Coupled-Mode Analysis of Two Slab Waveguides Fabricated with Vanadirum Oxide and Lithium Niobate

(VO_2와 $LiNbO_3$로 집적된 2개의 슬랩 도파로에 관한 결합모드 해석)

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要 約

두 개의 이웃하는 슬랩 도파로에서 광 모우드의 결합에 관한 이론적 해석을 하여 컴퓨터 시뮬레이션 하였다. 도파로 1은 리듐 니오베이트($LiNbO_3$), 도파로 2는 바나듐 옥사이드(VO_2)로 설정하였다. 이 결과로 (1)두 개의 도파로가 동일하든지 혹은 각 도파로 내의 모우드들이 동일한 위상을 갖을 경우 한 도파로에서 다른 도파로로 전력을 완전히 전달할 수 있음을 알았고 (2)한 도파로는 손실이 있고 또 다른 도파로는 손실이 없을 경우라도 위상 정합이 이루어질 때는 전력전달이 가능함을 알았다. 또 결합의 세기에 따라 전력 감쇠가 변화함을 볼 수 있었다.

Abstract

An analysis and numerical computations relating to the coupling of optical modes between two neighboring waveguides are discussed. The waveguides are fabricated with Lithium Niobate as guide 1 and Vanadium Oxide as guide 2. In a waveguide system that incorporates two lossless guides, a complete transfer of power from one guide to another can occur when the waveguides are (1) identical or (2) the modes in each guide have identical phase constants. Here we discuss the coupling effects when the guides are dissimilar with respect to both geometry and losses. In thers results, we show that power transfer can occur between the two guides, one lossy and the other lossless, provided the phase matched condition is satisfied. When properly coupled, the power attenuation varies according to the amount of coupling.

I. Introduction

This is a study of the characteristics of coupled waveguides fabricated by paralleling two dielectric waveguides. One guide will be fabricated in a semiconductor material such as Lithium Niobate ($LiNbO_3$) and the other guide consisting of Vanadium Oxide (VO_2) will be fabricated the semiconductor material and determing the effects of the coupling of the optical fields. The basic material is shown in Fig. 1. Under the phase matched condition, the optical field in the semiconductor couples very strongly to the modes of the thermochromic material. [1]-[2] Phase matched condtion means that the propagation constant of the mode in the semiconductor material is identical to the propagation constant of the

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mode in the thermochromic thin film waveguide. When the phase matched condition is not satisfied, there is little or no coupling between the two guides. [3]-[5]

The basic idea behind this study is to determine the necessary geomatries of parallel coupled waveguides which produce strong coupling from the semiconductor guide to the Vanadium Oxide material. [6]-[7] Because the Vanadium Oxide is a very lossy material in both of its bistable states (For example, at an optical wavelength of 1μm, the power absorption of Vanadium Oxide in the cold state is approximately 30,000/cm while in its hot state, it is about 90,000/cm.), optical field coupling from the semiconductor guide to the Vanadum Oxide film will be strongly attenuated. Consequently upon inputing light to the semiconductor guide under the non phase-matched condition, light in the guide will not couple to the thin film and will propagate with only slight attenuation. On the other hand, when the phase matched condition is satisfied, the light will be attenuated strongly. [8] The resulting device acts as a switch.

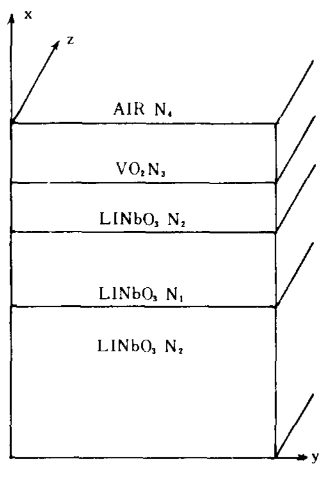


Fig. 1. The Basis Structure of the Coupled Waveguide System.

