Scattering of Sound by a Flexible Cylindrical Cavity

매질이 다른 무한 실린더에 의한 음의 산란

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ABSTRACT

The pressure waves scattered by an infinite cylindrical cavity filled with air in a homogeneous medium have been calculated for the incident plane pressure waves. For ka = 1/2, 1, 2, 4, 10 and 20, the scattered pressure waves are plotted, where k is the wave number and a is the radius of the cylindrical hole. As an indicator of the directivity of the scattering pattern, we have defined the angle at which the magnitude of the scattered pressure wave decreases by a half(6 dB) with respect to that of the forward peak scattered pressure wave. This angle depends strongly on the values of ka and the distance r, and the angle can be used for the detection of the location and the size of the cavity in a homogeneous medium.

요 약

산란된 파동의 형태는 그 산란체의 기하학적 형상 및 위치에 따라 변한다. 본 논문에서는 매질 내에 임피던스를 가지는 무한 실린더가 있는 경우 평면 입사파에 대한 산란된 파동을 유도하였고 그 결과로부터 산란된 파동의 방향성을 나타내는 지표로서 전방 산란파의 peak값에 대해 산란파의 크기가 반으로 떨어지는 각도를 구하였다. 이 각도를 검토해 본 결과 입사파의 주파수 및 거리에 따라 그 각의 변화가 뚜렷함을 볼 수 있었다. 따라서 매질 내에 어떠한 산란체가 있는 경우 이 각도가 산란체의 위치 및 형상을 추정하는데 유용하게 적용될 수 있을 것이라 생각된다.

I. INTRODUCTION

In detecting a certain object by acoustical waves, it is very important to know the scattering pattern caused by an object because the scattering patterns of a wave have informations about size and location of a scattering object. In the cases that the scattering objects are acoustically rigid or soft, the scattered wave has been derived in many books and papers.[3,4] However, in practice, the scattering object is neither acoustically rigid nor soft. In this paper, the scattered pressure wave caused by an infinite cylindrical cavity with impedance is derived in series form of the Bessel and Neumann functions and cal-

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culated by computer. Here, the incident wave is assumed to be plane and monochromatic. Although the analysis has been performed in frequency domain, the results can be used in time domain using the Fourier transform.

II. THEORY

Suppose we put p_i and q_{in} for the pressure and normal velocity component of the incident wave, p_s and q_{sn} for the scattered wave in the external medium, and $\bar{p}$ and $\bar{q}_n$ for the quantities in the internal medium(medium in cylindrical cavity). In the following all functions and quantities referring to the internal medium are denoted by a bar on top.

In cylindrical coordinates, the incident plane wave p_i can be expanded into a series of cylindrical waves[Fig.1] for matching to the boundary conditions.

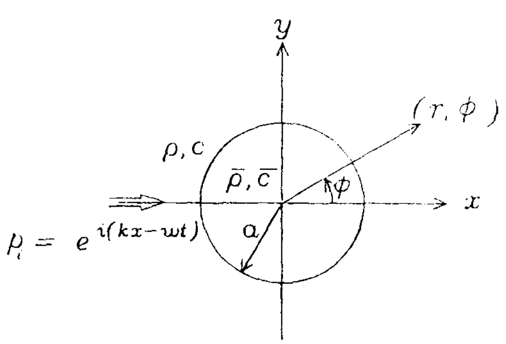


Fig. 1. Cylinder with a plane wave incidence from ϕ=180 direction

$$p_i = e^{i(kx-\omega t)} = e^{i(kr\cos\phi-\omega t)} = \sum_{m=0}^{\infty}\epsilon_m i^m J_m((kr)\cos(m\phi)\, e^{-i\omega t} \qquad (\text{II}-1)$$

where, ϵ_m is the Neumann coefficient which has the value 1 for m=0 and 2 for m>0, i is the imaginary unit($=\sqrt{-1}$)and J_m is the Bessel function of order m. And the scattered pressure wave p_s can be obtained from the wave equation:

$$\nabla^2 p_s = \frac{1}{c^2}\frac{\partial^2 p_s}{\partial t^2} \qquad (\text{II}-2)$$

