SEVERAL CONTINUITIES OF FUNCTIONS ON CONVERGENCE SPACES

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1. Introduction

The purpose of the present note is to introduce some new classes of functions on convergence spaces. These classes contain the class of continuous functions. For topological spaces, Levine, Noiri, Papp, Reilly, Vamanamurthy, and Singal ([8], [9], [10], [11], [12] and [13]) investigated some properties of these classes.

Kent ([4]) introduced a convergence structure on a nonempty set. Kent and Richardson ([5], [6], and [7]) investigated some properties of convergence spaces.

For a nonempty set $S$, $F(S)$ denotes the set of all filters and $P(S)$ the set of all subsets of $S$. For each $s \in S$, $\hat{s}$ is the principal ultrafilter containing $\{s\}$.

A convergence structure on $S$ is defined to be a function $\varphi$ from $F(S)$ into $P(S)$, satisfying the following conditions:

1. for each $s \in S$, $s \in \varphi(\hat{s})$ ;
2. if $\phi$ and $\psi$ are in $F(S)$ and $\psi \supseteq \phi$, then $q(\psi) \supseteq q(\phi)$ ;
3. if $s \in q(\phi)$, then $s \in q(\psi \cap \hat{s})$.

The pair $(S, \varphi)$ is called a convergence space. If $s \in \varphi(\phi)$, then we say that $\phi$ $\varphi$-converges to $s$. The filter $V_{\varphi}(s)$ obtained by intersecting all filters which $\varphi$-converges to $s$ is called the $\varphi$-neighborhood filter at $s$. If $V_{\varphi}(s)$ $\varphi$-converges to $s$ for each $s \in S$, then $(S, \varphi)$ is called a pretopological space.

Let $C(S)$ be the set of all convergence structures on $S$, partially ordered as follows: $q_1 \leq q_2$ if and only if $q_1(\phi) \supseteq q_2(\phi)$, for all $\phi \in F(S)$.

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The set function $\Gamma_q:P(S)\rightarrow P(S)$ is defined on all subset $A\subseteq S$ by

$$\Gamma_q(A) = \{s \in S \mid \text{there exists an ultrafilter } \psi \text{ } q\text{-converging to } s \text{ with } A \in \psi\}.$$  

$\Gamma_q$ is a closure operator in the topological sense, except idempotency.

The set function $I_q:P(S)\rightarrow P(S)$ defined recursively for every subset $A$ of $S$ as follows:

$$I_q(A) = \{s \in A \mid A \in V_q(s)\}$$

$I_q$ has all of the properties of a topological interior operator except idempotency.

If $\psi$ is any filter on $S$, $\Gamma_q(\psi)(I_q(\psi))$ is the filter generated by $\{\Gamma_q(F) \mid F \in \psi\}$.

A function $f$, mapping a convergence space $(S,q)$ onto a convergence space $(T,p)$, is said to be continuous if $f(\psi) \text{- } p\text{-converges to } f(s)$ whenever $\psi \text{ } q\text{-converges to } s$.

Let $X$ be a nonempty set, $(X,\alpha,q_\lambda)$ a convergence space, and for each $\lambda \in \Lambda$, $f_\lambda$ a function $X$ onto $(X,\alpha,q_\lambda)$. The initial convergence structure $q$ on $X$ induced by the family $\{f_\lambda | \lambda \in \Lambda\}$ is defined to be a function from $F(X)$ into $P(X)$ satisfying the following condition:

for each $x \in X$ and $\psi \in F(X)$, $x \in q(\psi)$ if and only if $f_\lambda(\psi)$ $q_\lambda\text{-converges to } f_\lambda(x)$ for each $\lambda \in \Lambda$.

Throughout the present paper, spaces always mean convergence spaces and $(\prod_{\lambda \in \Lambda} X, q^\Lambda)$ the initial convergence space induced by the family $\{P_\lambda | \lambda \in \Lambda\}$, $P_\lambda: \prod_{\lambda \in \Lambda} X_\lambda \rightarrow X_\lambda$ is the canonical projection.

$(\prod_{\lambda \in \Lambda} X, q^\Lambda)$ is called the product space of a family $\{(X_\lambda,q_\lambda) | \lambda \in \Lambda\}$ of convergence spaces.

A space $(S,q)$ is said to be regular if $\Gamma_q(\psi)$ $q\text{-converges to } s$ whenever $\psi$ $q\text{-converges to } s$.

