전단에 파괴되는 철근콘크리트 보의 해석적 연구

FRACTURE ANALYSIS OF REINFORCED CONCRETE BEAMS FAILING IN SHEAR

金 宇*
Kim, Woo

요 약

이 연구는 철근콘크리트보의 전단파괴 매카니즘에 대한 근본적인 성질을 밝히기 위해서 전단균열의 생성 및 진행과정을 해석적으로 연구하였다. 유한요소法에 파괴역학(fracture mechanics)을 결합시킨 program을 이용하여서, 철근 콘크리트 보에서 균열이 진행함에 따라 바뀌지는 內部應力상태와 균열정점에서의 stress intensity factors 등을 조사하여서, 전단균열의 생성 및 진행의 근본적인 이유를 밝히고자 하였다. 해석 결과로 밝혀진 사실들을 간단한 실험으로 비교 검증하였다.

Abstract

The behavior of shear crack is investigated analytically to get a better understanding of the fundamental nature of shear failure mechanism in reinforced concrete beams. Emphasis is placed on the exploration of the major cause of the initiation and the propagation of an inclined shear crack in reinforced concrete beams without web reinforcement. By utilizing a finite element method incorporated into a fracture mechanics, the quantitative response of reinforced concrete beams with varying amounts of cracking is examined. Progressions of the cracks are simulated. The analysis gives the information of the state of the stresses at various cracking stages. The results are compared with the experimental results.

INTRODUCTION

The importance of obtainining a knowledge of the fundamental nature of shear failure in reinforced concrete beams has brought forth a large number of laboratory investigations. However, the complexity of the problem is so great that as yet the bahavior of beams subjected to combined shear and moment has not been fully clarified. Since the basic variables governing the shear strength of reinforced concrete beams were appraised by Talbot in 1909, the general nature of the mechanism of shear failure in all its various aspects has emerged only recently. Although a tremendous number of papers have been published on the shear failure, it is not feasible to make a comprehensive literature review, dealing with all factors, into orderly body of knowledge because the individual contributions are resistant in many cases. Up to now, the best

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* 정회원, 전남대학교 토목공학과 조교수, 공학박사.

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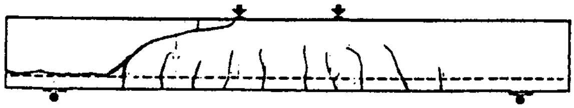
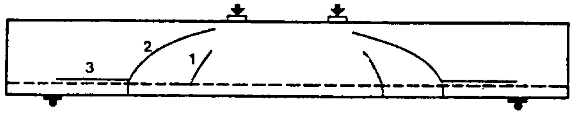
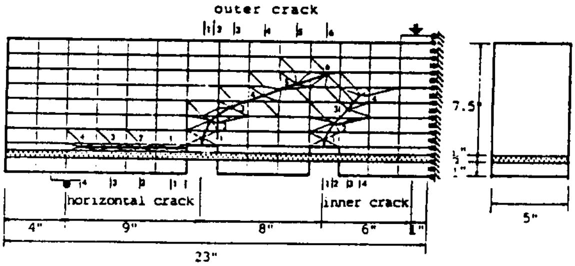
review on this topic may be the report of ASCE-ACI Joint Committee 426[1]. According to this report, the shear failure mechanism of reinforced concrete beams is charaterized by the occurrence of inclined shear cracks. The inclined shear crack in the web of a beam may develop either before or after a flexural crack occurs nearby. The first type of inclined shear crack is usually referred as an web-shear crack, which is well defined as a principal diagonal tension crack. The second type is generally identified as a flexural-shear crack. Although flexural-shear crack is the most common type of crack observed in reinforced concrete beams, the mechanism by which it forms is not entirely understood. Therefore, the main concern in this study is an inclined shear crack as a flexural-shear crack.