Excluding the time dependent term $e^{-i\omega t}$, the wave equation becomes the Helmholtz equation:

$$\nabla^2 p_s + k^2 p_s = 0, \quad (k=\omega/c) \qquad (\text{II}-3)$$

Put $p_s = R(r)\Phi(\phi)$, then Eq.(II-3)is divided into the following two ordinary differential equation,

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dR}{dr}\right)+\left(k^2-\frac{m^2}{r^2}\right)R=0, \qquad (\text{II}-4a)$$

$$\frac{d^2\Phi}{d\phi^2}+m^2\Phi=0 \qquad (\text{II}-4b)$$

The solutions of Eq.(II-4 a) are $H_m^{(1)}(kr)$ and $H_m^{(2)}(kr)$, the first and second kind Hankel functions of order m. But since the scattered pressure wave is an out-going wave, the second kind Hankel function, $H_m^{(2)}(kr)$ must be rejected. And the solution of Eq.(II-4 b) are sin $(m\phi)$ and cos $(m\phi)$ But sin $(m\phi)$ is not symmetric about x-axis, then must be discarded. Also, m is not symmetric about x-axis, then must be discarded. Also, m is integer because cos $(m\phi)$, has same value at $\phi=0, 2\pi, 4\pi, \ldots$ Therefore, the scattered pressure wave p_s is given by

$$p_s = \sum_{m=0}^{\infty} A_m H_m^{(1)}(kr)\cos(m\phi)\, e^{-i\omega t} \qquad (\text{II}-5)$$

where A_m are arbitary constants. Also, since the Neumann function becomes infinity at r=0, for the internal medium in the cavity, the pressure field in the internal medium of the cavity, $\bar{p}$ is given by

$$\bar{p} = \sum_{m=0}^{\infty} \bar{A}_m J_m(\bar{k}r)\cos(m\phi)\, e^{-i\omega t}, \qquad (\text{II}-6)$$

where $\bar{A}_m$ are arbitrary constants.

And the normal velocity components for

the incident and scattered pressure waves are given by

$$q_{in} = \frac{1}{i\omega\rho}\frac{\partial p_i}{\partial r}$$

$$= \frac{k}{i\omega\rho}\sum_{m=0}^{\infty} \epsilon_m i^m J_m(kr)\cos(m\phi)e^{-i\omega t}, \quad (\text{II}-7)$$

$$q_{sn} = \frac{1}{i\omega\rho}\frac{\partial p_s}{\partial r}$$

$$= \frac{k}{i\omega\rho}\sum_{m=0}^{\infty} A_m H_m^{(1)\prime}(kr)\cos(m\phi)e^{-i\omega t} \quad (\text{II}-8)$$

Also, for the internal medium of the cavity, the normal velocity component is given by

$$\bar{q}_n = \frac{1}{i\omega\bar{\rho}}\frac{\partial \bar{p}}{\partial r}$$

$$= \frac{\bar{k}}{i\omega\bar{\rho}}\sum_{m=0}^{\infty} \bar{A}_m J_m(\bar{k}r)\cos(m\phi)e^{-i\omega t} \quad (\text{II}-9)$$

In Eq.(II-7), (II-8) and (II-9), ρ and $\bar{\rho}$ are the densities of the external and the internal medium, and the prime denotes the derivative with respect to its argument. The arbitrary coefficients A_m and $\bar{A}_m$ are determined from the boundary conditions.

