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최적 조립에 관한 연구

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ON THE OPTIMAL ASSEMBLY

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축의 지름이 구멍의 지름보다 큰 경우, 축을 구멍에 조립하는 작업이 용이 하도록 하는 방법을 연구하였다. 이 조립작업의 성능평가를 위한, 새로운 多重目的函數를 고안하고 그에 관련된 설계변수들의 최적값을 찾아보았다.

1. Introduction

Assembly work needs the most labour in the general production process, but it is difficult to set up a systematic theory because it's phenomena are very complex and it is dependent on experience. But, we have to know the physical phenomena of the assembly so that we may make a design in which the ease of assembly work is considered [1].

According to one research result [1], the assembly of peg and hole with clearance has the most frequency among the all kinds of assembly works. By such reasons, Whiteny, D. E. [2] presented the results of theory and experiment for assembly of peg and hole with clearance in two dimensonal case, which had conducted at CSDL (Charles Stark Draper Lab., MIT). And, Gustavson, R. E. [3] also carried out some development about the theory of mating of peg and hole with clearance in three dimensional case.

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Besides the assembly of peg and hole with clearance, the force fit assembly of peg and hole has a high frequency among the all kinds of assembly works [1]. Hence, the research about such a kind assembly is also important. By such reasons, some Russian engineers have already studied about the improvement of the force fit performance and, especially, Andeev, G. Y. [4] explained the force fit assembly work of peg and hole of two dimensional case by mentioning that such an assembly can be treated as the assmbly with positive clearance if it is assumed that the assembly is happened in the thermal state and that hole diameter will be increased when the hole is heated. He also presented the theory related to minimum clearance and the clearance for freedom from wedge as the part of those research resullts.

Seltz, D. S. [7], also, mentioned that the assembly work with clearance less than 0. 025㎜ is difficult to be carried out on account of such a problem as the dimensional variation, and presented a devices, the lastest version of the IRCC (Instrumented Remote Center Compliance), as a result of his study for solving such a difficulty. The force fit assembly work has the similiar difficult problem as mentioned by him.

As the above mentioned facts, there are some researches for theoretical treatment of assembly work of peg and hole, but there is no research about the determination of optimum values of design variables related to peg and hole assembly work.

Especially, on considering the mechanical assembly product connected by the force fit, its mechanical performance, i. e. transmissible torque, is closely related to the inferference between assembled parts And, such an interference is related to the assembly cost. In addition to this, the ease for the design of automatic assembly machine is also related to the assembly cost. But, the more increment of the mechanical performance and the ease of design of the automatic assembly machine, the more increment of assembly cost is required. Hence, it is necessarily to be carried out studying about what values of some design variables related in the assembly phenomena have to be allocated in order to decrease the assembly cost and to increase the mechanical performance and the ease of design of the automatic assembly machine.

The first purpose of this paper is to formulate the objective function which governs the performance of the force fit assembly work. The second purpose is to find the optimum values of the related design variables. By doing this, it is the third purpose to show a procedure for improving the efficiency of the force fit assembly work.

In the procedure for such a treatment, the nonlinear objective function subject to the nonlinear inequality and equalty constrains is obtaind. So, the algorithm of Box, M. J. [6], which is used broadly in this case, is em·ioyed as the optimization method.

2. The relationship between the interference and the mechanical performance and heating cost.

(1) The relationshop between the interference and the mechanical performance.

The method of force fit joint often is used rather than the mechanical connecting elements (ex. key etc.) when the shaft connected to hub in order to transmit the power. In this case, the designer has to give the interference defined by following equation as a design variable, at initial design stage, so that the internal pressure may exist between the inner hole boundary of the hub and the outer hole boundary of the shaft.

interference ≌ basic size of shaft dia-basic size of hole dia.

It is noted that the term "basic size" is used in the definition of interference instead of only shaft dia. and hole dia. which can be often met in general book. Here, the basic size means the exact theoritical size.

That is, mathematically,

$$i = d_s - d_h \qquad (1)$$

Where

i = initial interference between basic size of shaft dia. and basic size of hole dia. before assembly (mm)

d_s = basic size of shaft dia. before assembly (mm)

d_h = basic size of hole dia. before assembly (mm)

At this time, this interference, i, has to be given so as to be between i_{min} and i_{max}

That is,

$$i_{min} \leqq i \leqq i_{max} \qquad (2)$$

where

i_{min} =min. interference required for the given mechanical performance.

i_{max} =max. interference which can be allocated without fracture within the used materials.