(a) Actual Crack Pattern of a Test Beam



(b) Symmetrical Idealization of Cracks



(c) Finite Element Mesh Layout

Fig. 1. Beam Model for Analysis

ANALYTICAL MODEL

The beam selected for the present analytical study is an actual test beam. The actual recorded crack pattern is shown in Fig.1a(Details are in Fig.6a). Since the actual recorded crack pattern is so complex, the analytical model cracks are idealized as shown in Fig.1b. Crack 1 represents one of flexural cracks in the shear span(termed “inner crack” in this study). Crack 2 is intended to represent either an inclined shear crack or the outermost flexural crack(termed “outer crack”). Horizontal cracking is idealized by crack 3 along the reinforcement. The finite element modeling is shown in Fig.1c. One of the basic assumptions used here is that the behavior of a reinforced concrete beam can be adequately represented in terms of a two-dimensional plane stress state. The reinforcement is assumed to be uniformly distributed across the width of the cross section so that the steel is transformed into a layer having a width equal to that of the beam by modifying the modulus of elasticity of the steel.

A perfect bond is assumed, but there is a finite length of steel with zero bond at each crack mouth. While this procedure may be criticized, it is felt that the essence of the problem has not been lost by such simple modeling.

The only nonlinear phenomenon taken into account in this study is the cracking of the concrete. In general, significant tractions exist between the two sides of a crack for some distance behind crack tip. These tractions are caused by aggregate interlocking and bridging across the crack. To model these tractions, interface elements are inserted in the crack as it propagates. These interface elements have shear and normal stiffnesses which are dependent on the crack opening at the location of each element. The stiffnesses are gradually reduced as the two faces of the crack move away from each other. The shear stiffness K_v is varied according to the model of Fenwick and Paulay [3] as shown in Fig.2a. This is

$$K_v = \{(467/u) - 8410\}\ (0.0225 f_c - 0.409)$$

where u=crack opening displacement.

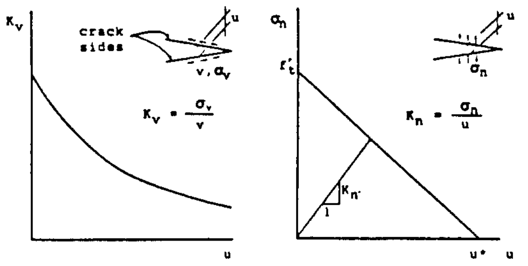
The normal stiffness K_n is varied according to the model of Catalano and Ingraffea [2] as shown in Fig.2b. This is

$$K_v = f'_t\{(1/u) - (1/u^*)\}$$

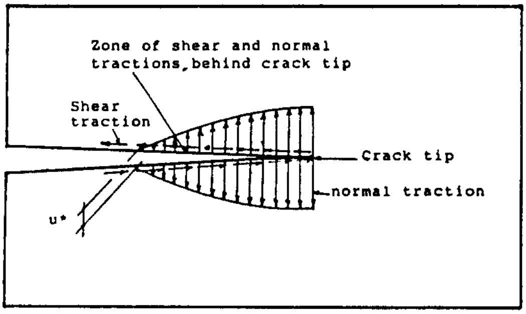
Where f'_t = ultimate tensile strength of the concrete used, here assumed $7\sqrt{f c}$

u^* = a limit value of u, and assumed to 0.004

An iterative scheme is employed in the program used, so that convergence of the results is achieved. In the way just described above, the tractions behind crack tip looks nonlinear as shown in Fig. 2c.



(a) Shear Stiffness Model (b) Normal Stiffness Model



(c) Nonlinear Distribution of Tractions Behind Crack Tip

Fig. 2. Nonlinear Interface Models Used for Concrete Cracking

Each crack propagates by a finite length (generally one element length) as marked by numbers in Fig.1c. For the sake of convenience, every crack stage is denoted by one letter and three digits, such as C134, C014, and C464. The first digit denotes the horizontal cracking stage(0 to 4 in Fig.1c), the second digit implies the outer cracking stage(0 to 6), and the third is the inner cracking stage(0 to 4). For example, C234 means the crack stage 234 at which 2 implies horizontal crack tip point, 3 means outer crack tip point, and 4 is inner crack tip point in Fig.1c.

PROGRAM USED

In this analytical work, a computer program named Finite Element Fracture Analysis(FEFA) is used. FEFA was developed at Cornell University by Saouma and Ingraffea [8]. The program is capable of automatically nucleating and propagating discrete cracks in a given finite element mesh. Triangular elements with self adjustment of nodes to the quarter point are always employed at the crack tips to ensure the condition of singularity. The stress intensity factors can be computed using the displacement correlation method [4], and the direction of crack propagation can be calculated from either $(\sigma\theta)_{max}$, $(S\theta)_{min}$, or $(G\theta)_{max}$ theories. The energy balanced approach is used to determine the stability of the crack. Furthermore, various types of interface element are included in the program for the study of the contact and crack problems.