The boundary conditions consist of the continuity of the pressure and the normal velocity component at the surface of the cylindrical cavity. Then the boundary conditions can be written in the following forms:

$$(p_i + p_s)\,|_{r=a} = \bar{p}\,|_{r=a}, \quad (\text{II}-10)$$

$$(q_{in} + q_{sn})\,|_{r=a} = \bar{q}_n\,|_{r=a} \quad (\text{II}-11)$$

where a is the radius of the cylindrical cavity. Provided the surface oscillation amplitude of the cylinder is very small so that it can be assumed that a is constant. Inserting Eq.(II-1) and Eq.(II-5)-(II-9) into Eq.(II-10) and Eq.(II-11), we can obtain the following two equations,

$$\sum_{m=0}^{\infty} \epsilon_m i^m J_m(ka)\cos(m\phi)e^{-i\omega t} + \sum_{m=0}^{\infty} A_m H_m^{(1)}(ka)\cos(m\phi)e^{-i\omega t}$$

$$= \sum_{m=0}^{\infty} \bar{A}_m J_m(\bar{k}a)\cos(m\phi)e^{-i\omega t}, \quad (\text{II}-12)$$

and

$$\frac{k}{i\omega\rho}\sum_{m=0}^{\infty} \epsilon_m i^m J_m'(na)\cos(m\phi)e^{-i\omega t}$$

$$+ \frac{k}{i\omega\rho}\sum_{m=0}^{\infty} A_m H_m^{(1)\prime}(ka)\cos(m\phi)e^{-i\omega t}$$

$$= \frac{\bar{k}}{i\omega\bar{\rho}}\sum_{m=0}^{\infty} \bar{A}_m J_m'(\bar{k}a)\cos(m\phi)e^{-i\omega t} \quad (\text{II}-13)$$

From above two equations, we can determine the coefficients A_m and $\bar{A}_m$. Since we are interested in the scattered pressure wave p_s, we solve for A_m. Then

$$A_m = \frac{\epsilon_m i^m \{J_m(\bar{k}a)J_m'(ka) - \alpha J_m'(\bar{k}a)J_m(ka)\}}{\alpha J_m'(\bar{k}a)H_m^{(1)}(ka) - J_m(\bar{k}a)H_m^{(1)\prime}(ka)} \quad (\text{II}-14)$$

where $\alpha = \frac{\bar{k}\rho}{k\bar{\rho}} = \frac{\rho c}{\bar{\rho}\bar{c}}$. If the cylindrical cavity is acoustically rigid, α becomes zero and the cylindrical cavity is acoustically soft, α becomes infinity.

Therefore, the scattered pressure wave p_s is given by

$$p_s = \sum_{m=0}^{\infty} \frac{\epsilon_m i^m \{J_m(\bar{k}a)J_m'(ka) - \alpha J_m'(\bar{k}a)J_m(ka)\}}{\alpha J_m'(\bar{k}a)H_m^{(1)}(ka) - J_m(\bar{k}a)H_m^{(1)\prime}(ka)}$$

$$\cdot H_m^{(1)}(kr)\cos(m\phi)e^{-i\omega t} \quad (\text{II}-15)$$

For short wavelengths, the values of ka, $\bar{k}$a and kr are large. Using the useful approximation formulae for the Bessel and Neumann functions and their derivatives:

$$J_m(z) \approx \sqrt{2/\pi z}\ \cos\left(z-\frac{m}{2}\pi-\frac{\pi}{4}\right)$$

$$Y_m(z) \approx \sqrt{2/\pi z}\ \sin\left(z-\frac{m}{2}\pi-\frac{\pi}{4}\right)$$

and

$$J'_m(z) \approx -\sqrt{2/\pi z}\ \sin\left(z-\frac{m}{2}\pi-\frac{\pi}{4}\right)$$

$$Y'_m(z) \approx \sqrt{2/\pi z}\ \cos\left(z-\frac{m}{2}\pi-\frac{\pi}{4}\right)$$

the coefficient A_m becomes

$$A_m \approx \epsilon_m i^m \frac{\{-\cos\bar{\chi}\sin\chi+\alpha\sin\bar{\chi}\cos\chi\}}{\{\cos\bar{\chi}\sin\chi-\alpha\sin\bar{\chi}\cos\chi\}-i\{\cos\bar{\chi}\sin\chi+\alpha\sin\bar{\chi}\cos\chi\}} \qquad (\text{II}-16)$$

where $\chi=ka-\frac{m}{2}\pi-\frac{\pi}{4}$ and $\bar{\chi}=\bar{k}a-\frac{m}{2}\pi-\frac{\pi}{4}$
Also, the first kind Hankel function becomes