Next, when the mechanical performance is considered as the torque since this is the case for the power transmission, it can be found that the relation between them is as follows. Firstly, it is defined the relative interference.

$$i_r \equiv \frac{i}{d_h} = \frac{d_s - d_h}{d_h} \quad (3)$$

where

i_r =relative interference (μ/mm)

d_s =basic size of shaft dia. before assembly (*mm*)

d_h =basic size of shaft dia. before assembly (*mm*)

And, the torque which a force fit assembly can transmit without relative displacement of the two parts is as follows [10]

$$T_d = \frac{\pi}{2} f p l \cdot d_h^2 = 2 \cdot Vol \cdot fp \quad (N \cdot mm)$$

where

f =the coefficient of friction

p =unit normal pressure between hole and shaft(N/㎟)

l =axial direction length of hole

d_h =basic size of hole dia. before(㎜) assembly (*mm*)

Vol =Volume of hole made by basic size of hole dia. d_h

$$= \frac{\pi d_h^2 l}{4}$$

As long as the deformation are elastic, has the following form [10]

$$p = p(i_r) = \beta \cdot i_r$$

where

$$\beta = \beta(E, E_1, D_h, d_s, d_h, d_i, \mu_1, \mu)$$

E =Young's modulus of the hub with hole

E_1 =Young's modulus of the shaft

D_h =the outer dia. of hub

d_i =the inner dia. of shaft

μ, μ_1 =Poisson's ratio of hub shaft, respectively

But, in general, the elastic-plastic deformation will take place in the shaft and the hub. In such case, there is no longer relationship of eq.(4) and p becomes a nonlinear function of i . So, it becomes a nonlinear function of i_r if it is expressed by the relative interference i_r of eq.(3). The value of this function, $p(i_r)$, is also increased as i_r is increased [10]

And, the above mentioned torque T_d can be expressed with i_r is as follows.

$$T_d = \frac{\pi}{2} f p(i_r) l d_h^2$$
$$= 2 \cdot Vol \cdot f \cdot p(i_r) \qquad (5)$$

And, it is kown that, as i_r is increased T_d is increased in spite of elastic or elastic-plastic. Hence, if $\frac{1}{T_d}$ is used as the indication for the disability of torque transmission, it can be said that the disability of torque is decreased as i_r is increased.

(2) The relation between the interference and the assembly cost

It is being considered the assembly work happened in the thermal state in this paper, becasuse it is often needed that the hole of the hub is made larger by heating in order to assemble the shaft into the hole of the hub easily. In this case, the heat required to make it larger and the dimension required to make it larger and the heating cost are as follows.

That is,

$$e = d_h \cdot t_c \cdot \triangle T$$
$$Q = \rho V c_p \cdot \triangle T$$

In practice, Q will be much larger than the amount calculated by the above equation, since there is a loss of heat and the equipment as the oil bath or furnance will be used in qrder to heat the considered material

$$A(e) = m \cdot Q$$

That is, there exists a following relationship.

$$A(e) = \frac{\rho V c_p m}{t_c} \cdot \frac{c}{d_h}$$

Where

t_c = thermal coefficient of linear expansion $\left(\frac{mm}{mm \cdot ^\circ C}\right)$
$\triangle T$ = the temperature difference between initial state and heated state of hole (℃)
Q = the required heat in order to make $\triangle T$ (joule)
ρ = density (kg/m^3)
c_p = specific heat $(kJ/kg \cdot ^\circ C)$
V = Volume of the hub with hole (mm^3)
$A(c)$ = the heating cost (dollar)

e = the radial direction length deformed by heating for assembly (mm)

m = conversion constant in terms of heating cost ($dollar/joule$)

If the relative length is defined as $e_r \equiv \frac{e}{d_h}$ (μ /mm), then this equation is expressed as follows.

$$A(e_r) = A \cdot e_r$$

Where

$A(e_r)$ = the heating cost (dollar)

$$A = \frac{\rho V c_p m}{t_c}$$

From now, it is defined that heating cost is assembly cost.

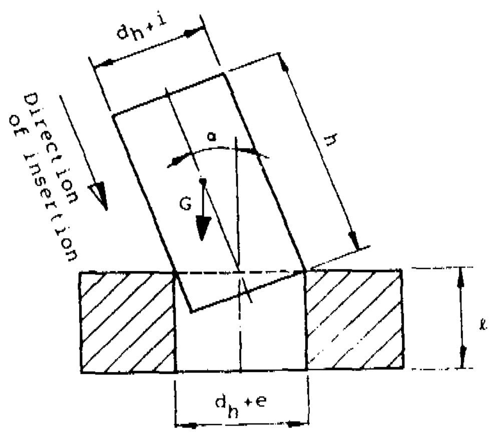

Fig. 1. The assemby of peg and hole with a clearance since the hole is heated.

