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Some Sufficient Conditions for Output Regulation of Uncertain Nonlinear Systems

(불확실 비선형 시스템의 출력제어를 위한 충분조건)

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要 約

불확실 시스템의 출력제어를 위하여 불확실한 모델구조에 요구되는 충분조건들을 제시한다. 이들 조건들은 과거의 불확실 시스템의 상태변수 제어를 위한 매칭조건과 외부교란의 출력 불간섭제어를 위한 조건들을 특수 경우들로 포함한다. 출력제어의 경우 과거 상태변수제어를 위해 불확실 모델구조에 요구되던 제한들이 상당히 완화될 수 있음을 보인다. 한 간단한 예를 들어 이들 새로운 조건들의 의미를 조명한다.

Abstract

In this note, we present some sufficient conditions on the structure of modelling uncertainties for output regulation of uncertain systems. It turns out that these conditions include as special cases the ordinary matching conditions for state regulation of uncertain systems and the disturbance decoupling condition. The previous restrictions on the structure of modelling uncertainties can be considerably relaxed in the case of output regulation. The significance of our result is illuminated through a simple example.

I. Introduction

An approach to robust state regulation pioneered by Gutman and Leitmann [7] has been explored by many authors [1-6, 8-12, 14-17]. An assumption popularly made in the prior literature is the so called matching condition on the structure of modelling uncertainties. It is shown in [14] that in some sense, the matching condition may be not only sufficient but also necessary for robust state regulation. In [9], we suggested, through an example, that the ordinary matching condition may be considerably weakened in the case of robust output regulation. Here, the observation is more extensively studied. The class of uncertain systems considered here has the form:

$$\dot{x}(t) = Ax(t) + Bu(t) + [MV(x,t) + NW(x,t)\ u(t)],\ y(t) = Cx(t), \qquad (1.1)$$

where $x(t) \epsilon R^n$ is the system state; $u(t) \epsilon R^m$ is the control input : $y(t) \epsilon R^s$ is the system output ; the continuous functions, $V : R^n \times [0, \infty) \to R^q$ and $W : R^n \times [0, \infty) \to R^{r \times m}$ are unknown but are upper-bounded by known

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scalar functions of x. Roughly speaking, the problem we treat is as follows:

(P) Find conditions on constant matrices M,N which guarantee the existance of a continuous feedback controller such that the system (1.1) with the controller has acceptable output regulation performance in the presence of the modelling uncertainties V and W.

The above problem (p) is solvable provided that the constant matrices M,N satisfy the ordinary matching condition (specifically speaking, there exist constant matrices $\hat{M} \in R^{mxq}$, $\hat{N} \in R^{mxr}$ satisfying $M = B\hat{M}$ and $N = B\hat{N}$). In [15], a robust output tracking problem is considered but under the ordinary matching condition. Some of the prior works present weaker conditions than the oridnary matching condition [2, 4, 17] or other sufficient conditions [3, 13, 18] for robust state regulation. However, these results have their own limitations in order to be applied to our problem (P). For instance, V,W in (1.1) should have forms linear [2, 3, 17, 18] or conebounded [13] with respect to the state x. The conditions proposed in [3,4] are abstract and hence do not specify explicitly acceptable structure of modelling uncertainties. The conditions in [13, 18] are not necessarily more general than the ordinary matching condition. In this note, we show that weaker conditions on M,N than the ordinary matching condition still solve the problem (P). Distrurbance decoupling considered by Wonham and Morse [19] is also useful in solving the problem (P). However, our condition is more general than the disturbance decoupling condition. To further illuminate the significance of our work a simple example will be discussed later.

