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On Minimum Time Joint-Trajectory Planning for the Cartesian Straight Line Motion of Industrial Robot

(산업용 로보트의 카르테시안 직선 운동을 위한 조인트-궤적의 최소 시간화)

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要 約

산업용 로보트의 조인트 영역(joint space)에서 선형보간(linear interpolation)에 의한 카르테시안(Cartesian) 직선운동의 근사는 많은 바람직한 잇점과 광범위한 응용성을 갖는다. 그러나 각각의 선형구간과 가속구간에서 그의 주행시간이 부적합하게 결정되면 최대의 입력 가능한 토크/힘을 이용하지 못하거나 혹은 비실현(unrealization)적인 조인트-궤적을 산출하게 된다. 본 논문에서 이러한 문제점을 효율적으로 해결할 수 있는 한 접근 방법이 제시된다. 이 결과의 유용성은 3개의 회전축을 가진 매니퓰레이터를 사용하여 입증된다.

Abstract

Approximation of a Cartesian straight line motion with linear interpolation in the joint space has many desirable advantages and applications. But inappropriate determination of the corresponding subtravelling and transition times makes such joint-trajectories violate the input torque/force constraints. An approach that can overcome this difficult and yield the joint-trajectories utilizing the allowable maximum input trque/force is established in this paper. The effectiveness of these results is demonstrated by using a three-joint revolute manipulator.

1. Introduction

Industrial robot is a computer controlled mechanical manipulator. Correspondingly, there are many issues that must be investigated to improve its productivity. One important issue is the minimization of the task execution time. In this paper a Cartesian straight line motion, which has many applications and desirable advantages, is considered as the specified task.

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The application of the conventional optimal control theory (including the well-known minimum principle of Pontryagin to the problem of minimizing the task execution time seems difficult because of the complicated dynamic equations and strong kinematic and dynamic constraints. An alternative way to study this problem is to divide the problem into two parts: (1) off-line minimum time trajectory planning; and (2) on-line trajectory tracking. Since several on-line path tracking schemes have been recently developed the minimization efforts are concentrated in the off-line trajectory planning stage.

It is well known that achieving the Cartesian straight line motion of the end-effector by controlling the joints is a nontrivial task. A practical alternative is to approximate this motion by placing enough intermediate points along the path and connecting linearly the adjacent points in the joint space [8]. The positional and orientational deviation resulted from these linear interpolations shall remain below certain prespecified deviation tolerances. This approximation has distinct advantages, including the efficiency of implementation, as mentioned by Brady as well [2]. It is to be noted that the method how to select the intermediate points is not repeated in the paper. After obtaining the intermediate points [8], it is next step to determine every subtravelling and transition times. Here subtravelling time means the between two adjacent intermediate points time and transition time means the time between two transition points (cf., Fig.3). Usually this determination [3] is based on the assumption of constant constraints of maximum joint velocities and accelerations. However, this approach yields one significant drawback that the resulting trajectories cannot utilize the maximum capacity of the manipulator. This is due to the fact that such constraints must be derived at the worst case for realizing the corresponding trajectories for any circumstances. Futhermore, it is often difficult and tedious to determine such constraints at the worst case because these are usually obtained experimentally for a given manipulator configuration. To avoid these defficulties and utilize the maximum capacity of the manipulator, the dynamic characteristics of the manipulator must be directly incorporated into the trajectory planning. Thus an approach to overcome the above-mentioned difficulties and also minimize the subtravelling and transition times subject to input torque constraints is established in this paper.

The organization of this paper is as follows. An efficient method to determine the minimum subtravelling time subject to input torque constraints is presented in Section 2. In Section 3, joint-trajectories based on quadratic polynomials are constructed at each transition and a method to determine the corresponding transition times is introduced. Computer simulations of the results developed in this paper are described using a three-joint revolute manipulator in Section 4. Conclusions are defered to Section 5.

II. Minimization of Subtravelling Time

The nonlinear dynamic characteristics of the manipulator are highly coupled. Two major approaches, called Lagrangian and Newton Euler methods, to formulate these dynamic equations are usually used. Since Lagrangian method provides the closed-form equations, this method is used in the following analysis.