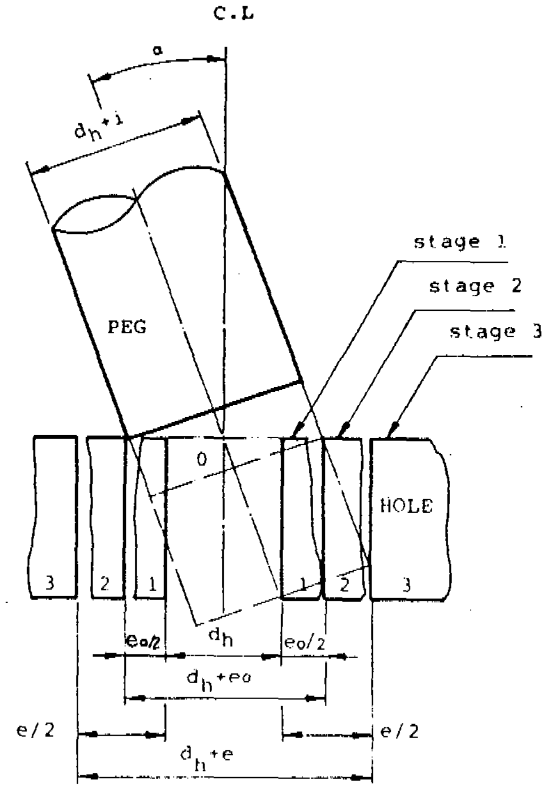

Fig. 2. The insertion process between peg and hole in heated assembly

3. Relationship among the assembly clearance and the interference and the α.

When it is considered the heated fit assembly of peg and hole in case of two dimension, the geometrical relationship between peg and hole can be seen in the Fig. 1 and Fig. 2. Here, it is assumed that the center of hole and the center of the peg meet point 0 during the assembly work, and the amount of the radial direction deformation of hole dia. is symmetry on centering around hole axis. The case of stage 1 of Fig. 2 means the state that hole is not heated. That is, this is the state which the peg dia. is greater than the hole dia. by interference i. More over, the peg should be inserted into the hole in the direction slanted by angle α caused by inaccuracy of the endefector of assembly machine. So, the peg can not be inserted into hole as can be seen in the stage 1. Hence, the hole dia. needs to be heated in order to increase the its dia. In order to determine how much e is needed, it needs to think the following things. First above all, the clearance between hole and peg is defined as follows.

$$c \equiv (d_h + e) - \frac{d_h + i}{\cos\alpha} \qquad (7)$$

Hence, $\frac{d_h + i}{\cos\alpha}$ is a projection of peg dia. on the surface which makes orthogonality with the axis of peg. In stage 1, $c<0$ is satisfied. The stage 2 is the case of $c=0$. In order for assembly work to advance more in the direction slanted by $\alpha, c>0$ must be satisfied. That is,

$$(d_h + e) - \frac{d_h + i}{\cos\alpha} > 0$$

So,

$$e > d_h\left(\frac{1 + i_r}{\cos\alpha} - 1\right) \qquad (8)$$

This is the condition which is absolutely nessessary for advance of the insertion of peg into hole and ensures the absence of any damage to the face edges of parts at the initial moment of assembly. And, e must be smaller than the limit value, e_{max} which the material can be heated without fracture. That is,

$$e \leqq e_{max}$$

And, in this 2-Dimensional peg-hole assembly work, the amount of $e,(e_c)$, which is the conditon for freedom from wedge in vertical assembly effected under the action of the weight G of peg (Fig. 1) is as follows [5].

$$e_c > (d_h + i)\frac{\sin(\alpha + \Psi) - \sin\Psi}{\sin\Psi\cos\phi} \sin\alpha \cdot \sin\left(\frac{\alpha}{2} + \phi\right) + i \qquad (9)$$

where

i = interference (*mm*)
d_h = basic size of hole dia. (*mm*)
Ψ = *arctan* $\left(\frac{d_h + i}{h}\right)$ (*radian*)
ϕ = *arctanf* (*radian*)
f = the coefficient of friction
α = the insertion angle caused

by inaccuracy of endeffector of the assembly machine. $(0 \leqq \alpha \leqq \alpha_{max})$

And, it is introduced the relative value of e and e_c as the following equations.

$$e_r \equiv \frac{e}{d_h} > \frac{1+i_r}{\cos\alpha} - 1 \qquad (10)$$

$$e \equiv \frac{e_c}{d_h} > (1+i_r) \frac{\sin(\alpha+\Psi)\sin\Psi}{\sin\Psi\cos\phi} \sin\alpha \cdot \sin(\frac{\alpha}{2}+\phi)+i_r \qquad (11)$$

Then, the followings can be defined for analysis of relationships among variables.