Some mathematical definitions are introduced, which are needed in the following sections. Let Ker C be the null space of C, that is, $\text{Ker } C \triangleq [\, x \in R^n : Cx = O \,]$. The it-h column vectors of B, M, N are denoted by B_i, M_i, N_i, respectively. Let B be the subspace spanned by B_i, i = 1, . . ., m. Let M be the subspace spanned by M_i, i = 1, . . ., q. Let N be the subspace spanned by N_i, i = 1, . . , r. Recall [19] that a subspace $\gamma \subset R^n$ is (A,B)- invariant if there exists $L \in R^{mxn}$ such that (A + BL) $\gamma \subset \gamma$ if and only if $A\,\gamma \subset \gamma + \beta$. Let γ^* be the supremal (A,B)-invariant subspace contained in Ker C. The maximum of the real parts of the eigenvalues of $A \in R^{\ell x \ell}$ is $\sigma_M(A)$. The transpose of $z \in R^{\ell}$ is z^T and its vector norm is $|z| \triangleq (z^T z)^{1/2}$ The matrix norm of $A \in R^{\ell x \ell}$ is $\| A \| \triangleq \max.\ (\,|Az| \,:\, z \in R^{\ell}, |z| = 1\,) = (\sigma_M(A^T A))^{1/2}$. The $(\ell x \ell)$ identity matrix is denoted by I_ℓ.

II. Main Result

First, we state some assumptions needed for the development in this section.

(C.1) There exist known scalar continuous functions $\rho_v : R^n \to [0, \infty)$ and $\rho_w : R^n \to [0, \infty)$ such that

$$|V(x, t)| \leqslant \rho_v(x), \ \ \|W(x,t)\| \leqslant \rho_w(x), \quad x \in R^n, t \in [0, \infty). \tag{2.1}$$

(C.2) There exists a positive integer $p \leqslant n$ and constant matrices $\hat{M} \in R^{mxq}$, $\hat{N} \in R^{mxr}$, $L \in R^{mxn}$, $T \in R^{pxn}$, $\bar{A} \in R^{pxp}$, $\bar{B} \in R^{pxm}$, and $\bar{C} \in R^{sxp}$ such that the linear system $(\bar{A}, \bar{B}, \bar{C})$:

$$\dot{\bar{x}} = \bar{A}\,\bar{x} + \bar{B}\,\bar{u}, \bar{y} = \bar{C}\,\bar{x} \tag{2.2}$$

is controllable and observable, and

$$\bar{A}T = T\,(A + BL),\ TB = \bar{B},\ \bar{C}T = C,\quad TM = \bar{B}\hat{M},\ TN = \bar{B}\hat{N}. \tag{2.3}$$

The meaning of the condition (C. 2) is as follows. Suppose that (C.2) is satisfied and consider the system (1.1) with the control law $u = Lx + \bar{u}$:

$$\dot{x} = (A + BL)x + B\bar{u} + [MV(x,t) + NW(x,t)(Lx + \bar{u})],\quad y = Cx. \tag{2.4}$$

Let $\bar{x} \triangleq Tx$. By (2.3), the input-output map of the system (2.4) is the same as that of the system:

$$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}\bar{u} + \bar{B}\,[\hat{M}V(x,t) + \hat{N}W(x,t)(Lx + \bar{u})],\ y = \bar{C}\bar{x}. \tag{2.5}$$

Note that in the system (2.5), the ordinary matching condition is satisfied.

Now, the rest of the assumptions needed in this section are introduced. Let $H \in R^{pxm}$ be the constant matrix such that

(C.3) $\bar{A}_H \triangleq \bar{A} + \bar{B}H$ is stable and has simple structure ($\bar{A}_H$ has p linearly indepedent eigenvectors).

Because $\bar{A}_H$ has a simple structure, it is possible to determine a nonsingular $P_H \in R^{pxp}$ so that

$$\tilde{A}_H \triangleq \bar{P}_H^{-1} \bar{A}_H \bar{P}_H = \text{diag}\, \Lambda_i, \quad (2.6)$$

were

$$\Lambda_i \triangleq \begin{cases} \begin{bmatrix} \sigma_i & -\omega_i \\ \omega_i & \sigma_i \end{bmatrix} \in R^{2x2},\ i = 1, \ldots, n_c, \\ [\lambda_i] \in R^{1x1},\ i = n_c + 1, \ldots, p\text{-}2n_c, \end{cases} \quad (2.7)$$

and $\sigma_i \pm j\omega_i$, $i = 1, \ldots, n_c$ and λ_i, $i = n_c + 1, \ldots, p\text{-}2n_c$ are respectively, the complex and real characteristic roots of $\bar{A}_H$. Let $\Pi_i \in R$, $i = 1, \ldots, p$ such that $\Pi_{2i-1} = \Pi_{2i}$, $i = 1, \ldots, n_c$. Let $\Pi \triangleq \text{diag}\, \Pi_i$. finally, we assume that