$$u_j = \sum_{k=1}^{n} D_{jk}(q)\ddot{q}_k + \sum_{k=1}^{n}\sum_{l=1}^{n} C_{jkl}(q)\dot{q}_k\dot{q}_l + g_j(q),$$

$$\text{for } j = 1, 2, \cdots, n. \qquad (1)$$

Here $q \in R^n$ is a generalized joint coordinate vector; $\dot{q}_k$ is the velocity of joint k; $\ddot{q}_k$ is the acceleration of joint k; u_j is the input torque applied at joint j; $D_{jk}(q)$ is the coupling inertial term between joints j and k (for k=j, effective inertial term at joint j); $C_{jkl}(q)$ is the Coriolis effect at joint j due to the velocities of joints k and ℓ (for ℓ=k, centripetal effect at joint j due to the velocity of joint k); and $g_j(q)$ is the gravity effect at joint j. It is to be noted that $D_{jk}(q)$, $C_{jk\ell}(q)$ and $g_j(q)$ are only position dependent terms. To obtain the minimum subtravelling time subject to input torque constraints, it is necessary to incorporate the joint positions, velocities and accelerations of the trajectories into the above dynamic equations.

Two adjacent points q^i and q^{i+1} are linearly interpolated in the joint space (cf., Fig. 1). The joint position $q(\gamma) \in R^n$ between q^i and q^{i+1} is described as follows.

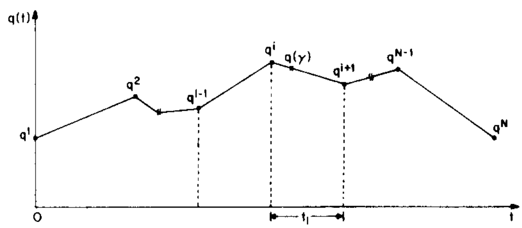

Fig. 1. Linear Joint Motion.

$$q(\gamma) = q^i + \gamma(q^{i+1} - q^i), \tag{2a}$$

$$\triangleq q^i + \gamma \triangle q^i. \tag{2b}$$

Here $\gamma \in [0,1]$ is a normalized time defined as $\gamma = \{t - \sum_{l=1}^{i} t_l\}/t_i$ with t_i as the subtravelling time between q^i and q^{i+1} and $\triangle q^i$ denotes the angular displacement between q^i and q^{i+1}. The corresponding joint velocities and accelerations are derived as follows.

$$\dot{q}(\gamma) \triangleq \frac{dq(\gamma)}{dt} = \frac{dq(\gamma)}{d\gamma}\frac{d\gamma}{dt} \triangleq \frac{\triangle q^i}{t_i}, \tag{3}$$

$$\ddot{q}(\gamma) \triangleq \frac{d^2q(\gamma)}{dt^2} = \frac{d^2q(\gamma)}{d\gamma^2}\left(\frac{d\gamma}{dt}\right)^2 + \frac{dq(\gamma)}{d\gamma}\frac{d^2\gamma}{dt^2} = 0. \tag{4}$$

Substituting (2), (3) and (4) into (1) yields

$$u_j = \frac{\sum_{k=1}^{n}\sum_{l=1}^{n} C_{jkl}(q)\triangle q_k^i \triangle q_l^i}{t_i^2} + g_j(q), \tag{5a}$$

$$\triangleq \frac{h_j(q)}{t_i^2} + g_j(q), \quad \text{for } j = 1, 2, \cdots, n. \tag{5b}$$

Here $\triangle q_k^i$ is the angular displacement between q_k^i and q_k^{i+1}. These equations are used to compute the torques that are required for executing the linear joint motions between two adjacent points. It can be seen from (5) that only the Coriolis and centripetal torques as well as gravity torques are remained effective in the dynamic equation (1).

By the statement of the torque constraints, it is assumed that

$$u_j^{min} \leq u_j \leq u_j^{max} (= -u_j^{min}), \quad \text{for } j = 1, 2, \cdots, n. \tag{6}$$

Using this assumption in (5), we have

$$\{u_j^{min} - g_j(q)\} \leq \frac{h_j(q)}{t_i^2} \leq \{u_j^{max} - g_j(q)\}, \quad \text{for } j = 1, 2, \cdots, n. \tag{7}$$

It is always assumed that all the actuators can supply enough torques to counteract the corresponding gravity effects, ie., $u_j^{max} \geq |g_i(g)|$, for $0 < \gamma < 1$ and j=1,2, ..., n. The inequalities (7), on using (6), can be simplified as follows.

$$t_i \geq \left\{\frac{|h_j(q)|}{u_j^{max} - \text{sgn}(h_j(q))\, g_j(q)}\right\}^{1/2} \triangleq t_{ij}. \tag{8}$$

Here sgn (x)= { 1 for x > o and -1 for x < o, respectivey }. Dividing both sides of (8) by $|\triangle q_j^i|$ and rearranging that result in

$$\frac{|\triangle q_j^i|}{t_i} \leq \frac{|\triangle q_j^i|}{t_{ij}} \tag{9}$$

From (9) the absolute value of allowable maximum velocity $\dot{q}_j^i(\gamma)$ of each joint j at some $\gamma \in [0,1]$ is determined as follows.