$c^r \equiv e_r$ *or* e_{cr}

$A \in \uparrow \equiv$ A is an element included in the set of the increasing functions.

$A - B \equiv$ A implies B.

Then, the followings are known.

d_h, h and f are assumed to be fixed constants.

From the eq. (10), and (11),

$i_r \in \uparrow$ and $\alpha \in \uparrow \rightarrow c^r \in \uparrow$ (a)

$i_r \in \uparrow$ and $\alpha \in \downarrow \rightarrow c^r \in \uparrow$ *or* $c^r \in \downarrow$ *or* $c^r = const.$ (b)

$i_r \in \uparrow$ and $\alpha = const. \rightarrow c^r \in \uparrow$ (c)

$\alpha \in \uparrow$ and $i_r \in \uparrow \rightarrow c^r \in \uparrow$ (d)

$\alpha \in \uparrow$ and $i_r \in \downarrow \rightarrow c^r \in \uparrow$ *or* $c^r \in \downarrow$ *or* $c^r = const.$ (e)

$\alpha \in \uparrow$ and $i_r = const. \rightarrow c^r \in \uparrow$ (f)

Also,

$c^r \in \uparrow \rightarrow i_r \in \uparrow$ and $\alpha \in \uparrow$ (g)

$c^r \in \uparrow \rightarrow i_r \in \uparrow$ and $\alpha \in \downarrow$ (h)

$c^r \in \uparrow \rightarrow i_r \in \uparrow$ and $\alpha = const.$ (i)

$c^r \in \uparrow \rightarrow i_r \in \downarrow$ and $\alpha \in \uparrow$ (j)

$c^r \in \uparrow \rightarrow i_r = const.$ and $\alpha \in \uparrow$ (k)

It is not known whether or not c^r is increased in Eq. (b) and (e). So, if ($i_r \in \uparrow$ and $\alpha \in \downarrow$) and ($i_r \in \downarrow$ and $\alpha \in \uparrow$) are excluded, the following conclusions can be obtained.

$$\left.\begin{array}{l} i_r \in \uparrow \text{ and } \alpha \in \uparrow \\ or \\ i_r \in \uparrow \text{ and } \alpha = const. \\ or \\ i_r = const. \text{ and } \alpha \in \uparrow \end{array}\right\} \Longleftrightarrow c^r \in \uparrow$$

And, from eq. (6) and eq. (7),

$c^r \in \uparrow \Longleftrightarrow A(e_r) \in \uparrow$ (m)

From eq. (5),

$T_d \in \uparrow \Longleftrightarrow i_r \in \uparrow$ (n)

And, if α is increased, the more positional error of the endeffecter of the automatic assembly machinc is permitted. And, eq. (11) can be treated more simply for manipulation if it is considered the $sin\alpha$. Hence, if $\frac{1}{\sin\alpha}$ is defined as the design cost of automatic assembly machine, it can be said that the design cost of the automatic assembly machine, $\frac{1}{\sin\alpha}$, is decreased as the α is increased. That is,

$$\frac{1}{\sin\alpha} \in \downarrow \ \Longleftrightarrow \ \alpha \in \uparrow \qquad \text{(o)}$$

In summerization, from eq. (l), (m), (n) and (o)

$$i_r \in \uparrow \text{ and } \alpha \in \uparrow \ \Longleftrightarrow$$
$$T_d \in \uparrow \text{ and } \frac{1}{\sin\alpha} \in \downarrow \text{ and} \qquad \text{(p)}$$
$$A(e_r) \in \uparrow$$

4. Formulation and Treatment of the optimization problem

(1) Formulation of optimization problem

On considering eq. (p), the problem to decrease the assembly cost and to increase the mechanical performance and to decrease the design cost of the automatic assembly machine is in conflict each other. Hence, the values of i_r and α and e_r which each term is compromised each other will be provided as their optimum value. So, there exists a problem about how much amount of i_r and α and e_r have to be allocated in order to compromise each term of the above mentioned problem. Such an problem is classified as one of the multicriteria optimization problems[12]. And, the following multicriteria objective can be formulated.

$$Minimize\ \Phi_1(i_r, \alpha, e_r) = w_1 u_1 \frac{1}{T_d(i_r)} + w_2 u_2 \frac{1}{\sin\alpha} + w_3 u_3 A e_r$$

where

w_1 = weighting coefficients given by the designer's subjective numerical values according to the importance of each term for the objective function. ($\Sigma w_i = 1$)

u_i = the constant for unification of the units of each term, i = 1, 2, 3,

When this kind of multicriteria objective function is used, there is a problem about how to choose w_i. In addtion to this problem, there is a problem which all terms should be expressed in equivalent units so that w_1. reflects closely the importance of the objective function [12, 13, 14]. That is, in case of Φ_1 the 2nd term and 3rd terms will be in (dollar) unit. But, the lst term will be in $(N \cdot mm)^{-1}$ or $(joule)^{-1}$.

