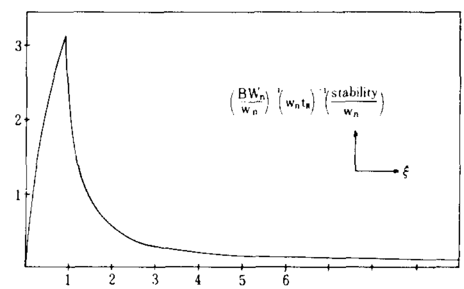


Fig. 7. Global Property of 2-nd Order PLL.

Therefore the optimum value of damping factor is 1.0.

The noise bandwidth and stability are proportional to the natural frequency and transient response time is inversely propertional to the natural frequency. In general, the natural frequency will be calculated so that the transient response time may be at most 10% of the signal duration time and the noise bandwidth may be small as possible and the circuit may be stable.

VII. Conclusion

The 2-nd order PLL which adopts the active-lag 1-st order as loop filter has been analysed on the basis of three criteria: transient response time, noise bandwidth, stability.. The conclusive result is shown in Fig. 7. in which the optimum value of damping factor is 1.0.

The transient response time is inversely proportional to the natural frequency as shown in Fig. 3. the noise bandwidth and stability are in parallel with the natural frequency in Fig. 4. Fig. 6. respectively.

References

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