2. $\theta$-continuous functions

A function $f$ from a space $(S,q)$ onto another space $(T,p)$ is said to be $\theta$-continuous at a point $s$ in $S$ if $I_\theta(V_\psi(f(s))) \subseteq f(I_\psi(\psi))$ whenever $\psi$ $q\text{-converges to } s$ $f$ is said to be $\theta$-continuous if $f$ is $\theta$-continuous at each $s \in S$.  

Clearly, continuous functions are always \( \theta \)-continuous.

The following properties are easily verified.

**Theorem 2.1.** Let \((T, p)\) be a pretopological space. If \( f: (S, q) \to (T, p) \) and \( g: (T, p) \to (R, r) \) are \( \theta \)-continuous, then \( g \circ f \) is so.

**Theorem 2.2.** Let \( f: (S, q) \to (T, p) \) be \( \theta \)-continuous. If \((T, p)\) is a pretopological regular space, then \( f \) is continuous.

**Lemma 2.3.** Let \( (\bigsqcup_{\lambda \in \Lambda} X_{\lambda}, q') \) be the product space of a family \( \{(X_{\lambda}, q_{\lambda}) | \lambda \in \Lambda\} \) of spaces. Then for each \( x = (x_{\lambda})_{\lambda \in \Lambda} \in \bigsqcup_{\lambda \in \Lambda} X_{\lambda}, \)

1. \( \prod_{\lambda \in \Lambda} V_{q_{\lambda}}(x_{\lambda}) \subseteq V_{q'}(x), \)
2. \( P_{x} (V_{q'}(x)) = V_{q_{\lambda}}(x_{\lambda}) \) for each \( \lambda \in \Lambda \) where \( P_{x} \) is the canonical projection.

**Proof.** (1) Let \( F \) be any element of \( \prod_{\lambda \in \Lambda} V_{q_{\lambda}}(x_{\lambda}) \). Then for each \( \lambda \in \Lambda \), there exists \( F_{\lambda} \in V_{q_{\lambda}}(x_{\lambda}) \) such that \( \prod_{\lambda \in \Lambda} F_{\lambda} \subseteq F \). Suppose that \( \prod_{\lambda \in \Lambda} F_{\lambda} \not\subseteq V_{q'}(x) \), then there exists a filter \( \psi \) \( q' \)-converging to \( x \) such that \( \prod_{\lambda \in \Lambda} F_{\lambda} \not\in \psi \). Therefore \( P_{\lambda}(\psi) \) \( q_{\lambda} \)-converges to \( x_{\lambda} \) for each \( \lambda \in \Lambda \) and there is a \( \mu \in \Lambda \) such that \( F_{\mu} \not\in P_{\mu}(\psi) \). But \( F_{\mu} \in V_{q_{\lambda}}(x_{\lambda}) \). Thus \( F \not\in V_{q'}(x) \).

(2) Since

\[
\prod_{\lambda \in \Lambda} V_{q_{\lambda}}(x_{\lambda}) \subseteq V_{q'}(x), \quad V_{q_{\lambda}}(x_{\lambda}) \subseteq P_{\lambda}(V_{q'}(x))
\]

for each \( \lambda \in \Lambda \). Let \( G_{\mu} \) be any element of \( P_{\lambda}(V_{q'}(x_{\lambda})) \) for each \( \mu \in \Lambda \), then there is \( F \in V_{q'}(x) \) such that \( P_{\mu}(F) \subseteq G_{\mu} \). Let \( \psi_{\lambda} \) be a filter which \( q_{\lambda} \)-converges to \( x_{\lambda} \) for each \( \lambda \neq \mu \). Then \( \prod_{\lambda \in \Lambda} \psi_{\lambda} \) \( q' \)-converges to \( x = (x_{\lambda})_{\lambda \in \Lambda} \). Since \( F \in V_{q'}(x), \) \( F \subseteq \prod_{\lambda \in \Lambda} G_{\lambda} \). Thus \( P_{\mu}(F) \in \psi_{\mu} \) and \( G_{\mu} \subseteq \psi_{\mu} \). Therefore \( G_{\mu} \subseteq V_{q_{\mu}}(x_{\mu}) \).

**Lemma 2.4** ([7]) If \( f: (S, q) \to (T, p) \) is continuous and \( A \subseteq S \), then \( f(A) \subseteq f(A) \).

**Theorem 2.5.** If \( f: (S, q) \to (\bigsqcup_{\lambda \in \Lambda} X_{\lambda}, q') \) is \( \theta \)-continuous, then \( P_{\lambda} \circ f \) is \( \theta \)-continuous for each \( \lambda \in \Lambda \).