$$H_m^{(1)}(z) \approx \sqrt{2/\pi z}\, e^{i\left(z-\frac{m}{2}\pi-\frac{\pi}{4}\right)}$$

$$=\sqrt{2/\pi z}\, i^{-m} e^{i\left(z-\frac{\pi}{4}\right)}$$

Therefore, the scattered pressure wave p_s becmes

$$p_s \approx \sqrt{2/\pi kr}\ e^{i\left(kr-\frac{k}{4}\right)} \cdot \sum_{m=0}^{\infty} \epsilon_m \frac{\{-\cos\bar{\chi}\sin\chi+\alpha\sin\bar{\chi}\cos\chi\}}{\{\cos\bar{\chi}\sin\chi-\alpha\sin\bar{\chi}\cos\chi\}-i\{\cos\bar{\chi}\sin\chi+\alpha\sin\bar{\chi}\cos\chi\}}$$

$$\cdot \cos(m\phi)\, e^{-i\omega t} \qquad (\text{II}-17)$$

III. CALCULATIONS AND RESULTS

We have derived the pressure waves scattered by an infinite cylindrical cavity with impedance. And, when the magnitude of the incident pressure wave is unit, we have calculated the scattered pressure wave for ka=1/2, 1, 2, 4, 10 and 20, and the distance r=5.5a, 5a, 10a and 20a, in the media. From the results of the Eq.(II-15), the scattered waves are plotted in the Fig.2 to Fig.5, in step of 5 degree. In calculation, 30 terms in series are evaluated, and this is confirmed to be sufficiently accurate, where the number of terms are dependent on the values of α, ka and $\bar{k}$a. The physical constants in calculation are listed in Table I.

Table I. Physical constants in calculation

	(Granite) External medium	(Air in cavity) Internal medium
Density	2700 Kg/m^3	1.21 Kg/m^3
Speed of sound	7610 m/s	343 m/s

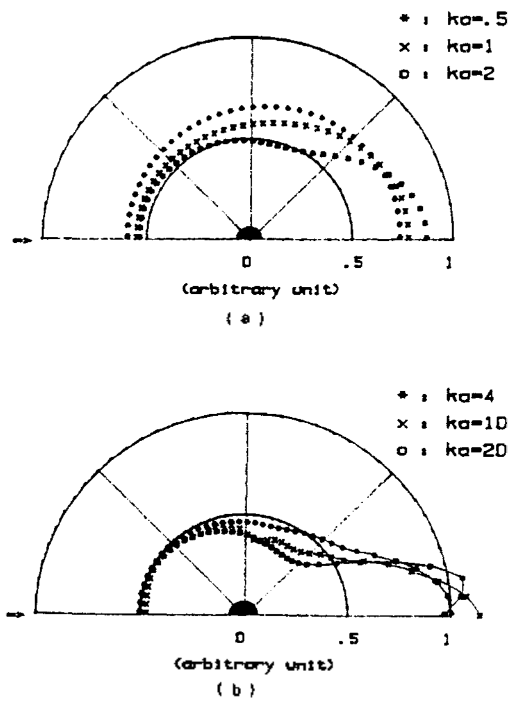
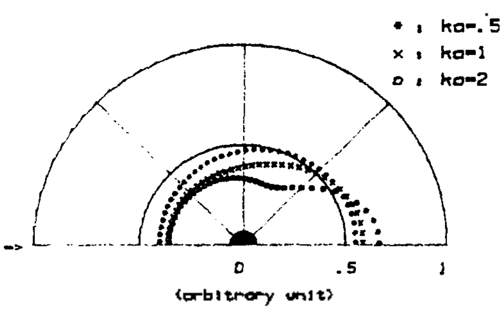


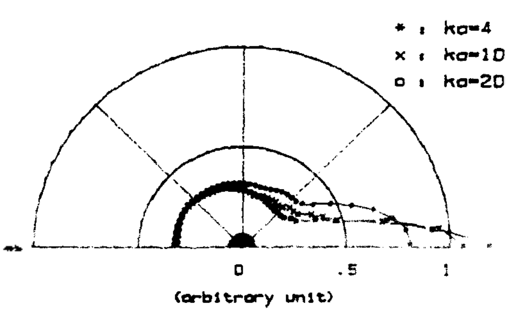
Fig. 2. Calculated scattered wave at r=2.5a.

