Modified Raised-Cosine Interpolation and Application to Image Processing

(변형된 상승여현 보간법의 제안과 영상처리에의 응용)

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要 約

새로운 보간함수로서 변형된 상승여현 보간함수가 제안되었다. 이 함수는 가중된 삼각함수와 상승여현 함수의 선형 조합으로 이루어졌으며 보간 오차를 초래하는 부엽(side lobe)들의 영향을 감소시킬 수 있다. 선형 연산자인 고차의 상승적분형 보간함수(higher-order convolutional function)들에서는 보간 오차가 현저히 줄어드나 주엽(main lobe)의 감쇄에 의한 해상도 오차를 수반한다. 제안된 보간함수 는 이와는 달리 부엽들의 감소에도 주엽은 잘 보존되었음을 알 수 있다. 영상 재구성과 영상확대에 제 안된 보간법을 적용한 결과 좋은 실용성을 보였다.

Abstract

A new interpolation function, named modified raised-cosine interpolation, is proposed. This function is derived from the linear combination of weighted triangular and raised-cosine functions to reduce the effect of side lobes which incur the interpolation error. Interpolation error reduces significantly for higher-order convolutional interpolation functions of linear operators, but at the expense of resolution error due to the attenuation of main lobe. However, the proposed interpolation function enables us to reduce the side lobes as well as to preserve the main lobe. To prove practicality, this function is applied in image reconstruction and enlargement.

I. Introduction

Interpolation is the process of estimating the intermediate values of a continuous event from the discrete samples. Because of the limited amounts of data associated with digital images, an efficient interpolation technique is essential to preserve image quality [1-3].

Several interpolation functions have been used in image processing[4-6]. The simplest one among linear operators is the nearest neighbor interpolation function, where the value of the new point is taken as the value of the old coordinate point which is located nearest to the new point. Another interpolation function frequently used is the linear interpolation function, where the new point is interpolated linearly between the old

points. The next most attractive function is the cubic B-spline function, which is four convolutions of the simple rectangular function. This function uses two points in each direction and is positive everywhere.

The choice of interpolation function to be used for image processing depends upon the task being performed. An understanding of the frequency spectra of the interpolation functions may help in the selection of proper interpolation method. The frequency spectrum of the nearest neighbor interpolation function has prominent side lobes, which contributes the significant aliasing errors. The linear interpolation function attenuates a significant amount of frequency components near the cut-off frequency, resulting in smoothing of the image. Cubic B-spline interpolation function has very small side lobes, but results in the most attenuation in the pass zone.

A new interpolation function suggested in this paper is a modified raised-cosine (MRC) interpolation function, which is a linearly combined expression of weighted raised-cosine and triangular functions. This gives in the spectrum the reduction of side lobes without broadening main lobe frequency response, whereas the inherent characteristic of linear operators requires attenuation of the main lobe with reduction of the side lobes.

II. Interpolation Functions

Two conditions are applied throughout this paper. First, the analysis pertains exclusively to one-dimensional problem. Two-dimensional interpolation is easily accomplished by performing one-dimensional interpolation in each dimension. Second, the data samples are assumed to be equally spaced.

If the signal is band-limited and sampling is done at a frequency satisfying the Nyquist sampling theorem, an original signal can be exactly reconstructed. Fig. 1. (a) shows the frequency spectrum of a signal satisfying the above conditions. To reconstruct the original signal exactly, it is apparent that the interpolation function of an ideal low-pass filter is necessary, as seen in Fig. 1. (b) and (c).