INITIATION OF SHEAR CRACK

The mechanism of beam shear failure is conceived as a modification of the internal force system, accompanied by associated deformations consistant with altered geometry of the member after cracks form. In view of the above, the general tendancy of the internal stress redistribution is examined as the cracks progress in order to explore the causes for the initiation of shear crack.

SHEAR STRESS LOCALIZATION

The shear stress distributions in concrete at some selected cracking stages are plotted in Fig. 3. After the formation of the cracks, the distribution pattern deviates form the parabolic shape. Inspection of Fig.3 provides some noteworthy information on the general trend of the shear stress redistribution as flexural cracks progress:

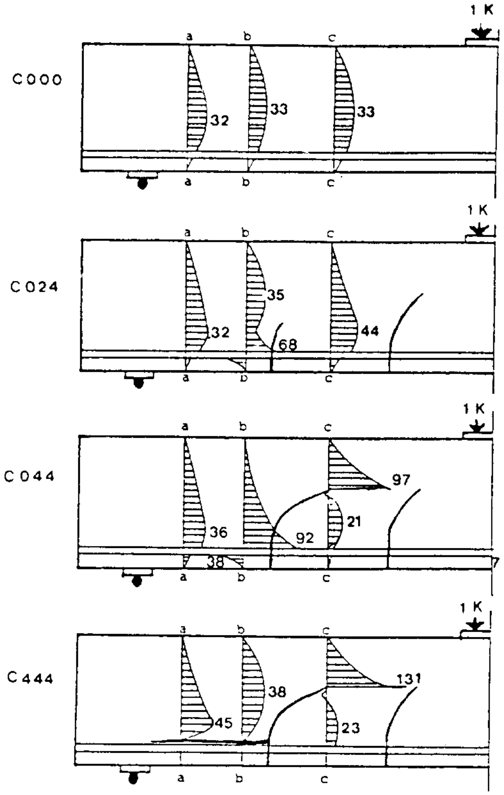


Fig. 3. Variation of Shear Stresses Distribution

(1) The location of maximum shear stress shiftes from the center of the section to the bottom side.

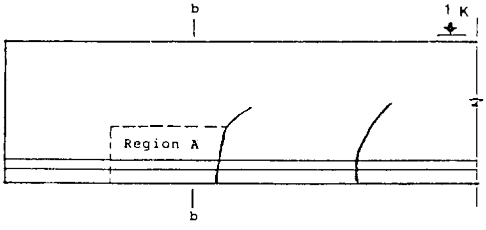
(2) Considerable shear stress localization takes place at the region next to the crack and above the steel. This shear stress localization becomes more pronounced as the cracks progress, as exemplified by the shear stress distribution at section b-b in Fig.3.

(3) Reverse directional shear stresses take place at the region below the steel and next to the crack.

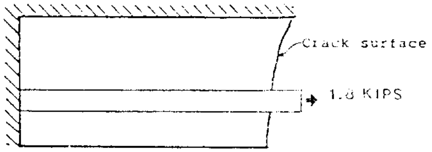
The important thing from the above is that the zone of the shear stress localization, where is slightly above the steel and near the previously developed flexural crack, is well coincide with the experimentally observed zone of shear crack initiation by former investigators [5, 6].

BOND-INDUCED SHEAR STRESS

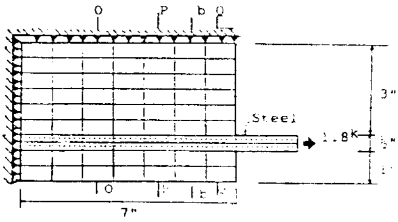
Since it is felt that the shear stress localization seems to have the responsibility for the initiation of an inclined shear crack, a simple analysis is performed to explore the fundamental reason for this stress localization. In order to explain conceptually, some portion of the beam denoted by A in Fig. 4a has been taken out to be a free body, and the two edges are approximately assumed to be fixed as shown in Fig.4b. Then, the portion A looks like an unsymmetrical fullout specimen having the special boundary condition. The finite element



(a) Overall Beam at Crack Stage 044



(b) Idealization of Region A



(c) Finite Element Model for Region A

Fig. 4. Analytical Model to Study on Mechanism of Bond-induced Shear Stress

modeling for the portion A is shown in Fig.4c. All properties are the same as those of the beam model in Fig.1c, but finer meshes are used. Only force applied is the steel tension force at the outer crack mouth which is obtained from the overall beam analysis of C044. This simple analysis intends to explore conceptually the effect of the steel tension force on shear distribution when the external shear force do not exist.