II. Theory and Derivation of Equations

It has been shown that a complete transfer of power from one waveguide to another is possible for specific geometries of two parallel waveguides. [9]-[10] The only requirement is that the phase velocities of the modes in both waveguides are identical. Two parallel waveguides can exchange energy if the field which is carried by one guide reaches or overlaps the field of the other guide. [11] Even though the two modes have identical phase velocities, their field distributions need not be identical.

Also, it is well known that lossless modes can have complete exchange of power between two guides, but when either or both of the guides have losses, it is not clear that power transfer can occur with the efficiency of that for the lossless case. In this paper, we show that power transfer can occur but the net power propagating down the guide will attenuate. Attenuation rates will depend upon the absorption coefficients of the two guides.

In the derivation presented here, the coupling coefficients will be assumed to be constant for guides having no functional dependence with respect to z. When the guides are lossless, the coupling coefficients are real, while for lossy guides, they are complex. We present an approximate derivation of the coupled wave equations for the case of the two parallel dielectric guides.

We use the semiconductor material lithium niobate as guide 1, but it is possible the use a different material. The primary condition is that the refractive index of the semiconductor material must have a value that lies between the refractive index of the Vanadium Oxide material in its hot and cold states.

The guide 2 is formed in the thermochromic thin film deposited over the semiconductor. The transverse distribution of the refractive index defining the two waveguides is shown in Fig. 2. For identical phase velocities ($\beta_1=\beta_2=\beta$), now we have

$$\gamma_1 = \alpha_1 + j\beta \qquad (1)$$
$$\gamma_2 = \alpha_2 + j\beta \qquad (2)$$

The electric field in each guide is given as

$$E_1(x,z) = \psi_1(x)\exp(-\gamma_1 z) \qquad (3)$$
$$E_2(x,z) = \psi_2'(x)\exp(-\gamma_2 z) \qquad (4)$$

When both guides are plased sufficiently close for coupling to occur, the total fields in the tandum guide can expressed as a linier superposition of the unperturbed fields of each guide, i.e.,

$$E(x,z) = A_1(z)E_1(x,z) + A_2(z)E_2(x,z) \tag{5}$$

Where $A_1(z)$ and $A_2(z)$ very slowly with respect to z.

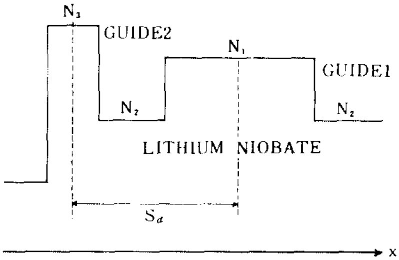


Fig. 2. Transverse Distribution of the Refractive Index.

1. Derivation of Coupled Mode Equations

For a transverse electric (TE) coupled mode we assume that $\kappa(x)$ is the dielectric constant of the total waveguide system while $\kappa_1(x)$ is the dielectric constant of guide 1 in the absence of guide 2. On the other hand, $\kappa_2(x)$ is the dielectric constant of guide 2 in the absence of guide 1. The electric field E_y satisfies Maxwell's equations so that we can write

$$\frac{\partial^2 Ey}{\partial z^2} + \frac{\partial^2 Ey}{\partial x^2} + K_o^2 \kappa(x)Ey = 0 \tag{6}$$

Using equation (5), we get

$$\frac{\partial^2 Ey}{\partial z^2} = \sum_{n=1}^{2} \frac{d^2 A_n}{dz^2} + 2\frac{dA_n}{dz}\frac{\partial^2 E_n}{\partial z} + A_n\frac{\partial^2 E_n}{\partial z^2} \tag{7}$$

Upon substituting (3) and (4) in the above equation, we get the following:

$$\frac{\partial^2 Ey}{\partial z^2} = \sum_{n=1}^{2} [A_n\gamma_n^2 - 2\gamma_n \frac{dA_n}{dz}]\psi_n(x)e^{-\gamma_n z} \tag{8}$$

and

$$\frac{\partial^2 Ey}{\partial x^2} = \sum_{n=1}^{2} - [\gamma_n^2 + K_o^2 \kappa_n(x)] \times A_n\psi_n(x)\exp(-\gamma_n z) \tag{9}$$