So, it is difficult to find the physical relation between two units. In order to reduce such a difficulty of the problem as the unifcation of units, the following objective function can be formulated.

$$\text{Minimize} \quad \Phi_2(i_r, \alpha, e_r) = M \cdot \frac{(Ae_r)^{w_1}}{(T_d(i_r))^{w_2}(\sin\alpha)^{w_3}}$$

where

w_i = weighting coefficients of the subjective values according to the importance of each term for objective function. ($\Sigma w_i = 3$)

M = Multiplication of contsants for unfification of units of each term.

In Φ_2, the optimum value of each design variable is not changed according to the value of M And, the value of Φ_2 can not be said by absolute value and can be said by relative value. Another possible formulation is as follows.

$$\text{Minimize} \quad \Phi_3(i_r, \alpha, e_r) = N \cdot \frac{1}{(T_d(i_r))^{w_i}} \cdot \left(\frac{1}{w_2 \sin\alpha} + w_3 Ae_r\right)$$

where

w_i = weighting coefficenths with the proprty mentioned in Φ_1 and Φ_2

N = Multiplication of coefficents for unification of units of each term

If the $\frac{1}{\sin\alpha}$ is one with non-exact coefficient Φ_3 then there is still same problem since the sensitiviy of each term can not be considered exactly.

Hence, the Φ_2 will be used as the objcctive function of this study. When this Φ_2 is written without subscript 2 and with the constraints in one place, the following optimization problem is formulated.

$$\text{Minimize} \quad \Phi(i_r, \alpha, e_r) = M \cdot \frac{(Ae_r)^{w_1}}{(T_d(i_r))^{w_2}(\sin\alpha)^{w_3}} \quad (12)$$

subject to

$$(i_r)_{min} \leqq i_r \leqq (i_r)_{max}$$
$$0 \leqq \alpha \leqq \alpha_{max}$$
$$0 \leqq e_r \leqq (e_r)_{max} \quad (13)$$
$$i_r \leqq \frac{1+i_r}{\cos\alpha} - 1 < e_r$$

$$i_r \leqq (1+i_r)\cdot\frac{\sin(\alpha+\Psi)-\sin\Psi}{\sin\Psi\cos\phi}$$

$$\sin\alpha\cdot\sin\left(\frac{\alpha}{2}+\phi\right)+i_r < e_r$$

where

w_i = weighting coefficients of the subjective numerical values according to the importance of each term for the objective function. ($\Sigma w_i = 3$)

M = Multiplication of constants for unification of units of each term

$$T_d(i_r) = \frac{\pi}{2}\, f\, p(i_r) l\, d_h^2 = 2\cdot Vol\ f\, p(i_r) \qquad (N\cdot mm)$$

$p(i_r)$ = the unit normal pressure, between peg and hole, depending on i_r which will exist in the contact boundary circle after assembling. (N/mm^2)

$$A = \frac{\rho V c_p m}{t_c} = \text{constant}$$

ρ = density (kg/m^3)

t_c = thermal coefficient of linear expansion ($mm/mm\,^\circ C$)

m = conversion constant in terms of heating cost (*dollar/joule*)

V = Volume of the hub with hole (mm^3)

c_p = specific heat ($kJ/kg\cdot{}^\circ C$)

$$\Psi = \arctan\left(\frac{d_h(1+i_r)}{h}\right) \quad (radian)$$

$$\phi = \arctan f \qquad (radian)$$

f = the coefficient of friction

(2) Investigation

The objective function is nonlinear. Constraints are nonlinear inequality and equality. Hence, this minimization problem is a constrained nonlinear optimization problem. In eq. (12), $p(i_r)$ will have the form by eq. (4) or another nonlinear equation of i_r according to whether the deformations are elastic or elastic-plastic. The specific description will be given by the specific problem. The upper and lower limits of i_r, $(i_r)_{min}$ and $(i_r)_{max}$ will be calculated by application of the theory of elasticity. The design variables are i_r, α, and e_r. The change of these design variables with time is not considered in this study. Since, there are condition for geometrical relationship and condition for wedging this problem can be treatd as two problems. After each problem is solved, the one side of which e_r value is larger, will be selected as the desired solution. So, this obtained solution is one such that the wedge phenomena will not hapened, as well as satisfying the geometrical relationships. And, the hole diameter (d_h) and the length of peg (h) will be regard as the constant value in this study.