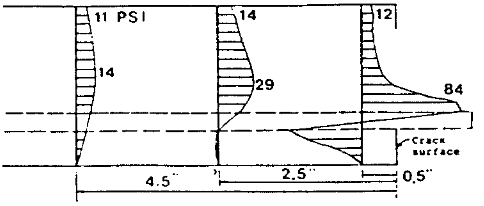
As a result of the steel tension force only, significant stress is induced in concrete as plotted in Fig. 5a. This shear stress is termed bond-induced shear stress because this is induced by the horizontal shearing action of the T-C force couple without any external shear force, and the force T is transferred through bond. The magnitude and distribution of this shear stress may be highly complex, and depends on various parameters, such as geometry and the interaction between steel and concrete. Within the results of this simple analysis, however, the followings could be said;

(1) As shown in Fig.5a, extremely high shear stresses are induced at the region behind the crack by the horizontal shearing action of T-C force couple after the formation of flexural crack.

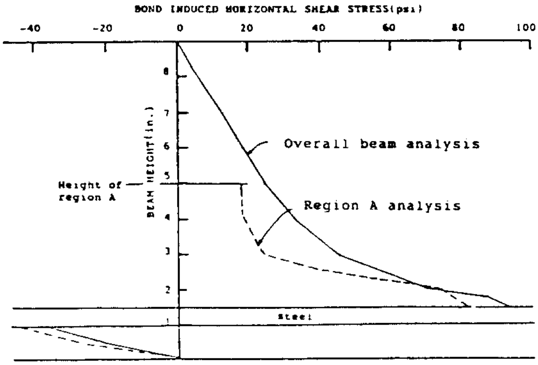
(2) At the sections close to the crack surface, the magnitude of the shear stress becomes higher and the distribution pattern becomes more localized.

(3) The bond-induced shear stress is major part in magnitude of the highly localized shear stress at section b-b.

This quantitative comparison clearly shows that the magnitute and the distribution shape of the shear stress are mainly governed by the bond-induced shear stress. For this reason, the shear stress distribution pattern tends to be distorted, and to has high value near the steel level as flexural crack extends.



(a) Shear Stress Distribution From Region A analysis



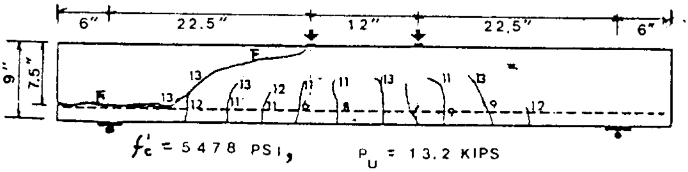
(b) Comparison of Shear Stresses at Section b-b

Fig. 5. Bond-induced Shear Stresses

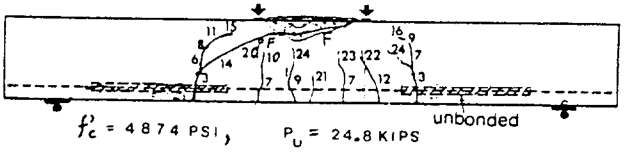
EXPERIMENTAL VERIFICATION

Since the analytical analysis indicates that the bond-induced shear stress, which is originated from the transferring the steel force to the concrete through bond due to the horizontal shearing action of T-C force couple, may have a major responsibility for the initiation of shear crack, a simple experiment is performed to verify it.