Upon simplifying the above equation, we get

$$\sum_{n=1}^{2} - 2\gamma_n \frac{dA_n}{dz} \psi_n(x)e^{-\gamma_n z} + K_o^2 A_n [\kappa(x)-\kappa_n(x)] \times \psi_n(x)e^{-\gamma_n z} = 0 \tag{10}$$

Multiplying by $\psi_m(x)\exp(\gamma_m z)$ on both sides of the above equation

$$\sum_{n=1}^{2} - 2\gamma_n \frac{dA_n}{dz} \psi_m(x)e^{(\gamma_m-\gamma_m)z} + K_o^2[\kappa(x)-\kappa n(x)]\, A_n\psi_m\psi_n(x)e^{\gamma_m-\gamma_n)z} = 0 \tag{11}$$

When m=1, equation (11) becomes

$$-2\gamma_1 \frac{dA_1}{dz} \psi_1^2(x) + K_o^2[\kappa(x)-\kappa_1(x)]A_1\psi_1^2(x) - 2\gamma_2\frac{dA_2}{dz}\psi_1(x)\psi_2(x)e^{(\gamma_1-\gamma_2)z} + K_o^2 \times [\kappa(x)-\kappa_2(x)]A_2\psi_1(x)\psi_2(x)e^{(\gamma_1-\gamma_2)z} = 0 \tag{12}$$

Upon integration with respect to the transverse dimension, we get

$$0 = -2\gamma_1\frac{dA_1}{dz} + A_2K_o^2[\int_{-\infty}^{\infty}[\kappa(x)-\kappa_2(x)] \times \psi_1(x)\psi_2(x)dx]\,e^{(\gamma_1-\gamma_2)z} \tag{13}$$

$$\frac{dA_1}{dz} = A_2C_1\exp[(\gamma_1-\gamma_2)z] \tag{14}$$

$$C_1 = \frac{K_o^2}{2\gamma_1} \int_{-\infty}^{\infty} [\kappa(x)-\kappa_2(x)]\psi_1(x)\psi_2(x)dx \tag{15}$$

Similarly, from equation (11), when m=2, we get

$$\frac{dA_2}{dz} = A_1 C_2 \exp[(\gamma_2-\gamma_1)z] \tag{16}$$

Where

$$C_2 = \frac{K_o^2}{2\gamma_2} \int_{-\infty}^{\infty} [\kappa(x)-\kappa_1(x)]\psi_1(x)\psi_2(x)dx \tag{17}$$

Equation (14) and (16) are the coupled wave equations while (15) and (17) are the coupling coefficients of the two guides.

2. Derivation of Coupled Ampulitude Equations

By making the transformations

$$a_1(z) = A_1(z)e^{-j\beta z} = A_1'(z)e^{-\gamma_1 z}$$

$$a_2(z) = A_2(z)e^{-j\beta z} = A_2'(z)e^{-\gamma_2 z} \tag{19}$$

to the new independent variables $a_1(z)$ and $a_2(z)$, the new differential equations become

$$\frac{da_1}{dz} = C_1 a_2(z) - \gamma_1 a_1(z) \tag{20}$$

$$\frac{da_2}{dz} = C_2 a_1(z) - \gamma_2 a_2(z) \tag{21}$$

Using (20) and (21), we get

$$\frac{d^2 a_1}{dz} + (\gamma_1+\gamma_2)\frac{da_1}{dz} + (\gamma_1\gamma_2 - C_1 C_2)a_1 = 0 \tag{22}$$