$$|\dot{q}_j^i(\gamma)| = |\triangle q_j^i| \left\{\frac{u_j^{max} - \text{sgn}(h_j(q(\gamma)))\, g_j(q(\gamma))}{|h_j(q(\gamma))|}\right\}^{1/2}, \tag{10a}$$

$$\triangleq \frac{|\triangle q_j^i|}{t_{ij}(\gamma)}. \tag{10b}$$

Also the absolute value of allowable maximum velocity $\dot{q}_j^i$ of each joint j between q^i and q^{i+1} is obtained as follows.

$$|\dot{q}_j^i| = \min_{\gamma} |\dot{q}_j^i(\gamma)|, \tag{11a}$$

$$= \frac{|\triangle q_j^i|}{\max_{\gamma} t_{ij}(\gamma)}. \tag{11b}$$

Thus the optimal subtravelling time t_{ij}^* of joint j between q^i and q^{i+1} is

$$t_{ij}^* = \frac{|\triangle q_j^i|}{|\dot{q}_j^i|},$$

$$= \max_{\gamma} t_{ij}(\gamma). \tag{12}$$

Because all n joints must reach the destination point q^{i+1} simultaneously, the desired optimal subtravelling time t_i^* between any q^i and q^{i+1} is determined as follows.

$$t_i^* = \max_j t_{ij}^*,$$

$$= \max_j \{\max_{\gamma} t_{ij}(\gamma)\}, \tag{13a}$$

$$= \max_{\gamma} \{\max_j t_{ij}(\gamma)\}. \tag{13b}$$

Thus t_i^* determined by (13) is the optimal subtravelling time during which all n joints can travel with maximum velocities between q^i and q^{i+1} without violating the input torque constraints. A simple and efficient algorithm to find this t_i^* using (13b) is summarized in Algorithm 1.

Algorithm 1 :

Step 1. Choose $d\gamma (\ll 1)$ and set $\gamma=0$ and $\bar{t}=0$.

Step 2. If $\gamma>1$, $t_i^*=\bar{t}$ and Stop. Otherwise compute $q(\gamma)$ by (2).

Step 3. Compute $t_{ij}(\gamma)$, for $j=1,2,\cdots,n$, by using (8) and find the maximum value t_{max}. If $t_{max}>\bar{t}$, set $\bar{t}=t_{max}$ and go to Step 4. Otherwise continue.

Step 4. Set $\gamma=\gamma+d\gamma$ and go to Step 2.

III. Determination of Minimum Transition Time

Linear interpolations of intermediate points in the joint space usually yield discontinuity in the joint velocity of adjacent segments (cf., Fig. 2), i.e., the vector of joint velocity of segment i-1 is not the same as that of segment i. To satisfy the continuity of the vectors of joint position and velocity at transition points, it is necessary to construct new joint trajectories during transition from segment i-1 to segment i. Paul [6] and Taylor [8] proposed a symmetric transition, where transition starts at $t(q^i)-\tau_i$ and ends at $t(q^i)+\tau_i$. The transition time $2\tau_i$ is usually determined as the time required to accelerate from maximum negative velocities to maximum positive velocities. With this determination of τ_i, the transition points q_α^i and q_β^i are specified as follows.

$$q_\alpha^i=q^i-\frac{\tau_i}{t_{i-1}}(q^i-q^{i-1}), \tag{14}$$

$$q_\beta^i=q^i+\frac{\tau_i}{t_i}(q^{i+1}-q^i). \tag{15}$$

Then quadratic polynomials are used to satisfy the boundary conditions (joint positions and velocities) at q_α^i and q_β^i. In this approach, as Brady [2] has pointed out, there is a possibility that τ_i becomes larger than t_{i-1} or t_i. This yields significant deviations from the desired linear joint motions. A possible way to overcome this difficulty and to determine the minimum transition time subject to input torque constraints is explained in the following.

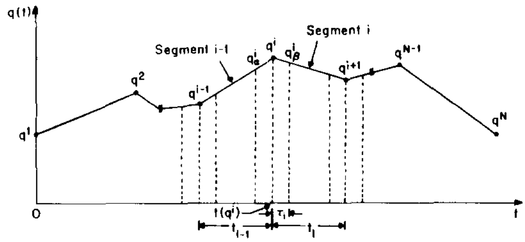

Fig.2. Symmetric Transition.

