渦巻노즐의 理論分析 (II)
－噴霧角 및 流量係數에 關하여－

Theoretical Analysis on The Swirl Type Nozzle (II)
—The Spray Angle and The Discharge Coefficient—

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摘　　要

噴霧量 및 流量係數에 關한 많은 硏究가 實驗結果資料에 기초하여 遂行되어졌다. 그러나 供試體의 노즐의 特性에 따라서 流線의 特性이 다르므로 많은 實驗結果 및 解析이 서로 상치되는 점이 많다.

本 硏究에서는 理論分析結果가 實驗結果와 다소 다르더라도 노즐 設計의 實際應用面에서 必要한 노즐構造의 基本機能을 理論的으로 分析, 理解시키고저 한다.

理論分析의 結果는 다음과 같다.

空洞面積은 噴口徑, 渦室徑, 中子導溝徑 및 中子導溝角에 관계되고 있으며, 특히 中子導溝角이 空洞現象에 큰 영향을 미친다.

$$r_r^6 - 3r_o^2 r_r^4 + \left[3r_o^4 + \frac{r_c^4 r_o^2}{(r_c - r_g)^2 \tan^2\theta} \right] r_r^2 - r_o^6 = 0$$

半徑方向 힘의 要素로 因한 流量係數(C_t)의 算出式은 아래와 같다.

$$C_t = \left[1 - \left(\frac{r_r}{r_o}\right)^2 \right]^{3/2}$$

噴霧角(α)은 空洞半徑 및 噴口半徑에 依하여 變化된다.

$$\alpha = 2\tan^{-1}\left(\frac{r_r}{\sqrt{r_o^2 - r_r^2}}\right)$$

噴霧角은 특히 渦室流線角의 영향을 많이 받음을 시사하고 있다.

I. Introduction

Numerous fundamental studies[1,2,3,4] have been carried out in order to relate the spray angle and the discharge coefficient to the nozzle dimensions on the basis of empirical performance data.

Although considerable research efforts have been directed toward the assessment of the effects of the many variables resulting from the variation in the sprayed liquid properties and the variation in the hydraulics of flow and also the changes in the ambient air surroundings of the spray, the difficulties encountered theoretically or experimentally limit the scope of investigation.

The results of various workers and their interpretations of experimental evidence conflicted in many instances since the sets of nozzles being used in the various experimental

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investigations differed in their flow characteristics.

It is necessary to understand theoretically the basic functions of the structures of the nozzle for practical applications of all nozzle design methods, even though theoretical results are not completely in accord with empirical results.

II. Theories on the spray angle and the discharge coefficient

All theories in this study on the spray angle and the discharge coefficient have been achieved in a nozzle system on the assumption that the steady rotation of the main stream about the nozzle axis describes a flow path with constant speed on the basis of frictionless flow, although the flows lie generally in the transitional critical velocity calculated in Reynolds number.

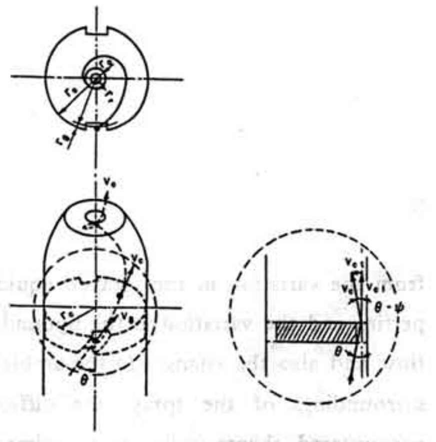

Fig. 1. Characteristics of flow in a swirl nozzle.