The above differential equation can now be solved. Assuming a solution of the form exp(pz) where $\gamma_1-\gamma_2=\alpha_1-\alpha_2$, (the phase matched condition is assumed), we obtain the following eigenvalue equation

$$P^2 + (\gamma_1+\gamma_2)P + \gamma_1\gamma_2 = C_1 C_2 \tag{23}$$

$$P = -\alpha - j\beta \pm j\triangle\beta \tag{24}$$

$$\alpha = \frac{\alpha_1 + \alpha_2}{2} \tag{25}$$

$$j\triangle\beta = [C_1 C_2 + \frac{(\alpha_1 - \alpha)^2}{4}]^{1/2} \tag{26}$$

Using (18), the coupled wave amplitude $A_1(z)$ can be written as

$$A_1(z) = \frac{1}{2}e^{-\alpha z} \quad a_1(0)[(1-\frac{\alpha_1-\alpha_2}{j2\triangle\beta})e^{j\triangle\beta z} + (1+\frac{\alpha_1-\alpha_2}{j2\triangle\beta})e^{-j\triangle\beta z} + a_2(0)[\frac{C_1}{j\triangle\beta}(e^{j\triangle\beta z} - e^{-j\triangle\beta z})] \tag{27}$$

Now using (19), we similarly obtain

$$A_2(z) = \frac{1}{2}e^{-\alpha z} \quad \alpha_2(0)[(1+\frac{\alpha_1-\alpha_2}{j2\triangle\beta})e^{j\triangle\beta z} - (1-\frac{\alpha_1-\alpha_2}{j2\triangle\beta})e^{-j\triangle\beta z}] + a_1(0)[(\frac{j\triangle\beta}{C_1} - \frac{(\alpha_1-\alpha_2)^2}{4C_1 j\triangle\beta}) \times (e^{j\triangle\beta z} - e^{j\triangle\beta z})] \tag{28}$$

Equations (27) and (28) represent the coupled wave amplitudes of the fields in guide 1 and guide 2. The coupling coefficients C_1 and C_2 are defined in (15) and (17).

III. Numerical Simulation

In this section we simulate the waveguide system (see Fig. 1) as a coupled switch.

First we choose the waveguide width of Vanadium Oxide as 0.12 microns and then find the phase velocity and all values of constants in guide 2. Next we select the index step $\triangle n$ in the Lithium Niobate guide (the waveguide is assumed to be 2 microns) for the same phase velocity as in guide 1 (i.e., $\beta_1 = \beta_2$ and $n_2 = n_1 + \triangle n$). The refractive indices and extinction coefficients of the various region used in the calculations as show in Table 1.

Table 1. The Refractive Indices and Extinction Coefficients.

Parameters of material			
Layer	Material	Refractive Indices	Extinction Coefficients
n_1	$LiNbO_3$	$2.2+\triangle n$	0.0
n_2	$LiNbO_3$	2.2	0.0
n_3	VO_2(cold)	3.0	0.5
n_4	VO_2(hot)	1.7	1.5
n_5	Air	1.0	0.0

1. Calculation of Coupled Coefficient C_1 and C_2

For guide 1 (separation distance S_d), the transverse wave function can be written as follows:

$$\psi_1(x) = \begin{cases} B_1 \exp\left[\gamma_1\left(\frac{d_1}{2} - x + S_d\right)\right]; x > \frac{d_1}{2} + S_d \\ B_2 \cos q_1 (x - S_d) \quad ; -\frac{d_1}{2} + S_d < x < \frac{d_1}{2} + S_d \\ B_3 \exp\left[\gamma_1\left(\frac{d_1}{2} + x - S_d\right)\right]; x < -\frac{d_1}{2} + S_d \end{cases} \tag{29}$$

The fields in guide 2 can also be written in terms of x as

$$\psi_2(x) = \begin{cases} A_1 \exp\left[P_2\left(\frac{d_2}{2} - x\right)\right]; x > \frac{d_2}{2} \\ A_2 \cos q_2 x + C_2 \sin q_2 x; \ -\frac{d_2}{2} < x < \frac{d_2}{2} \\ A_3 \exp\left[\gamma_2\left(\frac{d_2}{2} + x\right)\right]; x < -\frac{d_2}{2} \end{cases} \tag{30}$$

We now calculate the coupling coefficients C_1 and C_2 as a function of the separation distance from equation (15) and (17).