Two simple reinforced concrete beams with two-point loads are tested.One is an ordinary normal beam(named control normal beam) for providing a basis of comparison for the subsequant test (see Fig.6a). The other one is a specially designed beam(named beam with unbonded bars), in which the bond between the steel and the concrete is eliminated along some portion of the shear span as shown in Fig.6b. The two beams have the same cross section with 5 inch width and 9 inch height, and the same steel ratio of 0.0165(ASTM Grade 60 2-No.5 bars used). No stirrup is provided. The concrete compressive strength f'_c of the control normal beam is 5470 psi, while f'_c of the beam with unbonded bars is 4874 psi. Both the beams have the same span length(a / d of 3).



(a) Control Normal Beam



(b) Beam with Unbonded Bars

Fig. 6. Comparison of Beam with Unbonded Bars with Control Normal Beam

Comparing the experimental results of the beam with unbonded bars with those of the control normal beam, two distinct differences are observed as shown in Fig.6:

(1)In the beam with unbonded bars, the inclined shear crack does not appear.

(2)The beam with unbonded bars produces a considerably higher load-carrying capacity Pu of 24.8 kips which is nearly two times higher than the Pu of 13.2 kips in the control normal beam.

Surprisingly, the elimination of the bond results in incredible increase of failure load as well as change of the failure mechanism. It is apparent from the results that when the bond is eliminated, the bond-induced shear stresses are not induced, thereby inclined shear crack can not be initiated. The important thing that should be noted is that the P_u of 24.8 kips is the full flexural load-carrying capacity of the beam. This means that the beam with unbonded bars fails not by shear but by flexure.

In view of the above, it would be concluded that the bond-induced shear stress is the major cause for shear failure in reinforced concrete beams.

PROPAGATION OF SHEAR CRACK

The general tendency of the internal stress redistribution pattern as flexural cracks progress has been studied to explore the mechanism of an inclined shear crack initiation. In this section, the effect of horizontal cracking on the inclined shear crack propagation into the shear-compression zone is examined from a fracture mechanics point of view. Emphasis is placed on the variation of the stress field at the head of an inclined shear crack as a horizontal crack progresses toward the support point.

CRACK TIP STRESS DISTRIBUTION

Since it is generally accepted that the trajectory of the crack follows the trajectory of the principal compression stress, the distribution pattern of the stress normal to the inclined line ahead of the outer crack tip is examined. The inclined line meams the straight line drawn from the outer crack tip to the load point as shown in Fig.7, which may be considered to be trajectory of the inclined shear crack in the shear compression zone.

Fig.7 shows the distributions of the stresses normal to the inclined line when both flexural cracks reach slightly above the mid depth of

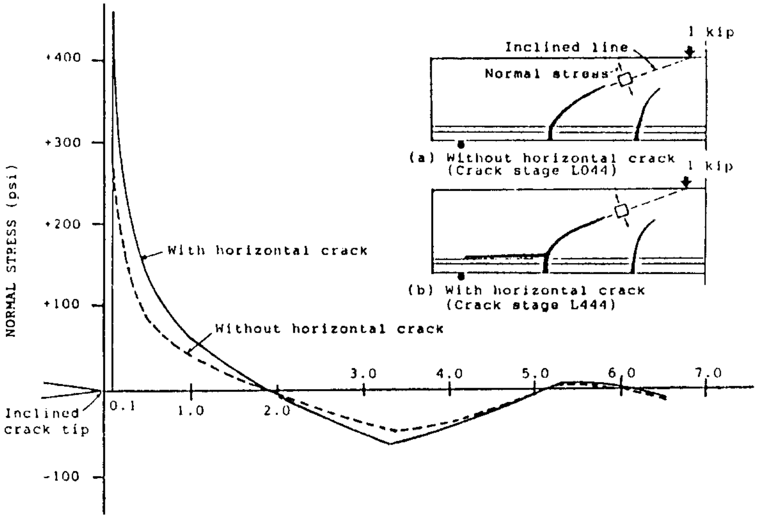


Fig. 7. Distribution of the Stresses Normal to the Inclined Line

the beam(C044 and C444). The normal stress distribution along this inclined line at C044(without horizontal crack shown in Fig.7a) is plotted as a dotted line, and that of at C444(with horizontal crack shown in Fig.7b) is plotted as a solid line in Fig.7. In both crack stages, the applied load is 1 kip. Inspection of these two curves provides three important points:

(1)Highly concentrated normal tension stresses exist ahead the crack tip. It may be noted that in actual concrete the high stress at the zone ahead of the crack are partly released by plastic deformation(the zone is conventionally defined as the process zone).