Referring to Fig. 3, the transition points q_α^i and q_β^i are prespecified as follows.

$$q_\alpha^i=q^i-k(q^i-q^{i-1}), \tag{16a}$$

$$q_\beta^i=q^i+k(q^{i+1}-q^i), \tag{16b}$$

where the scalar k ($\ll 1$) is selected to yield a small transition interval. It is to be noted that only q_β^1 and q_α^N are prespecified at the first and the last transitions, respectively. It is assumed that t_i^* (i = 1,2, ..., N-1) is already determined from Algorithm 1 by replacing q^i and q^{i+1} with q_β^i and q_α^{i+1} respectively. For convenience the superscript * of t* will be omitted in the following.

The velocity boundary conditions to be satisfied at the ith-transition are as follows.

$$\dot{q}_\alpha^i=\frac{(q_\alpha^i-q_\beta^{i-1})}{t_{i-1}}, \tag{17a}$$

$$\dot{q}_\beta^i=\frac{(q_\alpha^{i+1}-q_\beta^i)}{t_i}. \tag{17b}$$

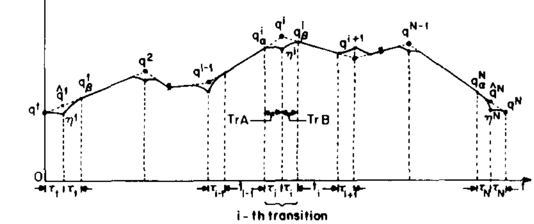

Fig. 3. New Determination of Transition. Here $\hat{q}^1=\frac{1}{2}(q^1+q_\beta^1)$ and $\hat{q}^N=\frac{1}{2}(q_\alpha^N+q^N)$.

Since this transition is not symmetric and there are four sets of boundary conditions (16a, 16b, 17a, and 17b) to be met at each transition, cubic polynomials are usually selected as the associated joint-trajectories. However, quadratic polynomials are chosen here in, because in general these polynomials result in much smaller transition time than the cubic polynomials. For this reason the ith-transition is decomposed into two different transitions of Tr A and Tr B, where two different constant accelerations are applied, respectively.

Referring to Fig. 3, the joint positions, velocities and accelerations at Tr A are established using the boundary conditions at q^i_α and the associated continuity conditions at point η^i. The η^i is a floating point which is on both Tr A and Tr B and obtained by substituting $\gamma_A = 1$ in (18) (Or $\gamma_B = o$ in (21). The resulting joint-trajectories for this portion of transition are as follows.

$$q(\gamma_A) = \tau_i\{\gamma_A \dot{q}^i_\alpha - \frac{\gamma_A^2}{4}(3\dot{q}^i_\alpha + \dot{q}^i_\beta)\} + \frac{\gamma_A^2}{2}(q^i_\beta - q^i_\alpha) + q^i_\alpha, \tag{18}$$

$$\dot{q}(\gamma_A) = \frac{\gamma_A}{\tau_i}(q^i_\beta - q^i_\alpha) - \frac{\gamma_A}{2}(3\dot{q}^i_\alpha + \dot{q}^i_\beta) + \dot{q}^i_\alpha, \tag{19}$$

$$\ddot{q}(\gamma_A) = \frac{1}{\tau_i^2}(q^i_\beta - q^i_\alpha) - \frac{1}{2\tau_i}(3\dot{q}^i_\alpha + \dot{q}^i_\beta). \tag{20}$$

Here $\gamma_A \in [0, 1]$ is defined as

$$\gamma_A = \{t - \sum_{l=1}^{i-1}(2\tau_l + t_l)\}/\tau_i.$$

Similarly the joint positions, velocities and accelerations at Tr B are obtained as follows.

$$q(\gamma_B) = \tau_i\left\{\frac{1}{4}(\dot{q}^i_\alpha - \dot{q}^i_\beta) - \frac{\gamma_B}{2}(\dot{q}^i_\alpha + \dot{q}^i_\beta) + \frac{\gamma_B^2}{4}(\dot{q}^i_\alpha + 3\dot{q}^i_\beta)\right\} + \gamma_B(q^i_\beta - q^i_\alpha) + \frac{\gamma_B^2}{2}(q^i_\alpha - q^i_\beta) + \frac{1}{2}(q^i_\alpha + q^i_\beta), \tag{21}$$

$$\dot{q}(\gamma_B) = \frac{1-\gamma_B}{\tau_i}(q^i_\beta - q^i_\alpha) + \frac{\gamma_B}{2}(\dot{q}^i_\alpha + 3\dot{q}^i_\beta) - \frac{1}{2}(\dot{q}^i_\alpha + \dot{q}^i_\beta), \tag{22}$$

$$\ddot{q}(\gamma_B) = \frac{1}{\tau_i^2}(q^i_\alpha - q^i_\beta) + \frac{1}{2\tau_i}(\dot{q}^i_\alpha + 3\dot{q}^i_\beta). \tag{23}$$