(C.4) There exists a known continuous scalar function $\gamma : R^n \to (0, \infty)$ such that

$$\| I_m + \hat{N}W(x,t) \| \geqslant \gamma(x),\ x \in R^n,\ t \in [0, \infty). \quad (2.8)$$

Using the notations in [9], $F(x, t) = Ax + MV(x, t)$, $G(x, t) = B + NW(x, t)$, $\hat{F}(x, t) = Ax$, $\hat{G}(x, t) = B$, $\triangle F(x,t) = MV(x,t)$, $\triangle G(x,t) = NW(x,t)$, and $\triangle E(x,t) = 0$. It can be easily verified that the above assumptions (C.1) - (C.4) imply the hypotheses of Theorem 2.1 in [8]. Specifically,

$$\alpha = -Lx,\ \beta = I_m,\ \triangle F^* = \hat{M}V,\ \triangle G^* = \hat{N}W,$$
$$S = I_m,\ \Phi = \hat{M}V + \hat{N}W(HT + L)x,$$
$$\phi = [\, \|\hat{M}\| \sigma_v + \|\hat{N}\| \sigma_w \,|(HT + L)\, x|\,] / \gamma. \quad (2.9)$$

Then, following the procedure indicated in [8], we can actually construct a robust controller:

$$u \triangleq K(x) \triangleq [L + \{H - \phi^2(x) \bar{B}^T (\bar{P}_H^{-1})^T \Pi \bar{P}_H^{-1} / \gamma(x)\} T]\, x \quad (2.10)$$

Consider the system (1.1) with the controller (2.10);

$$\dot{x} = Ax + MV(x, t) + [B + NW(x, t)]\, K(x),$$
$$y = Cx. \quad (2.11)$$

Let $x(0) \in R^n$. Assume (2.11) has a solution $x : [0, \infty) \to R^n$. Let

$$\mu \triangleq -\sigma_M(\tilde{A}_H),\ \sigma \triangleq \|\bar{C}\bar{P}_H \Pi^{-1/2}\| / 2\mu^{1/2}, \text{ and}$$
$$\sigma_0 \triangleq 2\mu^{1/2} |\Pi^{-1/2} \bar{P}_H^{-1} T x(0)|. \quad (2.12)$$

Then, it can be shown [8] that, for $t \in [0, \infty)$,

$$|(y(t)| \leqslant \begin{cases} \delta, & \delta_0 \leqslant 1, \\ \delta[1 + (\delta_0^2 - 1) e^{-2\mu t}]^{1/2}, & \delta_0 > 1 \end{cases} \quad (2.13)$$

Thus, $|y(t)| \to \delta$ as $t \to \infty$. Furthermore, δ can be made as small as is desired, by increasing the free parameters Π_i in (2.10).

We have shown that under the assumptions (C.1)–(C.4), the problem (p) is solvable. The assumption (C.2) is, however, not easy to verify directly. The following theorem presents a condition that is equivalent to (C.2) but that can be directly checked.

Theorem 2.1. The condition (C.2) holds if and only if.

$$\alpha, \eta \subset \beta + \gamma^* \quad (2.14)$$

The ordinary matching condition requests:

$$\alpha, \eta \subset \gamma^* \quad (2.15)$$

We see that (2.14) is more general than (2.15). However, (2.14) is equivalent to (2.15) if and only if $\gamma^* \subset \beta$. In particular, it is trivially satisfied when Ker $C = \{0\}$. (which means that the output consists of full states). This observation reveals that in the case of full state regulation, (2. 14) just recovers the ordinary matching condition. For this reason, (2. 14) may be called the matching condition for robust output regulation. Now, we give the proof of Theorem 2.1.