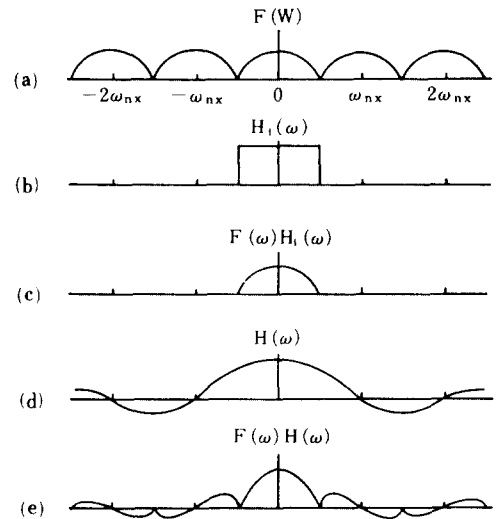Interpolation of the ideal low-pass filter in reality is impossible, so that the feasible proper

Fig.1. Power spectra for ideal and nonideal reconstruction.
(a) Sampled image.
(b) Ideal interpolation function.
(c) Ideal interpolator reconstruction.
(d) Nonideal interpolation function.
(e) Nonideal interpolator reconstruction.

interpolation function should be substituted for the purpose of reconstruction (Fig. 1. (d)). As seen in Fig. 1. (e), actual interpolation function may not only attenuate frequency spectrum of zero order, causing a loss of image resolution, but also permit higher order spectral modes, causing aliasing effect.

There are some conditions required for interpolation function. First, the function should be symmetry. Second, the amplification of the function must be unity. Third, the function has to be zero outside of subinterval.

The simplest interpolation function existed is so called, nearest neighbor interpolation, where each new pixel is given by convolving the sampled value with a rectangular function (Fig. 2. (a)).

$$h_N(x) = \text{rect}(x), \quad (0, 0.5) \qquad (1)$$

Convolution with rectangular function in spatial domain is equivalent to multiplying the signal by a sinc function in the frequency domain. Sinc function is considered as a very poor low-pass filter since it has prominent side lobes.
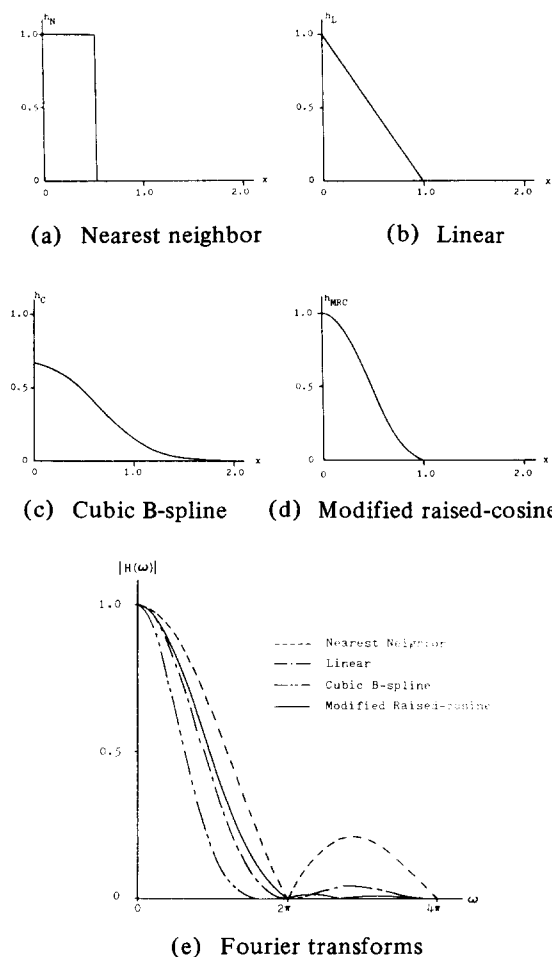
(a)　Nearest neighbor　　　(b)　Linear

(c)　Cubic B-spline　　　(d)　Modified raised-cosine

(e)　Fourier transforms

Fig.2. Interpolation waveforms and their Fourier transforms.

The next function frequently used is the linear interpolation. Linear interpolation performs convolution of the sampled value by a triangular function (Fig. 2.(b)).

$$h_L(x) = 1 - x, \quad (0, 1) \qquad (2)$$

It does attenuate near the cut-off frequency, resulting in smoothing of the image, and it does pass a significant amount of energy above the cut-off frequency.

