Simple Expressions of Electromagnetic Responses for Coaxial Electric and Magnetic Dipole-Dipole Systems

Hee Joon KIM
Department of Applied Geology, National Fisheries University of Pusan
Pusan 608-737, Korea

Introduction

Over 70 % of the earth's surface is covered by the seawater, and this immense area shows a largely unexplored resource base. Until recently, little was known on the ocean-floor environment which was assumed to be essentially barren. However, recent discoveries of manganese nodules and polymetallic sulfides on the sea floor have called great interest in the possibility of deep-sea mining. These deposits were mapped visually with submersibles and acoustically with high-precision bathometers. Although these methods have been able to examine surficial geology, they cannot estimate adequately the extent of deposits and the structure of regional geology in which they are found. Sea-floor resistivity mapping using the transient electromagnetic (EM) method is one of the few geophysical tools available for this purpose.

Since the electric conductivity of the sea floor is usually much less than that of the seawater above it, special attention is required in applying the EM method in the ocean. Generally, the EM field diffused through the sea floor reaches the receiver first. The signal diffused through the seawater arrives later, and ultimately the measured field approaches a static limit. Edwards and Chave (1986) showed that the transient coaxial electric dipole-dipole (ERER) system is particularly useful for determining sea-floor conductivity. Cheesman et al. (1987) found that the transient coaxial magnetic dipole-dipole (HRHR) system is also useful. These systems can detect the EM diffusion through the sea floor well. In other words, the EM responses for the systems vary significantly with the conductivity of the sea floor.

The transient EM response can be obtained from the frequency EM response, and the frequency response over a layered earth is expressed as Hankel transforms of order 0 and 1 (Cheesman et al., 1987). Therefore the two kinds of Hankel transforms should be evaluated numerically. In this note, modified expressions of the frequency responses for the ERER and HRHR systems over a layered earth is derived by transforming the Bessel function of order 0 to that of order 1.

Frequency em response

An analytic solution of frequency electric response for the ERER system can be found in Edwards and Chave (1986). In terms of Laplace variable s, the frequency response E(s) is

$$E(s)=\frac{j(s)}{2\pi}\left[F_{TM}^{E}(s)+F_{PM}^{E}(s)\right], \qquad (1)$$

where j(s) is the Laplace transform of electric dipole moment. The functions F_{TM}^{M} and F_{PM}^{E} are given by

$$F_{TM}^{E}=\frac{1}{2\sigma_0}\int_0^{\infty}(R_{TM}-1)u_0 \times[\lambda J_0(\lambda\rho)-J_1(\lambda\rho)/\rho]\,d\lambda, \qquad (2)$$

and

$$F_{PM}^{E}=-\frac{k_0^2}{2\sigma_0\rho}\int_0^{\infty}(R_{PM}+1)/u_0\,J_1(\lambda\rho)\,d\lambda, \qquad (3)$$

where

$u_0^2=\lambda_2+k_0^2$,
$k_0^2=i\omega\mu_0\sigma_{0,}$
ρ=horizontal separation of transmitter and receiver [m],
$\omega=2\pi f$, f>0 [Hz],
$\mu_0=4\pi \times 10^{-7}$ [H/m],

σ_0 = seawater conductivity [S/m].

The R_{TM} and R_{PM} are reflection coefficients for the independent toroidal (TM) and poloidal (PM) modes, respectively. For a double half-space model (a contact of seawater and sea floor), they are

$$R_{TM} = (u_0\sigma_1 - u_1\sigma_0)/(u_0\sigma_1 + u_1\sigma_0),$$

and

$$R_{PM} = (u_0 - u_1)/(u_0 + u_1),$$

where subscripts 0 and 1 indicate the seawater and the sea floor, respectively.

Any Hankel transform of order 0 can be transformed into its corresponding Hankel transform of order 1 (Das, 1984). Using this domain transformation, (2) becomes

$$F^{E}_{TM} = -\frac{1}{2\sigma_0\rho}\int_0^\infty \Big[\lambda u_0 \frac{dR_{TM}}{d\lambda} + \Big(\frac{\lambda^2}{u_0} + u_0\Big) \times (R_{TM} - 1)\Big] J_1(\lambda\rho)\, d\lambda, \quad (4)$$

Consequently,

$$E(s) = -\frac{j(s)}{4\pi\sigma_0\rho}\int_0^\infty \Big\{\lambda u_0 \frac{dR_{TM}}{d\lambda} + \frac{1}{u_0} \times [k_0^2(R_{TM} + R_{PM}) + 2\lambda^2(R_{TM} - 2)]\Big\} J_1(\lambda\rho)\, d\lambda, \quad (5)$$