Bernoulli's equation derives the following equation on the same elevation level,

$$\frac{P_o}{\gamma} + Z_1 = \frac{P_g}{\gamma} + Z_1 + \frac{V_g^2}{2g} = Z_1 + \frac{V_o^2}{2g} \quad (1)$$

where, P_o = gage pressure

P_g = pressure in swirl groove

V_g = velocity in swirl groove

V_o = velocity at the tip of the orifice

γ = specific weight of liquid

g = acceleration of gravity

From Eq. (1), the velocity at the tip of the orifice can be arranged,

$$\frac{P_o}{\gamma} = \frac{V_o^2}{2g} \rightarrow \quad V_o = \left(\frac{2gP_o}{\gamma}\right)^{1/2} \quad (2)$$

The discharge Q at the orifice is expressed from the equation of continuity of flow;

$$Q = CA_oV_o = C\pi r_o^2 \sqrt{\frac{2gP_o}{\gamma}} \quad (3)$$

where, C = coefficient of discharge

A_o = orifice area

r_o = orifice radius

g = acceleration of gravity

Also the discharge Q is computed by using the actual flow area of the orifice and the longitudinal velocity $V_{o\ell}$ at the orifice,

$$Q = \int_{r_r}^{r_o} 2\pi r\, dr. V_{o\ell} = \pi V_{o\ell}\,(r_o^2 - r_r^2) \quad (4)$$

where, r_r = the radius of the cavitational area at the orifice

The principle of the natural vortex is used for the relationship between the radius of the orifice and the cavitational radius.

$$V_{ot} \cdot r_o = V_{rt} \cdot r_r$$

$$r_r = r_o \cdot V_{ot}/V_{rt} \qquad (5)$$

where, V_{ot} = tangential velocity of the flow at the orifice

V_{rt} = tangential velocity of the flow at the cavitational area

In Eq. (5), the total energy is assumed to be transformed into the tangential velocity V_{rt} on the cavitational area because the flow does not exist actually in this cavitational area.

$$V_{rt} = (\frac{2gP_o}{\gamma})^{1/2} \qquad (6)$$

Meanwhile, the main velocity V_o at the orifice is divided into the longitudinal velocity $V_{o\ell}$ and the tangential velocity V_{ot}.

$$\frac{P_o}{\gamma} = \frac{V_o^2}{2g} = \frac{V_{o\ell}^2}{2g} + \frac{V_{ot}^2}{2g}$$

$$V_{ot} = [2g(\frac{P_o}{\gamma} - \frac{V_{o\ell}^2}{2g})]^{1/2} \qquad (7)$$

Substituting Eqs. (6) and (7) into Eq. (5),

$$r_r = r_o \frac{1}{(\frac{2gP_o}{\gamma})^{1/2}} [2g(\frac{P_o}{\gamma} - \frac{V_{o\ell}^2}{2g})]^{1/2}$$

$$r_r^2 = r_o^2 \frac{1}{(\frac{2gP_o}{\gamma})} [2g(\frac{P_o}{\gamma} - \frac{V_{o\ell}^2}{2g})]$$

$$\frac{V_{o\ell}^2}{2g} = \frac{P_o}{\gamma}(1 - \frac{r_r^2}{r_o^2}) \qquad (8)$$

Also the equation of continuity of flow at the swirl chamber derives another discharge equation because the longitudinal velocity $V_{c\ell}$ is supposed to be the steady flow towards the orifice as a component of the main stream velocity in the swirl chamber.

$$Q = V_{c\ell} \cdot A_c$$

$$V_{c\ell} = Q / A_c \qquad (9)$$

where, $V_{c\ell}$ = the longitudinal velocity in the swirl chamber

$A_c = \pi r_c^2$: the cross-sectional area of the swirl chamber

Eq. (9) is reformed by putting Eq. (4),

$$V_{c\ell} = \frac{Q}{A_c} = \frac{1}{\pi r_c^2} \pi V_{o\ell}(r_o^2 - r_r^2)$$

$$= V_{o\ell}(\frac{r_o^2 - r_r^2}{r_c^2}) \qquad (10)$$

where, r_c = radius of the swirl chamber

The relationship between the longitudinal velocity $V_{c\ell}$ and the tangential velocity V_{ct} in the swirl chamber is recognized as shown in Figure 1.

$$\tan(\theta - \psi) = V_{ct} / V_{c\ell}$$

$$\rightarrow V_{ct} = V_{c\ell}\tan(\theta - \psi)$$

Substituting Eq. (10) into this equation,

$$V_{ct} = \tan(\theta - \psi)\, V_{o\ell}(\frac{r_o^2 - r_r^2}{r_c^2})$$

$$V_{ct} = \tan\theta\, V_{o\ell}(\frac{r_o^2 - r_r^2}{r_c^2}) \qquad (11)$$

where, θ = the swirl groove angle

In this equation, the frictional angle ψ may be neglected because it is very small in comparison with the swirl groove angle θ.