where

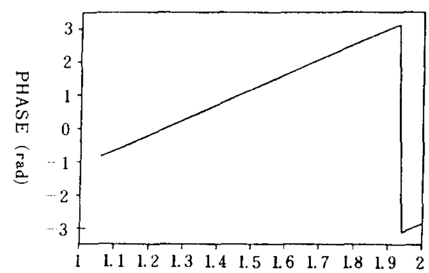
$$K(x) - K_2(x) = n_1^2 \text{ for } -\frac{d_1}{2} + S_d \le x \le \frac{d_1}{2} + S_d \tag{31}$$

$$K(x) - K_2(x) = n_1^2 \text{ for } -\frac{d_1}{2} \le x \le \frac{d_1}{2} \tag{32}$$

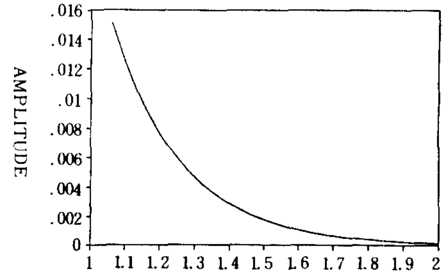
Using a three-layer waveguide program, we obtain the necessary data for the calculations of coupling coefficients. The separation distance S_d is varied from 1.1 to 2 microns. The amplitude and phase of C_1 is shown in Fig. 3. as a function of S_d. The corresponding results obtained for C_2 vs S_d are shown in Fig. 4.

2. Calculation of Coupled Amplitudes

From the previous theoretical results presented earlier, the functional dependence of $A_1(z)$ and $A_2(z)$ can be estimated provided initial conditions dependent upon the launching of an Optical wave in the semiconductor material.



(a)



(b)

Fig.3. Coupling Coefficient C_1 vs. Separation distance S_d.

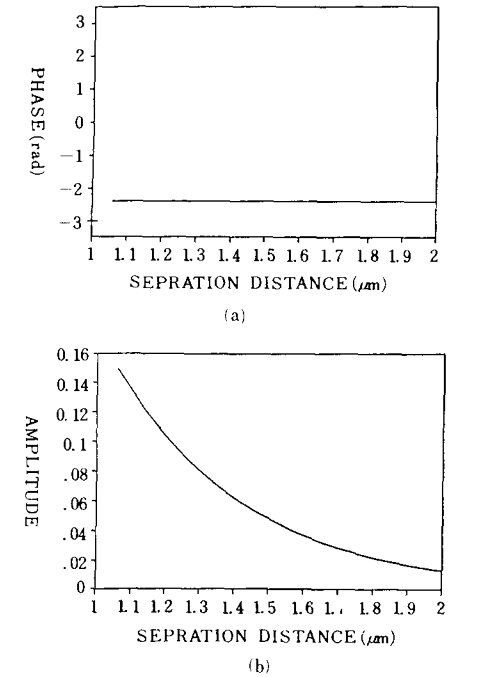


Fig.4. Coupling Coefficient C_2 vs. Separation Distance S_d.

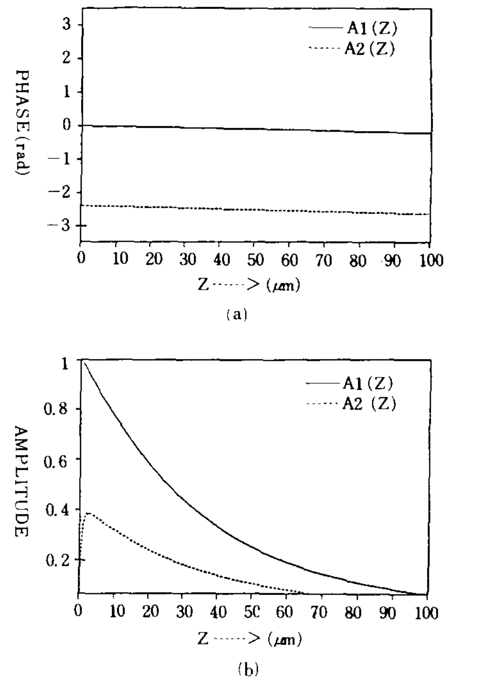


Fig.5. The Amplitude and Phase of Coupled Wave $A_1(z)$ and $A_2(z)$ at $S_d = 1.1\ \mu m$.