(2)Up to about 2 inches ahead the crack tip, normal tension stresses exist althouth the zone is in compression from overall bending action.

(3)The horizontal cracking amplifies considerably the magnitude of the stress. Comparison between the two curves in Fig.7 shows that the distribution patterns are basically identical, but the value of the stresses is amplified on the order of 1.74. The value of 1.74 results purely from an 8 inch (equivalent to 1.03 where d is the effective depth) penetration of the horizontal crack.

The amplification factor of the stress field ahead the crack tip is fundamentally nothing more than the well-known stress intensity factors K_I and K_{II}. Therefore, it is felt that a study on the variation of the stress intensity factor at various cracking stages is a more convenient way to examine the effect of horizintal cracking on the inclined shear crack propagation. Thus, the following part treats this topic.

EFFECT OF HORIZONTAL CRACKING ON SHEAR CRACK EXTENSION

In order to study the effect of horizontal cracking on the inclined shear crack tip stress field, the variation of the stress intensity factors K_I and K_{II} are examined as the length of horizontal crack varies when the two flexural cracks are fixed at certain crack stages. These K_I and K_{II} of each crack tip are used to calculate the crack extension starting load V_{start} of the corresponding crack by using $(\sigma_\theta)_{max}$ failure criteria. The$(\sigma_\theta)_{max}$ failure criteria states that the crack extension starts in the plane perpendicular to the direction of greatest tension when$(\sigma_\theta)_{max}$ reaches a critical constant:

$$\cos\frac{\theta}{2}\left\{\frac{K_I}{\sqrt{\pi}}\cos^2\frac{\theta}{2}-\frac{3K_{II}}{2\sqrt{\pi}}\sin\theta\right\}=K_{IC}$$

where K_I=stress intensity factor in opening,
K_{II}=stress intensity factor in shear,
K_{IC}=critical stress intensity factor (material toughness), here assumed to be 1250 psi $\sqrt{in}$. for concrete.

Then, the variation of V_{start} of the cracks are examined as the horizontal crack propagates.

CASE I

This study examines the variation of V_{start} when both the inner and the outer crack tips are located near the mid depth of the beam (crack stage X44 where X is varied from 0 to 4). The only variable to study is the length of the horizontal crack which progresses from the outer crack mouth to the support as in Fig.8. The horizontal crack extends by the length of 2 inches along the steel at each stage.

Fig.8 shows the variations of the V_{start} of inner and the outer crack. Comparison of V_{start} of the outer crack with V_{start} of the inner crack indicates a noteworthy fact: The outer crack extension starting load decreases abruptly as the horizontal crack reaches to near the support. In other words, the 8 inch extension of horizontal crack results in 43% reduction in V_{start} of the outer crack (here, the outer crack represents the inclined shear crack). This implies that the magnitude of the stress at head of the outer crack (shown in Fig.7) is amplified by the order of 1.74. While, the stress at the head of the inner crack is affected negligibly.

CASE II

This case study is planned to confirm the tendency obtained in Case I. Every condition is the same as in Case I except for the geometry of the outer