The conservation of angular momentum also derives the relation between the turning flow radius $(r_c - r_g)$ in the swirl chamber and the cavitational radius.

$$V_{rt} \cdot r_r = V_{ct}(r_c - r_g)$$

$$\therefore r_r = (r_c - r_g)\, V_{ct}/V_{rt}$$

Substituting Eq. (6) for V_{rt} and Eq. (11) for V_{ct},

$$r_r = (r_c - r_g)\,\frac{1}{\left(\frac{2gP_o}{\gamma}\right)^{1/2}}\tan\theta\, V_{o\ell}\left(\frac{r_o^2 - r_r^2}{r_c^2}\right) \quad (12)$$

Therefore Eqs. (8) and (12) can be used to find the cavitational radius r_r and the longitudinal velocity $V_{o\ell}$ at the orifice.

Substituting Eq. (8) into Eq. (12),

$$r_r = (r_c - r_g)\,\frac{1}{\left(\frac{2gP_o}{\gamma}\right)^{1/2}}\tan\theta\left[\frac{2gP_o}{\gamma}\left(1 - \frac{r_r^2}{r_o^2}\right)\right]^{1/2}\left(\frac{r_o^2 - r_r^2}{r_c^2}\right)$$

$$r_r^2 = \frac{(r_c - r_g)^2\tan^2\theta}{r_c^4 r_o^2}\,(r_o^2 - r_r^2)\,(r_o^4 - 2r_o^2 r_r^2 + r_r^4)\;\frac{(r_c - r_g)^2\tan^2\theta}{r_c^4 r_o^2}$$

$$(r_r^6 - 3r_o^2 r_r^4 + 3r_o^4 r_r^2 - r_o^6) + r_r^2 =$$

$$\frac{(r_c - r_g)^2\tan^2\theta\, r_o^4}{r_c^4}\; r_r^6 - 3r_o^2 r_r^4 + \left(3r_o^4 + \frac{r_c^4 r_o^2}{(r_c - r_g)^2\tan^2\theta}\right) x$$

$$r_r^2 - r_o^6 = 0 \quad (13)$$

Eq. (13) can be reformed into the cubic equation by putting $R = r_r^2$,

$$R^3 - 3r_o^2 R^2 + \left[3r_o^4 + \frac{r_c^4 r_o^2}{(r_c - r_g)^2\tan^2\theta}\right] x$$

$$R - r_o^6 = 0 \quad (13)$$

This equation can be utilized to predict the cavitational radius which is available in the calculations of the spray angle and the discharge coefficient by substituting only known factors such as the swirl groove angle or the swirl chamber flow angle, the orifice diameter, the swirl chamber diameter and the swirl groove diameter. The coefficient C_t of discharge due to the tangential velocity with the cavitational area can be derived by equating two Eqs (3) and (4).

$$C_t = \frac{V_{o\ell}}{r_o^2\left(\frac{2gP_o}{\gamma}\right)^{1/2}}\,(r_o^2 - r_r^2) \quad (14)$$

Eq. (14) is rewritten by substituting Eq.(8) for $V_{o\ell}$,

$$C_t = \frac{\left[\frac{2gP_o}{\gamma}\left(1 - \frac{r_r^2}{r_o^2}\right)\right]^{1/2}}{\left(\frac{2gP_o}{\gamma}\right)^{1/2}}\left[1 - \left(\frac{r_r}{r_o}\right)^2\right]$$

$$= [1 - (\frac{r_r}{r_o})^2]^{3/2} \qquad (15)$$

The coefficient C of the actual discharge must include both the coefficient C_t of the discharge due to the tangential velocity and the coefficient C_f of the frictional loss caused by the fluid passing a non-swirl groove nozzle.