**Proof.** For each \( s \in S \), let \( \psi \) \( q' \)-converge to \( s \), then, by Lemma 2.3 and 2.4,
\[ \Gamma_\alpha (V_\psi (f(s))) \subseteq f(\Gamma_\alpha (\psi)), \]
\[ \Gamma_\alpha (V_\alpha (P_\lambda f(s))) = \Gamma_\alpha (P_\lambda (V_\psi (f(s)))) \]
\[ \subseteq P_\lambda (\Gamma_\alpha (V_\psi (f(s)))) \subseteq P_\lambda (f(\Gamma_\alpha (\psi))). \]

Thus \( P_\lambda \cdot f \) is \( \theta \)-continuous.

**Corollary 2.6.** Let \( f:(S,q) \rightarrow (T,p) \) be a function and \( g:(S,q) \rightarrow (S \times T, r) \) given by \( g(s) = (s, f(s)) \), be its graph, where \( r \) is the product convergence structure of \( q \) and \( p \). If \( g \) is \( \theta \)-continuous, then \( f \) is \( \theta \)-continuous.

A function \( f \) from a space \( (S,q) \) onto a space \( (T,p) \) is said to be strongly \( \theta \)-continuous at a point \( s \) in \( S \) if \( V_\alpha (f(s)) \subseteq f(\Gamma_\alpha (\psi)) \) whenever \( \psi \) \( q \)-converges to \( s \). \( f \) is called strongly \( \theta \)-continuous if it is strongly \( \theta \)-continuous at each \( s \) in \( S \).

Strongly \( \theta \)-continuous functions are always \( \theta \)-continuous.

**Lemma 2.7** ([5]). If \( f:(S,q) \rightarrow (T,p) \) is continuous, then \( f(V_\alpha (s)) \subseteq V_\psi (f(s)) \) for all \( s \) in \( S \).

**Theorem 2.8.** Let \( f:(S,q) \rightarrow (T,p) \) be strongly \( \theta \)-continuous.

1. If \( (T,p) \) is a pretopological space, then \( f \) is continuous.
2. If \( g:(T,p) \rightarrow (R,r) \) is continuous, then \( g \cdot f \) is strongly \( \theta \)-continuous.

**Proof.** (1) Clear.

(2) Let \( \psi \) \( q \)-converge to \( s \), then \( V_\alpha (f(s)) \subseteq f(\Gamma_\alpha (\psi)) \) and \( g(V_\alpha (f(s))) \subseteq g(f(\Gamma_\alpha (\psi))) \). Since \( g \) is continuous, by Lemma 2.7, \( V_\alpha (g(f(s))) \subseteq g(V_\alpha (f(s))) \). Thus \( g \cdot f \) is strongly \( \theta \)-continuous.

**Theorem 2.9.** Let \( f:(S,q) \rightarrow (\bigcup_{\lambda \in \Lambda} X_\lambda, q') \) be a function. Then \( f \) is strongly \( \theta \)-continuous if and only if \( P_\lambda \cdot f \) is strongly \( \theta \)-continuous for each \( \lambda \) in \( \Lambda \).

**Proof.** Let \( \psi \) \( q \)-converge to \( s \), then \( V_\alpha (f(s)) \subseteq f(\Gamma_\alpha (\psi)) \). For each \( \lambda \) in \( \Lambda \), since \( P_\lambda \) is continuous,

\[ V_\alpha (P_\lambda (f(s))) \subseteq P_\lambda (V_\alpha (f(s))) \subseteq P_\lambda (f(\Gamma_\alpha (\psi))). \]

Thus \( P_\lambda \cdot f \) is strongly \( \theta \)-continuous.
Conversely, let \( P_\lambda \cdot f \) be strongly \( \theta \)-continuous for each \( \lambda \in \Lambda \) and \( \psi \) \( q \)-converges to \( s \), then \( V_{\theta, \lambda} (P_\lambda \cdot f (s)) \subseteq P_\lambda (f (\Gamma_\psi (s))) \). Suppose that there is \( N \in \mathcal{V}_q \cdot (f (s)) \) such that \( N \not\in f (\Gamma_\psi (s)) \). Then for each \( F \in \psi \), since \( f (\Gamma_\psi (F)) \cap N \), there exists \( \mu \in \Lambda \) such that \( P_\mu (f (\Gamma_\psi (F))) \subseteq P_\mu (N) \). And

\[
P_\lambda (V_{\theta, \mu} (f (s))) = V_{\theta, \mu} (P_\lambda (f (s))),
\]

\[
P_\lambda (N) \in V_{\theta, \mu} (P_\lambda (f (s))).
\]

Therefore \( P_\lambda (N) \in P_\lambda (f (\Gamma_\psi (s))) \). Thus there exists \( F \in \psi \) such that \( P_\lambda (f (\Gamma_\psi (F))) \subseteq P_\lambda (N) \).