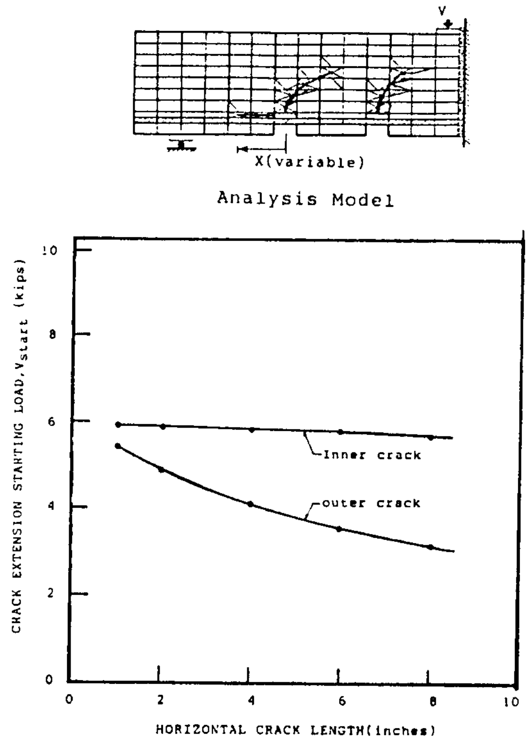


Fig. 8. Variation of V_{start} in CASE I

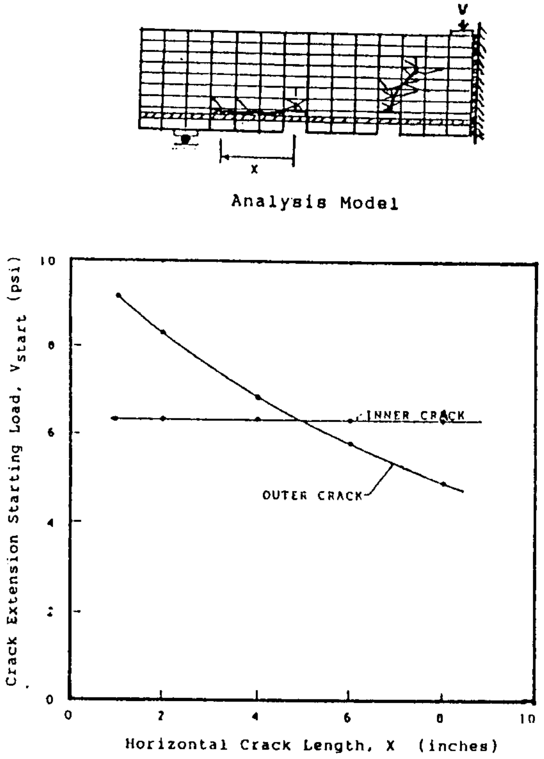


Fig. 9. Variation of V_{start} in CASE II

crack. In this case, the outer crack is shorter than that in Case Ⅰ, and crack stage is CX14 as shown in Fig.9. The horizontal crack is extended in the same manner as in Case Ⅰ, and V_{start} as both the outer crack tip and the inner crack tip is examined at every horizontal cracking stage. The variations of V_{start} of the two upward cracks are plotted against the length of horizontal crack.

The figure shows that V_{start} of the outer crack is decreased by 47% as the results of the 8 inch penetration of the horizontal crack along the steel. This means that the stresses at the head of the outer crack increase 1.87 times by the penetration of the horizontal crack. During the progression of the horizontal crack, the inner crack tip stress field is not affected. This tendency is the just same as the results of Case Ⅰ.

From the results of Case Ⅰ and Ⅱ, it would appear that the horizontal cracking results in considerable increase in the inclined shear crack tip stresses. Thus, the inclined shear crack becomes more and more critical as the horizontal crack progresses. From the above results, it can be said that the action triggering the propagation of the shear crack into the compression zone is the horizontal cracking along the bars.

EXPERIMENTAL VERIFICATION

With fracture mechanics theories and FEFA as tools, the effect of horizontal cracking on the shear failure mechanism has been briefly studied. Although the simple modeling of the beam and the applicability of fracture mechanics to concrete structures may be criticized, these analytical investigation gives new insights.

In order to check whether the findings above is correct or not, a reinforced concrete beam is tested. In the beam a prestresses external stirrup is provided at the outer third section of either span(see Fig. 10) to prevent horizontal cracking along the main reinforcement(This beam is named beam with no horizontal crack). The material properties and the geometry of the beam are the same as those of the control normal beam described before.

Comparing the results of this beam with those of the control normal beam, the manner by which the inclined shear crack initiates is the same, and the shear cracking loads are the same. The rates of the shear crack propagation into the compression zone after the inclined shear crack starts, however, are greatly different from each other as demonstrated the cracking load stages by numbers in Fig.10. In the control normal beam, the inclined shear crack penetrates whole section without any additional load, thus the inclined shear cracking load represents the failure load of 13.2 kips. However, in the beam with no horizontal crack, the inclined shear crack extends very slowly, and requires high additional