The next complex interpolation function is cubic B-spline interpolation. The cubic B-spline is four convolutions of the simple rectangular function. Mathematically, the cubic B-spline can be written as

$$h_C(x) = \begin{cases} x^3/2 - x^2 + 4/6, & (0, 1) \\ \\ -x^3/6 + x^2 - 2x + 8/6, & (1, 2) \end{cases}$$

$$(3)$$

It is reasonably good low-pass filter. However, it is positive in the whole interval from 0 to 2 in space domain (Fig. 2. (c)); therefore, it smoothes somewhat more than is necessary below the cut-off frequency. It does have very good efficiency in the stop band.

III. Modified Raised-Cosine Interpolation

The raised-cosine function is used for the window function in signal processing. This function can be written as

$$h_{RC}(x) = 0.5 + 0.5 \cos \pi x, \quad (0, 1) \qquad (4)$$

Satisfied with the constraints of interpolation function, the raised-cosine function can also be used as an interpolation function.

From the frequency spectra of triangular and raised-cosine functions, it is evident that they have partly opposite signs in the side lobes. This situation immediately suggests that weighted raised-cosine and triangular functions can be used to reduce certain effects of the side lobes, so that it may reduce the aliasing effect of an image. The outcome can be called modified raised-cosine interpolation function.

The proposed interpolation function can be written by

$$h_{MRC}(x) = \xi(1 - x) + (1 - \xi)(0.5 + 0.5 \cos\pi x),$$

$$(0, 1) \quad (5)$$

where, $\quad 0 \leqslant \xi \leqslant 1$.

The Fourier transform of the proposed function is

$$H(f) = \xi \frac{\sin^2(\pi f)}{(\pi f)^2} + (1 - \xi)\frac{\sin(2\pi f)}{2\pi f}$$

$$\cdot [1/(1 - 4f^2)] \qquad (6)$$

where f denotes the frequency component of x.

The aim of the proposed interpolation function is to reduce effect of side lobes, using

$$\int_1^\infty H(f)df = 0 \qquad (7)$$

as a measure.

Solving Eq. (7) gives ξ approximately equal to 0.24. Therefore, the complete form of interpolation function is given by

$$h_{MRC}(x) = 0.24(1-x) + 0.76(0.5+0.5\cos\pi x)$$

$$= 0.62 - 0.24x + 0.38\cos\pi x, \quad (0, 1) \qquad (8)$$

Eq. (8) satisfies the conditions of interpolation function, so that it can be used as a new interpolation technique (Fig. 2. (d)).

Frequency spectra of nearest neighbor, linear, cubic B-spline and modified raised-cosine functions in logarithmic scales are also shown in Fig. 3. The modified raised-cosine function shows reduction of side lobes but moderate main lobe.
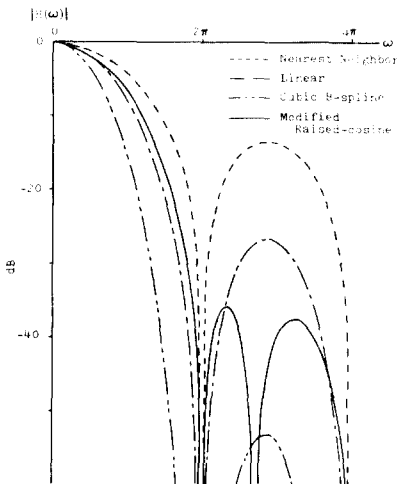

Fig.3. Logarithm of fourier transforms of interpolation functions.

The performances of interpolation functions can be compared numerically[1]. The utilization of nonideal interpolation functions leads to a potential loss of image resolution and to an introduction of high spatial frequency artifacts. The effect of an imperfect reconstruction filter

may be conveniently analyzed by examination of the frequency spectrum of a reconstructed image. The resolution loss resulting from the use of a nonideal reconstruction function h(x) may be quantitatively specified as follows. The resolution error is given by

$$ER_r = \frac{E_i - E_a}{E_i} \qquad (9)$$

where E_i is the energy of the ideally interpolated image and E_a represents the energy of the non-ideally interpolated image within Nyquist sampling band limits. Each term in Eq.(9) is given by