Here $\gamma_B \in [0, 1]$ is defined as

$$\gamma_B = \{t - \sum_{l=1}^{i-1}(2\tau_l + \tau_i + t_l)\}/\tau_i.$$

It can be seen from (18) and (21) that the joint positions are linearly proportional to τ_i at some fixed $\gamma_{A or B} \in [0,1]$. The corresponding joint positions, velocities and accelerations at the first and the last transitions are easily determined from these equations by setting $\dot{q}^1_\alpha$ and $\dot{q}^N_\beta$ to zero.

In order to prevent excessive deviation of η^i from q^i, it is necessary to find $\bar{\tau}_i$ which minimizes the following error function.

$$e^i \triangleq \| q^i - \eta^i \|, \tag{24a}$$

$$= \| q^i - \frac{1}{2}(q^i_\alpha + q^i_\beta) - \frac{\tau_i}{4}(\dot{q}^i_\alpha - \dot{q}^i_\beta) \|, \tag{24b}$$

where $\|\cdot\|$ is the Euclidien norm of $(\cdot)$. It is to be noted that q^1 and q^N in (24b) are replaced with $\hat{q}^1 = (q^1 + q^1_\beta)$ and $\hat{q}^N = (q^N_\alpha + q^N)$, respectively. Clearly $e^i = o$ implies η^i is perfectly matched to q^i.

Introducing the following short-hand notation,

$$\delta q^i \triangleq q^i - \frac{1}{2}(q^i_\alpha + q^i_\beta), \tag{25a}$$

$$\delta \dot{q}^i \triangleq \frac{1}{4}(\dot{q}^i_\alpha - \dot{q}^i_\beta), \tag{25b}$$

then (24b) can be rearranged as follows.

$$e^i = \{\sum_{j=1}^{n}(\delta q^i_j - \delta \dot{q}^i_j \tau_i)^2\}^{1/2}. \tag{26}$$

Here δq^i_j and $\delta \dot{q}^i_j$ are the j-th components of δq^i and $\delta \dot{q}^i$, respectively. To find $\bar{\tau}_i$ differentiating e^i with respect to $\bar{\tau}_i$ yields,

$$\frac{\partial e^i}{\partial \tau_i} = \frac{\sum_{j=1}^{n}(\delta \dot{q}^i_j)^2 \tau_i - \sum_{j=1}^{n}\delta q^i_j \delta \dot{q}^i_j}{\{\sum_{j=1}^{n}(\delta q^i_j - \delta \dot{q}^i_j \tau_i)^2\}^{1/2}}. \tag{27}$$

By setting $\partial e^i / \partial \tau_i = o$, we can find $\bar{\tau}_i$ as follows.

$$\bar{\tau}_i = \frac{\sum_{j=1}^{n}\delta q^i_j \delta \dot{q}^i_j}{\sum_{j=1}^{n}(\delta \dot{q}^i_j)^2}. \tag{28}$$

In order to avoid the excessive deviations of (18) and (21) from q^i, $\bar{\tau}_i$ can be selected as a transition time. But there is no guarantee that this $\bar{\tau}_i$ can satisfy the input torque constraints. In order words, when the manipulator makes transition at Tr A and Tr B for $\bar{\tau}_i$ the input torques for the trajectories can meet or violate the corresponding constraints. Even if $\bar{\tau}_i$ meets the above constraints, one can not ascertain that $\bar{\tau}_i$ is a desirable transition time from the point of view of minimum time, because there may exist smaller transition times than $\bar{\tau}_i$. Thus an approach to determine transition time τ_i^*, which is desirable from both the trajectory deviation and the minimum time aspects, is developed subsequently.

It can be seen from (18), (19) and (20) (or (21), (22) and (23)) that q at some $\gamma_{A\,or\,B} \in [0,1]$ is linearly proportional to τ_i, and $\dot{q}$ and $\ddot{q}$ are proportional to τ_i^{-1} and τ_i^{-2}, respectively. If the transition time and the transition interval are sufficiently small, the variations of q with respect to τ_i may be ignored with compared to those of $\dot{q}$ and $\ddot{q}$. Thus the variation of input torques with respect to τ_i will be dominated by those of the $\dot{q}$ and $\ddot{q}$. Consequently, it may be assumed that the input torques at the small transition interval are proportional to $O(\tau_i^{-1})$. With this assumption, a criterion to determine τ_i^* is established as follows.