**Corollary 2.10.** Let \( f : (S, q) \to (T, r) \) be a function and \( g : (S, q) \to (S \times T, r) \) given by \( g(s) = (s, f(s)) \), be its graph. Then \( f \) is strongly \( \theta \)-continuous if and only if \( g \) is strongly \( \theta \)-continuous.

Let \( \prod_{\lambda \in \Lambda} X_\lambda, q' \) (resp. \( \prod_{\lambda \in \Lambda} Y_\Lambda, p' \)) be the product space of a family \( \{ (X_\lambda, q_\lambda) \mid \lambda \in \Lambda \} \) (resp. \( \{ (Y_\lambda, p_\lambda) \mid \lambda \in \Lambda \} \) of spaces. For each \( \lambda \in \Lambda \), \( f_\lambda : (X_\lambda, q_\lambda) \to (Y_\Lambda, p_\lambda) \) is a function, \( f_\lambda (\prod_{\lambda \in \Lambda} X_\lambda, q') \to (\prod_{\lambda \in \Lambda} Y_\Lambda, p') \) is the function defined by \( f(x) = (f_\lambda(x_\lambda))_{\lambda \in \Lambda} \) for each \( x = (x_\lambda)_{\lambda \in \Lambda} \in \prod_{\lambda \in \Lambda} X_\lambda \).

By Theorem 2.9, the following corollary is easily obtained.

**Corollary 2.11.** \( f \) is strongly \( \theta \)-continuous if and only if \( Q_\lambda \cdot f \) is strongly \( \theta \)-continuous for each \( \lambda \in \Lambda \), where \( Q_\lambda : \prod_{\lambda \in \Lambda} Y_\lambda \to Y_\Lambda \) is the canonical projection.

**Lemma 2.12.** \( \Gamma_\psi (\prod_{\lambda \in \Lambda} F_\lambda) = \prod_{\lambda \in \Lambda} \Gamma_{\psi_\lambda} (F_\lambda) \).

**Proof.** Let \( x = (x_\lambda)_{\lambda \in \Lambda} \in \prod_{\lambda \in \Lambda} \Gamma_{\psi_\lambda} (F_\lambda) \), then \( x_\lambda \in \Gamma_{\psi_\lambda} (F_\lambda) \) for each \( \lambda \in \Lambda \). There exists an ultrafilter \( \psi_\lambda \) on \( X_\lambda \) such that \( \psi_\lambda q_\lambda \)-converges to \( x_\lambda \) and \( F_\lambda \in Q_\lambda \). Since \( \prod_{\lambda \in \Lambda} \psi_\lambda \) is an ultrafilter containing \( \prod_{\lambda \in \Lambda} F_\lambda \) and \( \prod_{\lambda \in \Lambda} \psi_\lambda q' \)-converges to \( x = (x_\lambda)_{\lambda \in \Lambda} \in \prod_{\lambda \in \Lambda} X_\lambda \).

Conversely, let \( x = (x_\lambda)_{\lambda \in \Lambda} \in \prod_{\lambda \in \Lambda} F_\lambda \), then there exists an ultrafilter \( \psi \) on \( \prod_{\lambda \in \Lambda} X_\lambda \) such that \( \psi q' \)-converges to \( x \) and \( \prod_{\lambda \in \Lambda} F_\lambda \in \psi \). For each \( \lambda \in \Lambda \), \( P_\lambda (\psi) \) is an ultrafilter, \( P_\lambda (\psi) q_\lambda \)-converges to \( x_\lambda \), and \( P_\lambda (\prod_{\lambda \in \Lambda} F_\lambda) = F_\lambda \in P_\lambda (\psi) \). Therefore \( x_\lambda \in \Gamma_{\psi_\lambda} (F_\lambda) \).
Theorem 2.13. \( f_{\lambda} \) is strongly \( \theta \)-continuous if and only if \( f_{\lambda} \cdot P_{\lambda} \) is strongly \( \theta \)-continuous for each \( \lambda \in \Lambda \).