$$E_i = \int_0^{\omega_{nx}/2} P(\omega_x)d\omega_x \qquad (10)$$

$$E_a = \int_0^{\omega_{nx}/2} P(\omega_x)|H(\omega_x)|^2 d\omega_x \qquad (11)$$

and $H(\omega_x)$ represents the frequency spectrum of the non-ideal interpolation function used. It is assumed that the power spectral density of the ideal image is given by

$$= P(\omega_x) \begin{cases} \sqrt{(\omega_{nx}/2)^2 - \omega_x^2}, & \omega_x^2 \le (\omega_{nx}/2)^2 \\ 0, & \omega_x^2 > (\omega_{nx}/2)^2 \end{cases}$$

$$(12)$$

where ω_x, ω_{nx} are frequency and Nyquist frequency in x-direction, respectively. The interpolation error contribution of higher-order components is assumed negligible. The interpolation error is given by

$$ER_{in} = \frac{E_t - E_a}{E_t} \qquad (13)$$

where E_t is the total energy of the non-lideally interpolated image and is given by

$$E_t = \int_0^\infty P_t(\omega_x)|H(\omega_x)|^2 d\omega_x \qquad (14)$$

where $P_t(\omega_x)$ represents the power spectral density of the sampled ideal image, and is given by the convolution of the power spectral density of the ideal image, $P(\omega_x)$ and the Fourier trans-

form of the Dirac sampling array.

Table 1 shows the resolution and interpolation errors obtained from the interpolation functions. Interpolation error reduces significantly for higher-order convolutional interpolation functions, but at the expense of resolution error. However, modified raised-cosine interpolation function provides relatively low resolution error as well as low interpolation error.

Table 1. Interpolation & Resolution errors.

Function	Resolution error ER_r (%)	Interpolation error ER_{in} (%)
Sinc	0.0	0.0
Nearest Neighbor	26.9	15.7
Linear	44.0	3.7
Cubic B-spline	63.2	0.3
Modified Raised-cosine	33.3	2.1

IV. Simulation and Results

1. Image reconstruction

Two-dimensional image reconstruction from one-dimensional projections is now widely used in the medical imaging. The mathematical method used almost widely is the filtered back-projection. It is easy to implement and usually gives the results sufficiently accurate for medical purposes.

The final step in the filtered back-projection involves an integral over rotation angle. Because the data are available only at the discrete angles and positions, the integral cannot be evaluated exactly and approximation must be made based on the available values of the integrand. The usual technique is to replace the integral with a discrete sum over available angles, each term in the sum being obtained by interpolation between the known values. Thus interpolation scheme is

quite important to determine the reconstructed image resolution, so that the selection of proper interpolation function is fairly important.

The Fourier method of reconstruction of two-dimensional density function f(x,y) is given by

$$f(x,y) = \int_0^\pi Q_\theta \ (x\cos\theta + y\sin\theta) \ d\theta \qquad (15)$$

where $Q_\theta \ (x\cos\theta + y\sin\theta)$ is the filtered projection value at angle θ and radial distance $x\cos\theta + y\sin\theta$. For computer implementation, the reconstructed picture may then be obtained by the discrete approximation to the integral.

$$f(x,y) = \frac{\pi}{N} \sum_{i=1}^{N} Q_{\theta_i} \ (x\cos\theta_i + y\sin\theta_i) \qquad (16)$$

where N angles, θ_i, are those for which the projections are known. The value of $x\cos\theta_i + y\sin\theta_i$ in Eq. (16) does not correspond to one of the values of $x\cos\theta + y\sin\theta$ in Eq. (15). However, this may be approximated by an interpolation. The proposed interpolation function is applied to have the values of f(x,y) in Eq. (16).

Shepp and Logan head phantom (Fig.4) was used in the simulation. The reconstructed values on the y =−0.605 line, which goes through the centers of the three small tumors, are shown in Fig.5.