Criterion:

(a) If $\bar{\tau}_i$ meets input torque constraints at Tr A and Tr B, a minimum transition time $\hat{\tau}_i (\leq \bar{\tau}_i)$ subject to the input torque constraints is to be found and τ_i^* is determined as $\tau_i^* = \omega\hat{\tau}_i + (1-\omega)\bar{\tau}_i$. Here $\omega \in [0, 1]$ is a weighting factor to represent relative importance between the trajectory deviation and the minimum time aspects.

(b) If $\bar{\tau}_i$ violates input torque constraints at Tr A or Tr B, a minimum transition time $\hat{\tau}_i (> \bar{\tau}_i)$ is to be determined and $\hat{\tau}_i$ becomes τ_i^*.

For this determination, a method to find $\hat{\tau}_i$ with $\bar{\tau}_i$ as an initial value is developed. The initial step of this method is to determine whether the input torques at Tr A and Tr B for the transition time $\bar{\tau}_i$ violate the corresponding constraints or not. If the constraints are met at Tr A and Tr B, an allowable minimum transition time $\tau_\eta (< \bar{\tau}_i)$ at the initial point ($\gamma_A = 0$ in (18)) of Tr A is determined by backward search method with $\bar{\tau}_i$ as an initial value. Then this τ_η is examind from $\gamma_{A\,or\,B} = 0$ to $\gamma_{A\,or\,B} = 1$ at Tr A and Tr B. Whenever the constraints are violated, τ_η will be increased to meet the constraints by forward search method. These two methods will be detailed later. Finally τ_η at end point ($\gamma_B = 1$ in (21)) of Tr B will be minimum time. Meanwhile, if the input torques for $\bar{\tau}_\eta$ violate the corresponding constraints at some point of Tr A or Tr B, the allowable minimum transiton time $\tau_\eta (> \bar{\tau}_i)$ at this point is determined by forward search method with $\bar{\tau}_i$ as an initial value. Then τ_η is examined from the point (where the constraints are violated) to the end point ($\gamma_B = 1$ in (21)) of Tr B. Whenever the constraints are violated, τ_η will be increased to meet the constraints by forward search method. τ_η at the end point of Tr B will be $\hat{\tau}_i$. The forward and the backward search methods, which are based on the well-known binary search methods [9], are detailed in the following.

FORWARD SEARCH METHOD :

Step 1. Choose ε (<<1) and m (>>1). Determine an initial value τ_α and compute $d\tau$ as $d\tau = \frac{\tau_\alpha}{m}$. Comment : $u_j(\tau)$ denotes the input torque of joint j at some fixed $\gamma_{AorB} \in [0, 1]$ of Tr A or Tr B, which is computed by (1) and (18) through (20) at Tr A and by (1) and (21) through (23) at Tr B with a transition time τ.

Step 2. Compute $u_j(\tau_\alpha)$, $u_j(\tau_\alpha + d\tau)$ and $\delta(j) = |u_j(\tau_\alpha) - u_j(\tau_\alpha + d\tau)|$. Repeat this step for $j = 1, 2, \cdots, n$.

Step 3. If $|u_j(\tau_\alpha)| > u_j^{max}$, compute $\Delta u(j) = |u_j(\tau_\alpha)| - u_j^{max}$ and $\psi(j) = \frac{\Delta u(j)}{\delta(j)}$. Otherwise $\psi(j) = 0$. Repeat this step for $j = 1, 2, \cdots, n$.

Step 4. Find $\psi_{max} = \max_j \psi(j)$.

Step 5. Set $\tau_\beta = \tau_\alpha + \psi_{max}\, d\tau$ and compute $u_j(\tau_\beta)$ for $j = 1, 2, \cdots, n$. If $|u_j(\tau_\beta)| \leq u_j^{max}$ for all j, go to Step 6. Otherwise repeat this step with $\tau_\alpha = \tau_\beta$.

Step6. Compute $\tau_w = \frac{(\tau_\alpha + \tau_\beta)}{2}$ and $u_j(\tau_w)$ for $j=1, 2, \cdots, n$. If $|u_j(\tau_w)| \le u_j^{max}$ for all j, go to Step7. Otherwise repeat this step with $\tau_\alpha = \tau_w$.

Step7. If $(u_j^{max} - |u_j(\tau_w)|) \le \varepsilon$ for some j, set $\tau_\eta = \tau_w$ and stop. Otherwise go to Step 6 with $\tau_\beta = \tau_w$.

BACKWARD SEARCH METHOD :

Step1. Choose ε (<<1) and m (>>1). Determine an inital value τ_α an compute $d\tau$ as $d\tau = \frac{\tau_\alpha}{m}$.