**Proof.** Let \( \psi \) \( q' \)-converge to \( x=(x_{\lambda})_{\lambda \in \Lambda} \), then \( P_{\lambda} (\psi) \) \( q_{\lambda} \)-converges to \( x_{\lambda} \) for each \( \lambda \in \Lambda \). Since \( f_{\lambda} \) is strongly \( \theta \)-continuous and \( P_{\lambda} \) is continuous,

\[
V_{\rho_{\lambda}} (f_{\lambda} (P_{\lambda} (x))) = V_{\rho_{\lambda}} (f_{\lambda} (x_{\lambda})) \subset f_{\lambda} (P_{\lambda} (\Gamma_{q_{\lambda}} (P_{\lambda} (\psi)))) \\
\subset f_{\lambda} (P_{\lambda} (\Gamma_{q} (\psi)))
\]

Thus \( f_{\lambda} \cdot P_{\lambda} \) is strongly \( \theta \)-continuous.

Conversely, let \( \phi_{\lambda} \) \( q_{\lambda} \)-converge to \( x_{\lambda} \) for each \( \lambda \in \Lambda \), then \( \Pi_{\lambda} \phi_{\lambda} \) \( q' \)-converges to \( x=(x_{\lambda})_{\lambda \in \Lambda} \). By Lemma 2.12,

\[
V_{\rho_{\lambda}} (f_{\lambda} (x_{\lambda})) = V_{\rho_{\lambda}} (f_{\lambda} (P_{\lambda} (x))) \subset f_{\lambda} (P_{\lambda} (\Gamma_{q'_{\lambda}} (\Pi_{\lambda} \phi_{\lambda})))) \\
= f_{\lambda} (P_{\lambda} (\Pi_{\lambda} \Gamma_{q_{\lambda}} (\phi_{\lambda}))) = f_{\lambda} (\Gamma_{q} (\phi))
\]

Thus \( f_{\lambda} \) is strongly \( \theta \)-continuous.

**Corollary 2.14.** \( f \) is strongly \( \theta \)-continuous if and only if \( f_{\lambda} \) is strongly \( \theta \)-continuous for each \( \lambda \in \Lambda \).

**Theorem 2.15.** Let \( f : (S,q) \rightarrow (T,p) \) be continuous. If \( (S,q) \) is a regular space, then \( f \) is strongly \( \theta \)-continuous.

A space \( (S,q) \) is said to be semi-regular if \( I_{q} (\Gamma_{q} (\psi)) \) \( q \)-converges to \( s \) whenever \( \psi \) \( q \)-converges to \( s \). Clearly, every regular space is semi-regular.

A function \( f : (S,q) \rightarrow (T,p) \) is said to be almost strongly \( \theta \)-continuous at a point \( s \in S \) if \( I_{p} (\Gamma_{p} (V_{\rho} (f(s)))) \subset f(\Gamma_{q} (\psi)) \) whenever \( \psi \) \( q \)-converges to \( s \). \( f \) is called almost strongly \( \theta \)-continuous if \( f \) is almost strongly \( \theta \)-continuous at each \( s \in S \).

Almost strongly \( \theta \)-continuous functions are always \( \theta \)-continuous.

The following theorem is easily consequence of the above definition.
Theorem 2.16. Let \((T, p)\) be a pretopological semi-regular space. If \(f: (S, q) \to (T, p)\) is almost strongly \(\theta\)-continuous, then \(f\) is strongly \(\theta\)-continuous.

3. \(\delta\)-continuous functions and the others functions

A function \(f\) from a space \((S, q)\) onto another space \((T, p)\) is said to be \(\delta\)-continuous (resp. almost continuous, weakly continuous) at \(s \in S\) if \(I_p (I_q (V_p (f(s)))) \subseteq f(I_q (\{f(s)\}))\) (resp. \(I_p (I_q (V_p (f(s)))) \subseteq f(\{f(s)\})\)) whenever \(\psi\) \(q\)-converges to \(s\). \(f\) is called \(\delta\)-continuous (resp. almost continuous, weakly continuous) if \(f\) is \(\delta\)-continuous (resp. almost continuous, weakly continuous) at each \(s \in S\).

By above definitions, the following statements are easily obtained.