From the Fig.2 to Fig.5, the forward peak occurs at ϕ =0° except the case of ka=20, r=2.5a, and the directivity is apparently appeared for the case of ka > 2. However, in the cases of ka=1/2 and 1, the directivity is very weak, nearly omnidirectional. And to represent the directivity of the scattered pressure wave, we have found the angle at which the magnitude of the scattered pressure wave decreases by a half (6 dB) with respect to that of the forward peak scattered pressure wave. But in the case of ka=20, r=2.5a the forward peak does not occur at $\phi=0^\circ$, however we have found the angle decreasing by a half(6 dB) with respect to the magnitude of the forward scattered pressure at $\phi=0^\circ$, therefore, the angle of ka=10, r=2.5a is smaller than that of ka=20, r=2.5a. This angle is shown in Table II.

The angle varies apparently with the value of ka and the distance r. Therefore, it is expected that the angle can be used for detection of the location and the size of the cavity in a homogeneous medium.



(a)

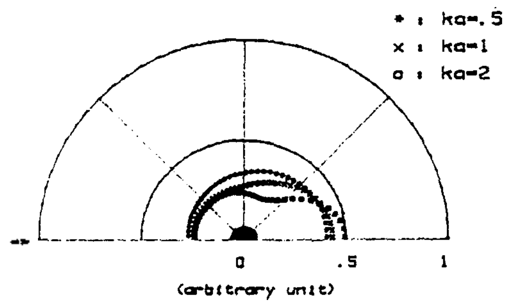


(b)

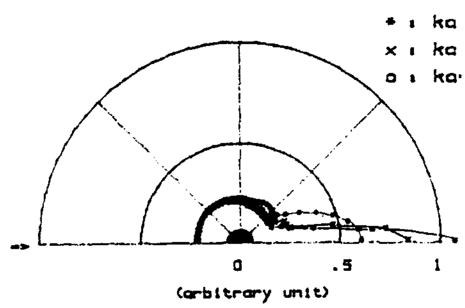
Fig. 3. Calculated scattered wave at r=5a.

Table II. Angle decreasing by half(deg.)

r/a \ ka	2	4	10	20
2.5	-	50	28	32
5	55	28	14	12
10	50	26	11	6.5
20	47	25	10.5	6

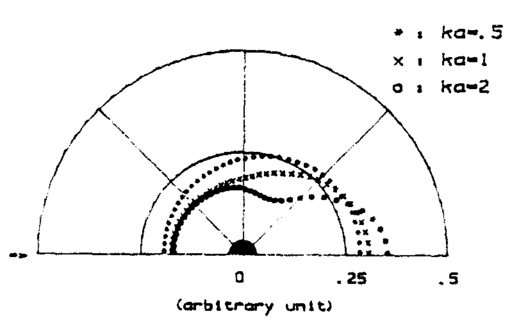


(a)

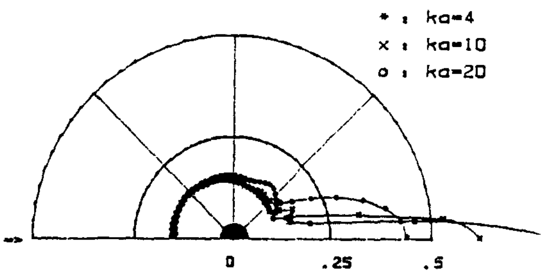


(b)

Fig. 4. Calculated scattered wave at r=10a.



(a)



(b)

Fig. 5. Calculated scattered wave at r=20a.

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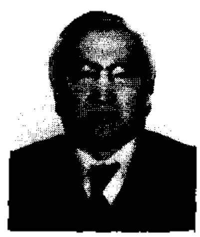

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