(3) Employed optimization algorithm.

The Complex method [6] developed by Box, M. J. is employed in order to treat this optimization problem, in this study. The reason is why, like other direct search methods, this method is, computationally, uncomplicated and, as easy to implement and quick to debug as to be broadly until recently [9], in addition to be able to treat the nonlinear objective function subject to nonlinear inequality constraints [8]

The brief procedure of the Complex method is as follows [6]

$$Maximize\ f\ (x_1, x_2, \cdots, x_n)$$

subject to

$$g_k \leqq x_k \leqq h_k, \quad k = 1, 2, \cdots m$$

Here, the $x_1, x_2, \cdots x_n$ are the independent variables. $x_{n+1}, x_{n+2}, \cdots x_m$ are functions of the independent variables. g_k and h_k are either constants or functions of the independent variables.

Step 1 : This method requires the generation of $k \geqq n+1$ points as the first complex. Here, 1 point is an initial feasible starting point. The remaining k -1 points are generated from the following equation.

$$x_i = g_i + r_i(h_i - g_i), \ i = 1,2,3,\cdots n$$

where r_i is a random number over the interval [0, 1]

Step 2 : These k points satisfy both explicit and implicit constraints. If the explicit constraints are violated, then the trial point is reset to a small value inside the violated limit. If some implicit constraints are violated, the point is moved halfway towards the centroid of the remiaining points. This process is repeated until all constraints are satisfied.

Step 3 : The function evaluated at each vertex. The vertex of lowest function is replaced by a point which is located at a distance α times as far from the centroid of the remaining points.

Step 4 : If this trial point is also the worst, it is moved one half the distance to the centroid of the remaining points. This procedure is repeated until the complex has not collapsed into the centroid.

Step 5 : The new point is checked against the constraints. If some constraints are violated, it is adjusted as before.

Step 6 : When the objective function values at each point are within a bound for some consecutive iterations, the program will stop.

Table 1(a). Mechanical properties of used materials

Material type	steel
component	carbon and low alloy ASTEM-A242
Young's Modulus E (GN/m^2)	200
Ultimate strength σ_u (MN/m^2)	480
Ultimate shear stress τ_u (MN/m^2)	350
Density ρ (kg/m^3)	7.86×10^3
Specific Heat C_p (kJ/kg$^\circ$C)	0.46
Thermal expansion coefficient t_c (mm/mm$^\circ$C)	11.7×10^{-6}
Poission's ratio μ	0.26 - 0.29

Table 1(b). The given data for geometrical realtionships and unity of units

d_h	5.0 (mm)	h	20.0 (mm)
(i_r)min	1.0 (μ /mm)	(i_r)max	100.0 (μ/mm)
α min	0.0 (rad)	α max	0.04 (rad)
(e_r)min	0.0 (μ /mm)	(e_r)max	6.0 (μ /mm)
f	0.5	D_h	10.0 (mm)
m	0.01 (cent/J)	M	1000

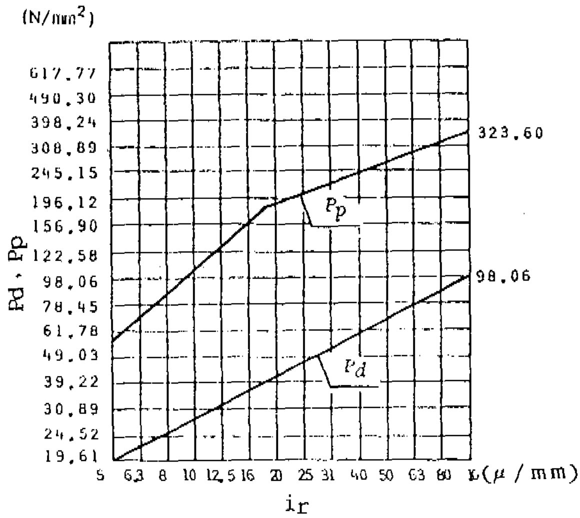

Fig. 3. The relation between relative interfernce and drifting force (or press-in force) log-log graph

5. Example

(1) Problem description

It is considered about the heated assembly between peg and hole as can be seen in Fig. 1 with the given data of mechanical properties of materials as Table 1(a). The data for geometrical relationships of assembly are given in Table 1(b). Firstly, it needs to know the relation between p and i_r in order to insert this relation into T_d. The result of one experiment about i_r $vs \cdot p_d$ about the peg and hole with the above mentioned material is shown in Fig. 3 [5]. Here, p_d is the drifting force and has the following relation.