Theorem 3.1. (1) Almost strongly \(\theta\)-continuous functions are always \(\delta\)-continuous and almost continuous
(2) Almost continuous functions are always weakly continuous
(3) \(\theta\)-continuous implies weakly continuous.

Theorem 3.2. Let \(f: (S, q) \to (T, q)\) and \(g: (T, p) \to (R, r)\) be two \(\delta\)-continuous. If \((T, p)\) is a pretopological semi-regular space, then \(g \cdot f\) is \(\delta\)-continuous.

Proof. Let \(\psi\) \(q\)-converge to \(s\), then \(I_p (I_q (V_p (f(s)))) \subseteq f(I_q (\{f(s)\}))\). Since \((T, p)\) is a pretopological semi-regular space, \(I_p (I_q (V_p (f(s))))\) \(p\)-converges to \(f(s)\). Thus
\[
I_r (I_q (V_r (g(f(s))))) \subseteq g(I_p (I_q (V_p (f(s)))))
\subset_R g(I_q (\{f(s)\}))
\]

Theorem 3.3. Let \((T, p)\) be a pretopological space. If \(f: (S, q) \to (T, p)\) is strongly \(\theta\)-continuous and \(g: (T, p) \to (R, r)\) is almost continuous, then \(g \cdot f\) is almost strongly \(\theta\)-continuous.

Proof. Let \(\psi\) \(q\)-converge to \(s\). Since \((T, p)\) is a pretopological space and \(f\) is strongly \(\theta\)-continuous,
\[
I_r (I_q (V_r (g(f(s))))) \subseteq g(V_p (f(s))) \subseteq g(I_q (\{f(s)\}))
\]
by almost continuity of \( g \). Thus \( g \cdot f \) is almost strongly \( \theta \)-continuous.

**Theorem 3.4.** Let \( (S,q) \) be a semi-regular space. If \( f:(S,q)\rightarrow(T,p) \) is almost continuous, then \( f \) is \( \delta \)-continuous.

**Proof.** Let \( \psi \) \( q \)-converge to \( s \), then \( I_q(\Gamma_q(\psi)) \) \( q \)-converges to \( s \).

A function \( f:(S,q)\rightarrow(T,p) \) is said to be super continuous at \( s \in S \) if \( V_{\psi}(f(s)) \subseteq f(I_q(\Gamma_q(\psi))) \) whenever \( \psi \) \( q \)-converges to \( s \). \( f \) is called super continuous if \( f \) is super continuous at each \( s \in S \).

The following properties are easily obtained by definitions.

**Theorem 3.5.** (1) Let \( (T,p) \) be a pretopological space. If \( f:(S,q)\rightarrow(T,p) \) is super continuous and \( g:(T,p)\rightarrow(R,r) \) is continuous, then \( g \cdot f \) is super continuous.

(2) If \( f:(S,q)\rightarrow(\prod_{\lambda \in \Lambda} X_{\lambda},q') \) is super continuous, then \( P_\lambda \cdot f \) is also super continuous for each \( \lambda \in \Lambda \).

(3) Let \( f:(S,q)\rightarrow(T,p) \) be a function and \( g:(S,q)\rightarrow(S \times T,r) \) given by \( g(s) = (s,f(s)) \), be its graph. If \( g \) is super continuous, then \( f \) is super continuous.

**Theorem 3.6.** If \( f:(S,q)\rightarrow(T,p) \) is \( \delta \)-continuous and \( (T,p) \) is a pretopological semi-regular space, then \( f \) is super continuous.

**Theorem 3.7.** Let \( (T,p) \) be a pretopological regular space and \( f:(S,q)\rightarrow(T,p) \) be a function.

(1) If \( f \) is \( \theta \)-continuous, then \( f \) is strongly \( \theta \)-continuous.

(2) If \( f \) is weakly continuous, then \( f \) is continuous.

(3) If \( f \) is almost continuous, then \( f \) is continuous.

Since a regular space implies a semi-regular space, by Theorem 2.8, 2.16, 3.1, and 3.7, the following properties are easily verified.

**Theorem 3.8.** Let \( f:(S,q)\rightarrow(T,p) \) be a function and \( (T,p) \) be a pretopological regular space. Then

(1) If \( f \) is almost strongly \( \theta \)-continuous, then \( f \) is continuous.

(2) Further, the following statements are equivalent,

(a) \( f \) is continuous.
(b) \( f \) is \( \theta \)-continuous.

(c) \( f \) is strongly \( \theta \)-continuous

(d) \( f \) is weakly continuous.

References


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