Reconstruction was made from 100 views and 128 rays in each view. The simulated skull is taken to have ventricles with density of 1.0 (water) and the tumors with densities of 1.03 and 1.04 in the gray matter of 1.02. The skull has density of 2.0 and consists of two big ellipses.
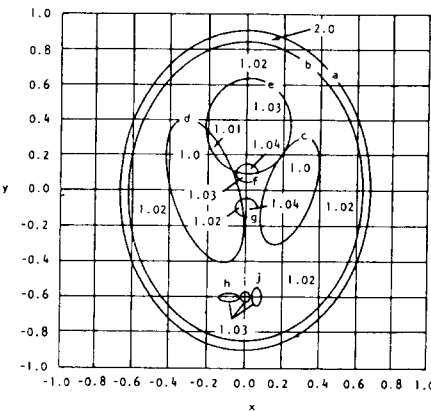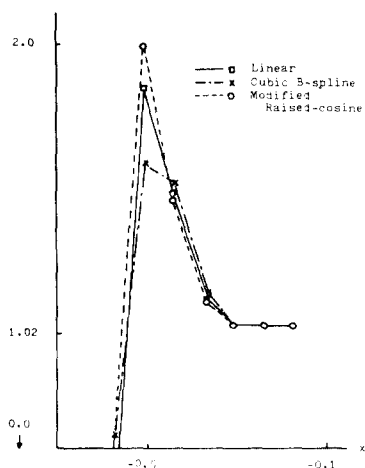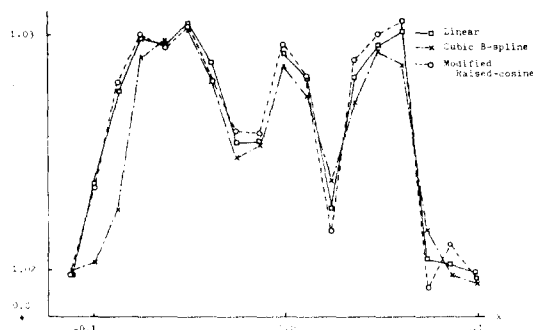

Fig.4. Shepp and logan head phantom.

(a) In the skull area

(b) In the three-tumor area

Fig.5. Reconstructed shepp and logan head phantom on the y=-0.605 line.

(a) Original COUPLE image

(b) Enlarged with nearest   (c) Enlarged with
     neighbor   linear

(d) Enlarged with cubic   (e) Enlarged with
     B-spline   modified
                                     raised-cosine

Fig.6. COUPLE image and their enlarged images with the various interpolation functions.

The sharpness of edges reconstructed with the modified raised-cosine is quite better as compared to the linear and cubic B-spline methods.

2. Image enlargement

COUPLE image is used to simulate and its selected size is 64x64 pixels. The magnified images, enlarged by four times, are 256x256 pixels in size. Fig.6 shows the results of magnified image with nearest neighbor, linear, cubic B-spline and modified raised-cosine interpolations.

There can be shown a significant amount of resoulution error with the nearest neighbor interpolation function. Interpolation error reduces significantly for linear, cubic B-spline interpolation functions, but at the expense of resosution error with increased burring phenomena in the resulting images. Application of the modified raised-cosine interpolation improves blurring phenomena with significantly reduced interpolation error. Visual result and sharpness of edges with the proposed interpolation is better as compared to the nearest neighbor, the linear and the cubic B-spline interpolations.

V. Discussion

The modified raised-cosine interpolation method extends over two interpixel distances, and has reasonably good pass and stop zone performances in the spectrum. This method gives better resolution than the nearest neighbor, the linear and the cubic B-spline interpolation methods in the final image quality, which can be proved in the application to image reconstruction and image enlargement.

The choice of an interpolation function to be used for image processing depends upon the task being performed. Zero-order interpolation with a square interpolation function results in a significant amount of resolution error. Interpolation error reduces significantly for higher-order convolutional interpolation function, but at the expense of resolution error. The modified raised-cosine interpolation function is more effective than the linear and the cubic B-spline in resolution error, and more effective than the nearest neighbor and the linear in interpolation error.

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