$$p_d = f \cdot p$$

And, this Figure is simplified diagram obtained by replacing the curves by straight lines in such a way that the error does not exeed ±10% From this figure, the following relation of i_r vs. p_d is found.

$$p = 8.26048 \cdot i_r^{0.53724} \quad (N/mm^2) \quad \text{(a)}$$

Hence,

$$T_d = 2431.7 \cdot i_r^{0.53724} \quad (N \cdot mm) \quad \text{(b)}$$

Next, in general, the value of i_{max} and i_{min} of interference i are calculated on the basis of eq. (2). But, those values were assumed as Table1 (b), in this example. For this assumed i_{min} the min. transmissible torque

$$(T_d)_{min} = 2431.7 \quad (N \cdot mm)$$

It is assumed that value of this $(T_d)_{min}$ is suitable min. transmissible torque in this example. For the assumed i_{max}, the compressive stress (σ) of shaft which occurs during the pressing-in operation and the shear stress (τ) induced by twisting of the shaft are calculated by the Fig. 1 and the following equation [5]

$$\sigma = 4 \cdot \frac{l}{d} p_p \quad (N/mm^2) \quad [5]$$

where p_p = pressing-in force

$$\tau = 8 \cdot \frac{l}{d} p_d \quad (N/mm^2)$$

where p_d = drifting force

So,

$$\sigma_{max} = 3883.2 \quad (N/mm^2)$$

$$\tau_{max} = 2353.4 \quad (N/mm^2)$$

These values are below the values of σ_u and τ_u of Table 1 (a). So, those values of i_{min} and i_{max} of Table 1 (b) can be used for the purpose of this example.

Next, in order to find the value of e_{max} of hole dia. which can be expanded by heat, it is assumed that the heating equipment has a capacity of 500 ℃ which the equipment can produce. By such data and maximum $\triangle T = 500$ ℃ is assumed,

$$e_{max} = d_h \, t_c \, (\triangle T)_{max} = 0.03 \, mm$$

where t_c is assumed as constant until 500 ℃ So,

$$(e_r)_{max} = \frac{e_{max}}{d_h} = 6 \, (\mu / mm)$$

Next, in order to find the value of

constant A

$$A = \frac{\rho V c_p m}{t_c} = 2.64316 \quad (cent)$$

In this example, set $w_1 = w_2 = w_3 = 1$ since each term is assumed as the equal importance. And, $M = 1000$ is assumed.

Then, the following optimization problem is obtained.

(2) Optimization Problem

$$Minimize \ \Phi(i_r, \alpha, e_r) = \frac{1}{2431.7 \cdot i_r^{0.53724}} \cdot \frac{1}{\sin\alpha} \cdot 2643.16 \cdot e_r \quad (14)$$

subject to

$$\begin{aligned} & 1 \leqq i_r \leqq 100 \\ & 0 \leqq \alpha \leqq 0.04 \\ & 0 \leqq e_r \leqq 6 \\ & i_r \leqq \frac{1+i_r}{\cos\alpha} - 1 < e_r \\ & i_r \leqq (1+i_r)\frac{\sin(\alpha+\Psi)-\sin\Psi}{\sin\Psi\cos\phi} \\ & \sin\alpha \cdot \sin(\frac{\alpha}{2}+\phi)+i_r < e_r \end{aligned} \quad (15)$$

where

$$\Psi = arctan\,(0.25 \cdot (1+i_r))$$

$$\phi = arctan\,0.5$$

Table 2. The initial feasible points

group	i_r	α	e_r
1	0.5000	0.0100	6.0000
2	1.0200	0.0200	4.0200
3	2.0250	0.0300	5.0500
4	3.0900	0.0010	5.0010
5	3.0900	0.0020	3.1010

Table 3. The several random numbers

group	r_1	r_2	r_3
A	0.3987	0.5165	0.8829
	0.9306	0.9059	0.8829
	0.7880	0.9408	0.1780
	0.7943	0.0019	0.4468
	0.4037	0.8764	0.7506
B	0.8764	0.9059	0.1780
	0.9306	0.8829	0.3987
	0.7880	0.3987	0.1780
	0.3987	0.0019	0.4468
	0.5163	0.8764	0.7506