$$C = C_t \cdot C_f \qquad (16)$$

The direction (spray angle) of the main stream at the orifice can be determined from th relationship between the tangential velocity V_{ot} and the longitudinal velocity $V_{o\ell}$.

$$\tan(\frac{\alpha}{2}) = \frac{V_{ot}}{V_{o\ell}} \qquad (17)$$

Substituting Eqs. (7) and (8) into Eq. (13),

$$\tan(\frac{\alpha}{2}) = \frac{(\frac{2gP_o}{\gamma})^{1/2}(\frac{r_r}{r_o})}{[\frac{2gP_o}{\gamma}(1 - \frac{r_r^2}{r_o^2})]^{1/2}} = \frac{r_r}{\sqrt{r_o^2 - r_r^2}}$$

$$\frac{\alpha}{2} = \tan^{-1}(\frac{r_r}{\sqrt{r_o^2 - r_r^2}})$$

$$\alpha = 2\tan^{-1}(\frac{r_r}{\sqrt{r_o^2 - r_r^2}}) \qquad (18)$$

The direction of the main stream is determined on the base of its central line and then may be slightly small in comparison with the actual spray angle, because the spray angle α is influenced with the diffusion to cause its enlargement a little more at the right front of the orifice.

Eq. (13) to predict the cavitational radius is very complicated in its calculation as a higher degree equation and the discharge is erratic due to the coefficient C of the actual discharge.

Therefore, if Eq.(4) can be replaced into an approximate discharge equation; $Q = \pi V_o(r_o^2 - r_r^2)$ in order to calculate the cavitational radius, a quadratic equation can be derived simply by using Eqs. (5), (6), (11) and the relation;

$$V_{ot}r_o = V_{ct}(r_c - r_g)$$

$$V_{ot} = V_{ct}(r_c - r_g) / r_o$$

Consequently,

$$r_r = r_o \frac{1}{(\frac{2gP_o}{\gamma})^{1/2}} \tan\theta (\frac{2gP_o}{\gamma})^{1/2} \times (\frac{r_o^2 - r_r^2}{r_c^2})(\frac{r_c - r_g}{r_o})$$

$$r_r = \tan\theta \; (\frac{r_o^2 - r_r^2}{r_c^2})(r_c - r_g)$$

$$r_r^2 \tan\theta + (\frac{r_c^2}{r_c - r_g})r_r - r_o^2 \tan\theta = 0$$

$$r_r = \frac{[(\frac{r_c^2}{r_c - r_g})^2 + 4r_o^2 \tan^2\theta]^{1/2} - (\frac{r_c^2}{r_c - r_g})}{2\tan\theta} \qquad (19)$$

Also another equation so as to predict the spray angle can be derived from the relationship between the tangential velocity V_{ot} at orifice and the main velocity V_o which is calculated from the approximate discharge equation:

$$Q = \pi V_o(r_o^2 - r_r^2)$$

$$\alpha = 2\left\{[90^\circ - [\cos^{-1}(\tan\theta)\left(\frac{r_o^2 - r_r^2}{r_c^2}\right) \times \left(\frac{r_c - r_g}{r_o}\right)]\right\} \quad \text{——(20)}$$

Two Eqs. (19) and (20) indicate obviously all factors to be concerned with the cavitational radius and the spray angle.