Step2. Compute $u_j(\tau_\alpha)$, $u_j(\tau_\alpha - d\tau)$ and $\delta(j) = |u_j(\tau_\alpha - d\tau) - u_j(\tau_\alpha)|$. Repeat this step for $j=1, 2, \cdots, n$.

Step3. Compute $\Delta u(j) = u_j^{max} - |u_j(\tau_\alpha)|$ and $\psi(j) = \frac{\Delta u(j)}{\delta(j)}$. Repeat this step for $j=1, 2, \cdots, n$.

Step4. Find $\psi_{min} = \min_j \psi(j)$.

Step5. Set $\tau_\beta = \tau_\alpha - \psi_{min}\, d\tau$ and compute $u_j(\tau_\beta)$ for $j=1, 2, \cdots, n$. If $|u_j(\tau_\beta)| \ge u_j^{max}$ for some j, go to Step 6. Otherwise repeat this step with $\tau_\alpha = \tau_\beta$.

Step6. Compute $\tau_w = \frac{(\tau_\alpha + \tau_\beta)}{2}$ and $u_j(\tau_w)$ for $j=1, 2, \cdots, n$. If $|u_j(\tau_w)| \le u_j^{max}$ for all j, go to Step 7. Otherwise repeat this step with $\tau_\beta = \tau_w$.

Step7. If $(u_j^{max} - |u_j(\tau_w)|) \le \varepsilon$ for some j, set $\tau_\eta = \tau_w$ and stop. Otherwise go to Step 6 with $\tau_\alpha = \tau_w$.

Using these forward and backward search methods and $\bar{\tau}_i$ as an initial value, the desired transition time τ_i^* is obtained as follows.

Algorithm 2 :

Step1. Choose $d\gamma$ and ω. Set $\gamma_1 = 0$, $\gamma_2 = 0$, $k=1$ and $Z=1$. Compute $\bar{\tau}_i$ by (28). Comments : γ_1 and γ_2 denote γ_A and γ_B, respectively; and $Z=0$ and $Z=1$ denote Criterion (a) and (b), respectively.

Step2. If $\gamma_k \le 1$, compute $u_j(\bar{\tau}_i)$ for $j=1, 2, \cdots, n$ and go to Step4. Otherwise continue.

Step3. If $k=1$, set $k=k+1$ and go to Step 2. Otherwise set $Z=0$ and $k=1$ and find τ_η by backward search method with $\tau_\alpha = \bar{\tau}_i$ at $\gamma_k = 0$ and go to Step5.

Step4. If $|u_j(\bar{\tau}_i)| \le u_j^{max}$ for all j, set $\gamma_k = \gamma_k + d\gamma$ and go to Step 2. Otherwise find τ_η by forward search method with $\tau_\alpha = \bar{\tau}_i$ and continue.

Step5. Set $\gamma_k = \gamma_k + d\gamma$. If $\gamma_k \le 1$, go to Step6. Otherwise go to Step7.

Step6. Compute $u_j(\tau_\eta)$ for $j=1, 2, \cdots, n$. If $|u_j(\tau_\eta)| \le u_j^{max}$ for all j, go to Step 5. Otherwise increase τ_η by forward search method with $\tau_\alpha = \tau_\eta$ and go to Step5.

Step7. If $k=1$, set $k=k+1$ and $\gamma_k = 0$ and go to Step 6. Otherwise set $\hat{\tau}_i = \tau_\eta$ and continue.

Step8. If $Z=1$, set $\tau_i^* = \hat{\tau}_i$ and stop. Otherwise $\tau_i^* = \omega\, \hat{\tau}_i + (1-\omega)\, \bar{\tau}_i$ and stop.

Remark 1 :

Summarizing all the results discussed in Sections 2 and 3, the procedure to determine the minimum subtravelling times (t_i^*, $i=1, 2, \cdots, N-1$) and the desired transition times (τ_i^*, $i=1, 2, \cdots, N$) is briefly described as follows, provided that all the intermediate points are already determined.

Step1. Choose ε, k, m, ω, $d\gamma$ and u_j^{max} ($j=1, 2, \cdots, n$).

Step2. Determine q_α^i and q_β^i ($i=1, 2, \cdots, n$) by (16a) and (16b).

Step3. Find t_i^* ($i=1, 2, \cdots, n-1$) between q_β^i and q_α^{i+1} by Algorithm 1.