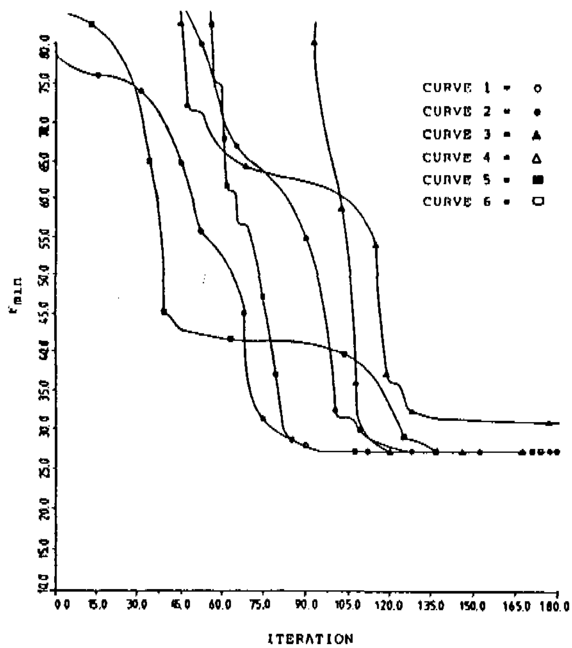

Fig.4. FMN - ITERATION

Table 4. Some combinations of table 2 and table 3.

curve no.	group no. of table 2	group no. of table 3
1	1	A
2	2	A
3	3	A
4	4	A
5	4	B
6	5	B

(3) Numerical Experiments

When the optimization problem, which is constrained with eq. (14) and (15), is run with the data of table 4, the following relations are examined.

(3.1) The change of F_{min} according to the change of inital feasible point and/or random number.

The 5 groups of the given initial feasible point of relative interference (i_r) insertion angle (α) and relative radial direction deformation (e_r) by heating and 2 groups of different random numbers can be seen in the table 2 and table 3, respectively. Some combinations of the groups of inital feasible point and the groups of random number can be seen in table 4. The values of F_{min} (FMN) periteration are seen in the Fig. n4, respectively. The curves which were drawn in case of having different feasible points only are curve 1, curve 2, curve 3 and curve 4, The curves, which were drawn in case of having same initial feasible points and different random numbers, are curve 4 and cruve 5. The curves, which were drawn in case of having same random

Table 5. The numerical results

Design variable \ Data	curve 1	curve 2	curve 3	curve 4	curve 5	curve 6
Optimum i_r	1.00000	1.00131	1.00001	1.00028	1.00076	1.00010
Optimum α	0.03942	0.04000	0.03990	0.03775	0.03669	0.03990
Optimum e_r	1.00156	1.00301	1.00160	1.00179	1.00236	1.00169
Shaft dia. d_h (mm)	5.00500	5.00501	5.00500	5.00500	5.00500	5.00500
Transmissible Torque (N-mm)	2431.70	2433.41	2431.71	2432.06	2342.69	2431.83
Design cost of assembly machine (cent)	25.37729	25.00737	25.06843	26.49487	27.26151	25.06908
Assembly cost (cent)	264.72812	265.11060	264.74017	264.78891	264.93970	264.76358
Min. objective func.	27.62706	27.24460	27.29200	28.84612	29.69002	27.29375

number and different initial feasible points, are curve 5 and curve 6. The curves, which were drawn in case of having different initial feasible points and different random number, are curve 6 and curve 4 (or curve 1, curve 2, curve 3)

(3.2) Results, Comparison and Discussion

The numerical results for this optimization problem can be seen in the table 5 and Fig. 4. As it can be seen in Fig. 4, the minimum values of F converges to one limit value. So, the optimum solutions were obtained.

As mentioned at the time when the objective is formulated, the importance for the objective funcion in each term (Torque, Assembly cost, Design cost of assembly machine) is equal to each other. So, since the value of objective function is lowest in curve 2, the data for design variable of curve 2 among all other data is best for applying for design. When the table 5 is examined, the value of i_r of curve 2 is largest. But, there is no sensitivity for the peg dia. and objective function and transmissible torque. As it can be seen in the Fig. 4, it needs many iteration for convergence for this objectve function.The used computer is VAX11/785.

6. Conclusion

Until now, the formulation of the objective function which governs the performance of the force fit assembly work and the treatment of finding the optimum values of the related design variables was carried out. The more detailed things are as follows.

1) This objective function was consisted of terms of the torque distransmissibility, the design cost of an automatic assembly machine and the assembly cost equation in consideration with the wedge and the required geometrical clearance when the force fit assembly was carried out on the thermal condition. So, the multicriteria optimization problem was treated.

2) The Complex method was redeveloped and used as the optimization algorithm for this problem.

The problems for further research are as follows.

(1) The consideration of dimensional tolerance in this problem.

(2) The consideration of dynamic process of this prolem.

(3) When the peg is working in compliant motion in this problem, adaptive or optimal control of peg under some constraints.

(4) The consideration of the case of assembly about non-rigid peg and non-rigid hole in this problem.

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