III. Applications of theories and discussion

Illustrative example 1 to predict the spray angle and the discharge coefficient under the conditions,

swirl chamber radius, $r_c = 1.6$ mm
swirl groove radius, $r_g = 0.21$ mm
orifice radius, $r_o = 0.6$ mm
swirl groove angle, $\theta = 51^\circ$

[Solution 1]

$$r_r^6 - 3r_o^2 r_r^4 + \left(3r_o^4 + \frac{r_c^4 r_o^2}{(r_c - r_g)^2 \tan^2\theta}\right) r_r^2 - r_o^6 = 0 \quad \text{——(13)}$$

$$R^3 - 3r_o^2 R^2 + \left[3r_o^4 + \frac{r_c^4 r_o^2}{(r_c - r_g)^2 \tan^2\theta}\right] R - r_o^6 = 0 \quad \text{——(13')}$$

$a = 1$

$b = -3r_o^2 = -1.08$

$c = \left[3r_o^4 + \frac{r_c^4 r_o^2}{(r_c - r_g)^2 \tan^2\theta}\right] = 1.1895$

$d = -r_o^6 = -0.046656$

$2q = (2b^3/27) - (bc/3a^2) + (d/a) = 0.288253$

$3p = (3ac - b^2) / 3a^2 = 0.8007$

$u = \sqrt[3]{-q + \sqrt{q^2 + p^3}} = 0.380985$

$v = \sqrt[3]{+q + \sqrt{q^2 + p^3}} = -0.7003644$

$y_1 = u + v = -0.319378 = R + (b / 3a)$

$R = 0.040622$

$r_r = R^{1/2} = 0.20155$ mm

The spray angle, α;

$$\alpha = 2\tan^{-1}\left(\frac{r_r}{\sqrt{r_o^2 - r_r^2}}\right)$$

$$= 2\tan^{-1}\left[\frac{0.20155}{\sqrt{(0.6)^2 - (0.20155)^2}}\right]$$

$$= 2\,(20^\circ\,10')$$

$$= 40^\circ 20'$$

The discharge coefficient, C_t;

$$C_t = \left[1 - \left(\frac{r_r}{r_o}\right)^2\right]^{3/2}$$

$$= \left[1 - \left(\frac{0.20155}{0.6}\right)^2\right]^{3/2}$$

$$= 0.8356$$

[Solution 2]

$$r_r = \frac{\left[\left(\frac{r_c^2}{r_c - r_g}\right)^2 + 4r_o^2\tan^2\theta\right]^{1/2} - \left(\frac{r_c^2}{r_c - r_g}\right)}{2\tan\theta}$$

$$r_r = \frac{[(\frac{1.6^2}{1.6-0.21})^2 + 4(0.6)^2(\tan 51)^2]^{1/2} - (\frac{1.6^2}{1.6-0.21})}{2\tan 51^\circ}$$

$= 0.21142\text{mm}$

$$\alpha = 2\tan^{-1}[\frac{0.21142}{(0.6)^2 - (0.21142)^2}]$$

$= 2\,(20^\circ 30')$

$= 41^\circ$

$$C_t = [1 - (\frac{0.21142}{0.6})^2]^{3/2}$$

$= 0.8197$

Illustrative example 2 to determine the spray angle α under the same conditions of example 1 with exemption of $\theta = 40^\circ$.

[Solution]

According to Eq. (19),

$$r_r = \frac{2.0989 - 1.8417}{2\tan 40} = 0.15325\text{mm}$$

$$\alpha = 2\tan^{-1}[\frac{0.15325}{(0.6)^2 - (0.15325)^2}]$$

$= 29^\circ 40'$

The cavitational radius and the spray angle could be calculated simply by using only known factors in the derived Eqs. (13) and (19).

Two cavitational radiuses were nearly the same results from two Eqs. (13) and (19), therefore Eq. (19) was easy in its utilization for the approximate calculation of the cavitational radius.

If the swirl groove angle is altered according to characteristics of the swirl groove or the swirl chamber, the finally changed angle must be substituted in the calculation of the cavitational radius.

The spray angles resulting from the examples 1 and 2 were evaluated to be small slightly in comparison with the actual spray angles, because they were calculated by only the mechanical factors involved in the nozzle system on the basis of the central line of the main stream.

If the physical properties of the liquid sprayed were considered the expansion theory together, the predicted spray angle would be enlarged slightly more until it would be likely to approach the real one.