Typical results for our calculations are shown in Fig. 5. to 7. Fig. 5. represents the amplitudes and phase of A_1 and A_2 as a function of distance along the waveguide. The separation distance is 1.1 microns. Fig. 6. is shown the amplitudes and phases of A_1 and A_2 with respect to z when S_d = 1.3 microns. Similarly, the amplitudes and phases of fields for S_d = 2 microns are shown in Fig. 7. As indicated in the calculations, when the separation distances are small, the optical field in the semiconductor couples very strongly to the modes of the thermochromic material.

IV. Conclusion

In this analysis and resulting calculations, we show that when the imaginary parts of the propagation constants of the Vanadium Oxide guide and semiconductor guide are identical then power transfer occurs. Since the thin film guide is very lossy, the net power propagating down the tandum guide will attenuate rapidly. The rate of attenuation is dependent

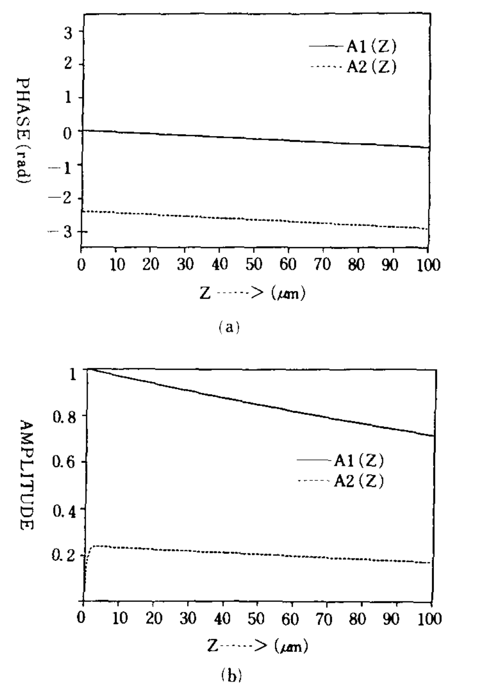


Fig.6. The Amplitude and Phase of Coupled wave $A_1(z)$ and $A_2(z)$ at $S_d = 1.3\mu m$.

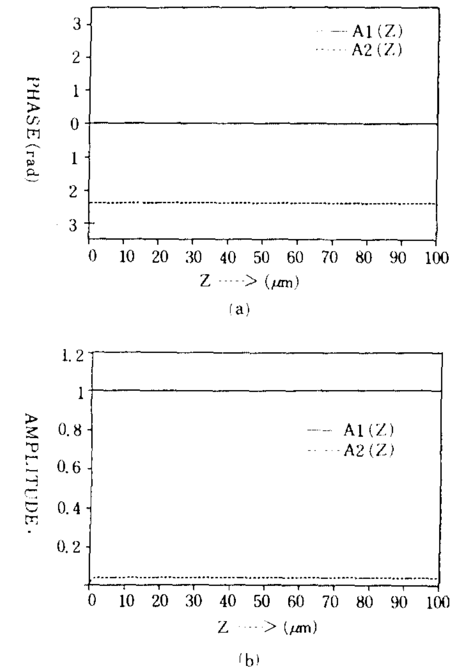


Fig.7. The Amplitude and Phase of Coupled wave $A_1(z)$ and $A_2(z)$ at $S_d = 2.0\ \mu m$.

upon the separation between the two guides. Generally, high Q resonant systems are very sensitive. In these coupled waveguides, obtaining phase matched conditions require very precise values of the waveguide thickness and refractive index.

Obtaining such precise waveguide geometries would require very close manufacturing tolerances. However, Vanadium Oxide as well as other thermochromic materials have refractive indices that range over rather wide values. Thus, the thermochromic materials could be biased and fine turned to a point that the phase matched condition could be satisfied.

If the tandum waveguide system was designed for applications where the input optical power was "high", then the Vanadium Oxide could be switched by the optical field itself. The sensitive of such a system of course be dependent upon the boundary conditions relating to the heat capacity and transmission of heat energy from the Vanadium Oxide films. Application area will include both power and spectrum analyzers.

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