Proof of Theorem 2.1 First, suppose that (2.14) is satisfied.
Then, for each $i = 1,..., q$ and $j = 1,..., r$, there exist $\hat{M}_i, \hat{N}_j \in R^m$ and $\zeta_i, \rho_j \in \gamma^*$ such that

$$M_i = B\hat{M}_i + \zeta_i \text{ and } N_j = B\hat{N}_j + \rho_j. \qquad (2.16)$$

Since γ^* is an (A,B)- invariant subspace contained in Ker C, there exists a constant matrix $L \in R^{m\times n}$ such that

$$C(A + bL)^k z = o, \quad z \in \gamma^*, k = 0, 1, ... \qquad (2.\ 17)$$

Let the linear system $(\bar{A},\bar{B},\bar{C})$ be a minimal realization of $G(s) \triangleq C(sI_n - A - BL)^{-1}B$. Let p be the dimension of the linear system $(\bar{A},\bar{B},\bar{C})$. Then, there exists a constant matrix $T \in R^{p\times n}$ satisfying the first three equalities in (2.3). Also, this with (2. 17) implies that

$$\bar{C}\bar{A}^k Tz = C(A + BL)^k z = o, \qquad (2.18)$$
$$z \in \gamma^*, \quad k = 0, 1, ...$$

Since the linear system $(\bar{A},\bar{B},\bar{C})$ is controllable and observable, (2. 18) implies that

$$Tz = 0, \quad z \in \gamma^*, k = 0, 1, ... \qquad (2.19)$$

By (2.16) and (2.19).

$$TM_i = TB\hat{M}_i = \bar{B}\hat{M}_i, i = 1, ..., q. \qquad (2.20)$$

$$TN_i = TB\hat{N}_i = \bar{B}\hat{N}_i, \ i = 1, ..., r. \qquad (2.12)$$

finally, taking $\hat{M} \triangleq [\hat{M}_1 ......\hat{M}_q]$ and $\hat{N} \triangleq [\hat{N}_1 ...... \hat{N}_r]$ completes all requirements for (C. 2).

Next, suppose (C.2) is satisfied. By (2.3),

$$C(A + BL)^k M = \bar{C}\bar{A}^k \bar{B}\hat{M} \text{ and } C(A + BL)^k N = \bar{C}\bar{A}^k \bar{B}\hat{N}, k = 0, 1, ... \qquad (2.22)$$

Let

$$\zeta_i \triangleq M_i - B\hat{M}_i, i = 1, ..., q. \qquad (2.23)$$

$$\rho_i \triangleq N_i - B\hat{N}_i, i = i ,....., r. \qquad (2.24)$$

By (2.3), (2.22), and (2.23)

$$C(A + BL)^k \zeta_i = C(A + BL)^k M_i - C(A + BL)^k B\hat{M}_i$$
$$= \bar{C}\bar{A}^k \bar{B}\hat{M}_i - \bar{C}\bar{A}^k \bar{B}\hat{M}_i = o, k$$
$$= 0, 1, .., i = 1,..., q. \qquad (2.25)$$

Similarly, from (2.3), (2.22) and (2.24),

$$C(A + BL)^k \rho_i = o, k = 0, 1, ..., r. \qquad (2.26)$$

Note that the subspace defined by

$$\gamma \triangleq (x \in R^n : C(A + BL)^k x = o, k = 0, 1, ...) \qquad (2.27)$$

is an (A,B)- invariant subspace contained in Ker C. By the definition of γ^*, $\gamma \subset \gamma^*$, This with (2.25) and (2.26) implies that

$$\zeta_i, \rho_j \in \gamma^*, i = 1, . . ., q, \ j = 1,..., r. \qquad (2.28)$$

By (2.23), (2.24), and (2.28),

$$M_i , N_j \in \beta + \gamma^*, i = 1,..., q, j = 1,..., r. \qquad (2.29)$$

This implies (2.14). Q.E.D.