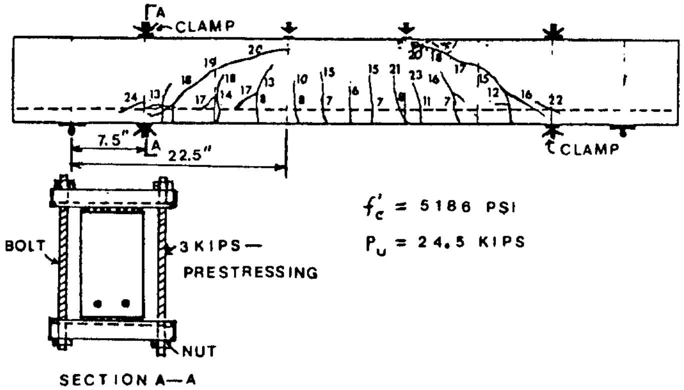


Fig. 10. Experimental Results of Beam with No Horizontal Cracking

load of 11.5 kips for full penetration of the section. Consequently, the failure load P_u is 24.5 kips which is almost two times larger than the P_u of 13.2 kips in the control normal beam. It may be noted that the P_u of the beam with no horizontal crack is the full flexural capacity of the section.

From the experimental results, it is obvious that the prevention of horizontal cracking results in small stress field at the head of the inclined shear crack, thereby, additional load is needed to extend.

CONCLUSIONS

The following conclusions on the shear failure mechanism are drawn from the results of the investigation and the discussion:

(1)Inclined shear crack is believed to be initiated mainly by bond-induced shear stress because it is highly magnified by the nature of bond after the formation of the outer flexural crack.

(2)The action triggering the propagation of the inclined shear crack into the compression zone is the horizontal cracking. It can be stated that the inclined shear cracking in the compression zone is a secondary phenomenon in shear failure mechanism of reinforced concrete beams.

(3)The conclusions presented above, based on the experimental and analytical investigations, consistantly indicate that external shear force on the shear span of a beam do almost nothing on beam failure. So-called shear failure of reinforced concrete beam is another type of flexural failure due to stress localization. Therefore, it may be noted that the conventional knowledge including code provisions on beam shear failure should need a serious reconsideration.

REFERENCE

1. ASCE-ACI Joint Committee 426, "The Shear Strength of Reinforced Concrete Members," Journal of the Strutural Division, ASCE, V. 99, NO. ST6, June 1973, pp. 1091-1188.
2. Catalano, D., and Ingraffea, A., "Concrete Fracture: A Linear Elastic Fracture Mechanics Approach," Report No. 82-1, Dept. of Structural Engineering, Cornell University, Ithaca, N.Y., 1982.
3. Fenwick, R.C., and Paulay, T., "Mechanisms of Shear Resistance of Concrete Beams," Journal of the Structural Division, ASCE, V. 94, No. ST10, Oct. 1968, pp. 2325-2350.
4. Ingraffea, A.R., "On Discrete Fracture Propagation in Rock Loaded in Compression," Proceedings, First International Conference on Numerical Methods in Fracture Mechanics, Swansea, 1978, pp.235-248.
5. Leonhardt, F., and Walther, R., "Contribution to the Treatment of Shear Problems in Reinforced Concre"("Beitrage zur Behandlung der Schubprobleme Stahlbetonbau"), Beton-und Stahlbetonbau (Berlin), V. 56, No.12, Dec. 1961.
6. Krefeld, W.J., and Thurston, C.W., "Studies of the Shear and Diagonal Tension Strength of Simply Supported Reinforced concrete Beams," ACI Journal Proceedings V.63, April 1966, pp.451-476.
7. Ngo, D., and Scordelis, A.C., "Finite Element Analysis of Reinforced Concrete Beams," ACI Journal, Proceedings V. 64, No.3, Mar. 1967, pp. 152-163.
8. Saouma, E.V., "Interactive Finite Element Analysis of Reinforced Concrete A Fracture Mechanics Approach," Ph.D. Thesis, Cornell University, Ithaca, N.Y., Jan. 1981.
9. Sih, G.C., and Ditommaso, A., "Fracture Mechanics of Concrete: Structural application and numerical calculation," Martinus Nijhoff Publishers, Dordrechet, The Netherlands, 1985, 276pp.
10. Walraven, J.C., "Mechanism of Shear Transfer in Cracks in concrete- A Survey of Literature," Report No. 5-78-12, Stevin Laboratory, University of Technology Delft, Holland, 1978.

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