Next, a frequency magnetic response for the HRHR system H(s) can be found in Cheesman et al (1987):

$$H(s) = \frac{m(s)}{4\pi}\,[F^{H}_{TM}(s) + F^{H}_{PM}(s)], \quad (6)$$

where m(s) is the Laplace transform of magnetic dipole moment,

$$F^{H}_{TM} = -\frac{k_0^2}{\rho}\int_0^\infty (R_{TM} + 1) / u_0 J_1(\lambda\rho)\, d\lambda, \quad (7)$$

and

$$F^{H}_{PM} = \int_0^\infty (R_{PM} - 1)\, u_0\, [\lambda J_0(\lambda\rho) - J_1(\lambda\rho)/\rho]\, d\lambda, \quad (8)$$

Using the domain transformation, (8) becomes

$$F^{H}_{PM} = -\frac{1}{\rho}\int_0^\infty \Big[\lambda u_0 \frac{dR_{PM}}{d\lambda} + \Big(\frac{\lambda^2}{u_0} + u_0\Big) \times (R_{PM} - 1)\Big] J_1(\lambda\rho)\, d\lambda, \quad (9)$$

Consequently,

$$H(s) = -\frac{m(s)}{4\pi\rho}\int_0^\infty \Big\{\lambda u_0 \frac{dR_{PM}}{d\lambda} + \frac{1}{u_0}[k_0^2(R_{PM} + R_{TM}) + 2\lambda^2(R_{PM} - 1)\Big\} J_1(\lambda\rho)\, d\lambda, \quad (10)$$

For the double half-space model, (4) and (9) become

$$F^{H}_{TM} = \frac{1}{\rho}\int_0^\infty \Big[\frac{\lambda^2(u_0^3\sigma_1 + u_1^3\sigma_0)}{u_0u_1(u_0\sigma_1 + u_1\sigma_0)^2} + \frac{u_0u_1}{u_0\sigma_1 + u_1\sigma_0}\Big] J_1(\lambda\rho)\, d\lambda, \quad (11)$$

and

$$F^{H}_{PM} = \frac{2}{\rho}\int_0^\infty \Big[\frac{\lambda^2(u_0 - u_1)}{u_1(u_0 + u_1)} + \frac{u_1(\lambda^2 + u_0^2)}{u_0(u_0 + u_1)}\Big] J_1(\lambda\rho)\, d\lambda, \quad (12)$$

Closed-form solution

For the double half-space model of $\sigma_1 >> \sigma_0$, closed-form solutions of (1) and (6) were found by Edwards and Chave (1986), and Cheesman et al. (1987), respectively. From these solutions, transient EM responses can be derived by using the Laplace transform. Since the closed-form solutions are only valid for $\sigma_1 >> \sigma_0$, for the other cases, a numerical procedure should be carried out. The numerical procedure for computing the transient EM response was described by Cheesman et al. (1987).

Between (11) and (12), which are derived in this note, only the poloidal part for the HRHR system (12) can be solved exactly. Rearranging (12) yields

$$F^{H}_{PM} = \frac{2}{(k_0^2 - k_1^2)\rho}\int_0^\infty \Big[3\lambda^2(u_1 - u_0) + (k_0^2u_1 - k_1^2u_0) + \lambda^2\Big(\frac{k_0^2}{u_1} - \frac{k_1^2}{u_0}\Big) + \lambda^4\Big(\frac{1}{u_1} - \frac{1}{u_0}\Big)\Big] J_1(\lambda\rho)\, d\lambda. \quad (13)$$

After tedious manipulation using standard integrals, one can obtain

$$F^{H}_{PM} = \frac{2}{\rho^5}\Big[\frac{k_0k_1\rho^3}{k_0 + k_1} + (-k_0\rho + 2)\Big(\rho^2 + 3\frac{k_0\rho + 2}{k_0^2 - k_1^2}\Big)e^{-k_0\rho} + (k_1\rho + 2)\Big(\rho^2 - 3\frac{k_1\rho + 2}{k_0^2 - k_1^2}\Big)e^{-k_1\rho}\Big] \quad (14)$$

While (14) agrees significantly with the expression (16) given by Cheesman et al (1987), (14) may be desirable because of its numerical robustness.

Concluding remarks

The ERER and HRHR system are capable of accurately measuring a relatively low conductivity of the sea floor in the presence of seawater (Edwards and Chave, 1986; Cheesman et al., 1987). In this note, modified expressions of the frequency EM responses for these systems over a layered earth are derived by transforming the Bessel function of order 0 to that of order 1. The modified expressions consist of only the Hankel transform of order 1 instead of a combination of the Hankel transforms of orders 0 and 1. This simpleness may be useful in the numerical evaluation of transient EM responses.

References

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