Now, let us consider another special case of (2.14):

$$\alpha, \eta \subset \gamma^*. \qquad (2.30)$$

Following the arguments in the first part of the proof of Theorem 2.1, we can easily see that (2. 30) satisfies (C.2), in particular, with $\hat{M} = 0$ and $\hat{N} = 0$. This implies that ϕ in (2.9) can be chosen zero. In this special case, the feedback system (2.11) has a stronger property than the one described in (2.4). That is, for $x(0) \in R^n$,

$$|y(t)| \leqslant |CTx(0)| e^{-\mu t}, \ t \geqslant 0. \qquad (2.31)$$

The special case (2.30) is a simple extension of disturbance decoupling considered by Wonham and Morse [19] to the problem (p). We have shown that the disturbance decoupling condition is a special case of our matching condition for roubst output regulation.

The computation of γ^* usually involves cumbersome calculations. See a computation algorithm for γ^* in [19]. The following theorem concerns a simpler condition than (2.14). First, we need some notations. Let C_i be the ith row vector of C. Let $E \in R^{n x \ell}$. The $d_i(E)$, i = 1,...,s are defined by the smallest nonnegative integer satisfying.

$$\begin{cases} C_i A^k E = 0, \ k = 0, 1, \ldots, (d_i - 1), \\ C_i A^{d_i} E \neq o. \end{cases} \qquad (2.32)$$

Let $Q \triangleq [C_1 A^{d_1(B)}, \ldots, C_s A^{d_s(B)}]^T$. Now we are ready to state the following theorem.

Theorem 2.2. Suppose that

$$d_i(B) \leqslant d_i(M), \ d_i(N), \ i = 1, \ldots, s, \qquad (2.33)$$

and that there exist constant matrices $L \in R^{mxn}$ $\hat{M} \in R^{mxq}$, and $\hat{N} \in R^{mxr}$ satisfying

$$Q(A + BL) = 0, \ Q(M - B\hat{M}) = o, \text{ and}$$
$$Q(N - B\hat{N}) = O. \qquad (2.34)$$

Then, (2.14) holds.

Because of limited space, the proof of Theorem 2.2. is omitted. Theorem 2.2 can be easily proved by observing that $C_i(A+BL)^{d_i(B)} M = C_i A^{d_i(B)} M = C_i (A+BL)^{d_i(B)} B\hat{M}$ and $C_i(A+BL)^{d_i(B)+1} = 0$, $i = 1, \cdots, s$. Although Theorem 2.2 is a specialization of Theorem 2.1, its hypotheses are still more general than the ordinary matching condition and the disturbance decoupling condition. In fact, (2.15) implies the hypotheses of Theorem 2.2 and (2.30) implies, instead of (2.33), that

$$d_i(B) < d_i(M), \ d_i(N), \ i = 1, \ldots, s. \qquad (2.35)$$

III. An Example

Consider the system (1.1) with n = 3, m = q = r = 1, s = 2, and

$$A = \begin{bmatrix} -1 & 1 & -1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad M = N = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad (3.1)$$

Note that the linear system (A,B,C) is controllable and observable. Clearly, the ordinary matching condition (2.15) is not satisfied. Simple computation shows that γ^* = Ker C. Consequently, the disturbance decouling condition (2.30) is not satisfied, either. Note, however, that our new matching condition (2.14) is satisfied. Actually, we can compute

$$p = 2, \ T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \bar{A} = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix},$$
$$\bar{B} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \bar{C} = I_2, \quad L = [\, 0 \ 0 \ 1 \,] \qquad (3.2)$$

to see that (C.2) is satisfied. Direct calculation yields $d_i(B) = d_i(M) = d_i(N) = 0$, i = 1,2. Consequently, Q = C. It can be easily checked that there is no $L \in R^{1X3}$ satisfying (2.34). Hence, Theorem 2.2 cannot be applied to this example.

Another interesting example is in [9]. For the example, the hypotheses of Theorem 2.2 are satisfied, while neither (2.15) nor (2.30) is satisfied.

IV. Conclusion

We have presented some sufficient conditions for output regulation of uncertain nonlinear systems. Our conditions are less restrictive than the prior conditions for state regulation of uncertain nonlinear systems. This is mainly because internal stability always implies output stability while the converse is not necessarily true. Therefore, our robust controller may not provide the internal stability of the closed-loop system.

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