Step4. Find τ_i^* ($i=1, 2, \cdots, n$) between q_α^i and q_β^i by Algorithm 2

IV. Computer Simulations

A Fortran program has been written on the Univac 1100 computer to simulate the proposed schemes for a PUMA *Unimation Inc., U.S.A) 500 series manipulator. For simplicity of the computation, only the first three joints of the manipulator are considered. Also seven intermediate points (two end points and five intermediate points) are selected, whose joint values are tabulated in Table 1. The input torque constraints u_j^{max} (j=1,2,3) and a scalar value k to specify the transitions points q^i and q^i are chosen as 100 Nm and 0.2, respectively. Also the weighting factor at each transition is set to one because we put our emphasis on the minimum time aspect.

Table 1. Joint Positions (rad) at Each Point.

	POINTS						
	q^1	q^2	q^3	q^4	q^5	q^6	q^7
Joint 1	0.4	0.8	1.2	1.6	2.0	2.4	2.8
Joint 2	0.5	0.83	1.16	1.5	1.8	2.17	2.5
Joint 3	0.2	0.57	0.93	1.3	1.6	2.03	2.4

Table 2. Minimum Subtravelling and Transition Times (sec).

	Transition time (τ_i)	Subtravelling time (ti)
i=1	0.0902	0.0707
2	0.0482	0.0706
3	0.0440	0.0626
4	0.0362	0.0483
5	0.0232	0.0344
6	0.0222	0.0302
7	0.0659	

Table 2. shows the minimum subtravelling and the transition times determined from our proposed schemes. It can be seen from Table 2 that the initial and the last transition times are greater than any other time. This result stems from the fact that the initial and the final velocities are zero and the velocity differences between the corresponding transition points are relatively large. Fig. 4. shows the joint positions consist of linear and quadratic polynomials.

It is to be noted that the deviations at each transition can be adjusted by selecting a proper weighting factor. Also, when the manipulator moves along the specified joint-trajectories with the determined subtravelling and transition times, the input torque applied at each joint is plotted in Fig. 5. From this figure it can be seen that at least one of three joints nearly utilizes its maximum torque during each movement.

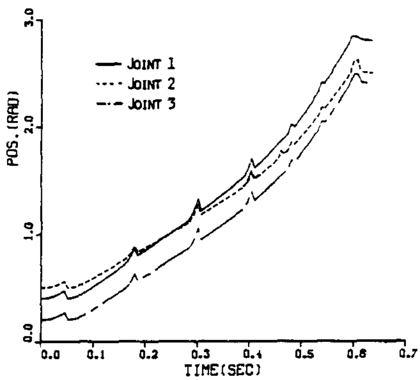

Fig. 4. Joint Positions.

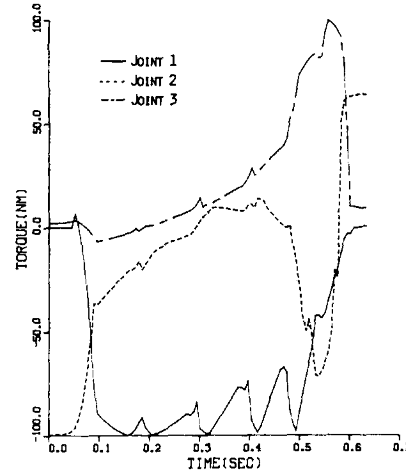

Fig. 5. The Input Torque Applied at Each Joint.

V. Conclusions

Recently many researches ([1], [4] , and [7] about the minimum time joint trajectory planning have been made. But most of the works put emphasis on satisfying only the time-optimality without considering several important criteria for the trajectory plannings (i.e., the efficiencies of computations, execution, visualization and smoothness). One way to meet these two important issues is to use the polynomials as the basis functions for the minimum time joint trajectory planning.

In this paper an approach to achieve a Cartesian straight line motion based on the linear approximations in the joint space is proposed. The proposed joint-trajectories consisting of linear and quadratic polynomials have the following desirable advantages: (1) these joint-trajectories are more effective from the minimum time aspect than any other joint-trajectories resulted from applications of the higher-order polynomials; (2) the linear polynomial portion of these joint-trajectories can greatly reduce the computational burden resulted from incorporating dynamic equations: (3) the quadratic polynomial portion (at each transition) of these joint-trajectories makes determination of each transition time independent of the corresponding transition interval and results in the joint-trajectoreis with the least deviations from the desired motion.

This paper also propses a method to determine the minimum subtravelling and transition times subject to input torque constraints. This method needs only one computation of dynamic equations at each point for determining the minimum subtravelling time. Also determination of transition time based on binary search method needs only few iterative computations of dynamic equations. Thus this proposed method can effectively determine the minimum subtravelling and transition times utilizing the allowable maximum input torques of the manipulator.

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