Illustrative example 3 to describe relationships between the swirl groove angle and the spray angle or the cavitation coefficient (ratio of r_r / r_o) or the discharge coefficient in range of the swirl groove angle from 0° to 90° under conditions:

swirl chamber radius, $r_c = 1.6$ mm
orifice radius, $r_o = 0.6$ mm
swirl groove radius, $r_g = 0.21$ mm

[Solution]

Eqs, (13), (15),(18) and (19) were used and the results were plotted as shown in Figure 2.

The spray angle and the cavitation coefficient increased together with increase of the

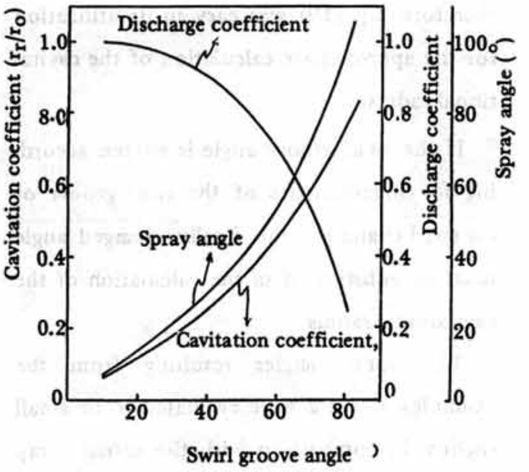

Fig. 2. Relationships between the swirl groove angle and the spray angle or the cavitation coefficient or the discharge coefficient.

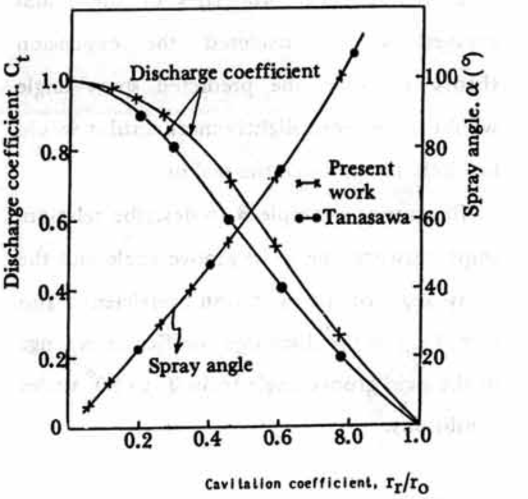

Fig. 3. Variations of C_t and α against (r_r/r_o).

swirl groove angle, while the discharge coefficient decreased with increase of the swirl groove angle, since the increase of the swirl groove angle caused directly the increasing of the tangential velocity in balance of decrease of the longitudinal velocity.

Also the cavitation coefficient caused the spray angle to enlarge greatly, but simultaneously it caused the discharge coefficient to shorten highly.

It is obvious that the swirl groove angle has a great effect on the discharge coefficient and the spray angle.

To compare the results of the present work with the previous work of Tanasawa(1951), the discharge coefficient and the spray angle of his studies were plotted against the cavitation coefficient together in Figure 3.

The present works seemed to have nearly the same tendencies with the results of Tanasawa's works, although the discharge coefficient of the present works was slightly higher at middle portions than that of Tanasawa's works.

IV. Conclusion

The cavitational radius could be readily computed with known factors so as to be adapted in the calculations of the spray angle and the discharge coefficient.

$$r_r^5 - 3r_o^2 r_r^4 + (3r_o^4 + \frac{r_c^4 r_o^2}{(r_c - r_g)^2 \tan^2\theta}) \times$$

$$r_r^2 - r_o^6 = 0$$

The discharge coefficient C_t only concerned to the tangential force component with the cavitational area was expressed as follows;

$$C_t = [1 - (\frac{r_r}{r_o})^2]^{3/2}$$

The spray angle was changeable according to the cavitational radius and the orifice radius,

$$\alpha = 2 \tan^{-1}\left(\frac{r_r}{\sqrt{r_o^2 - r_r^2}}\right)$$

This equation implies that the swirl chamber flow angle has effect on the spray angle more greatly than other factors.

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