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Estimation of the Actual Working Time by Interval Linear Regression Models with Constraint Conditions

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제약부 구간 선형 회귀모델에 의한 실동시간의 견적

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Abstract

The actual working time of jobs, in general, is different to the standard time of jobs. In this paper, in order to analyze the actual working time of each job in prodcution, we use the total production amount and the necessary total working time. The method which analyzes the actual working time is as follows. In this paper, we propose the interval regression analysis for obtaining an interval linear regression model with constraint conditions with respect to interval parameters. The merits of this method are the following: 1) it is easy to obtain an interval linear model by solving a LP problem to which the formulation of proposed regression analysis is reduced, 2) it is easy to add constraint conditions about interval parameters, which are a sort of expert knowledge. As an application, within a case which has given certain data, the actual working time of jobs and the number of workers in a future plan are estimated through the real data obtained from the operation of processing line in a heavy industry company. It results from the proposed method that the actual working time and the number of workers can be estimated as intervals by the interval regression model.

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1. Introduction

The objective of time study is to determine reliable standards for all works[1]. Time study is used to measure work, and its result is called the standard time for the work[2]. The standard time is the time being necessary to bring a work to completion using standard job methods and conditions[3]. However, in actual, the standard time of jobs is not equal to the actual working time of jobs by means of variation of working environment and conditions etc..

In order to analyze the actual working time, we use the following novel linear regression analysis. As the method of regression analysis based on the concept of possibility[4], the possibilistic linear regression model with fuzzy prameters, has been proposed by Tanaka et al.[5] and discussed in detail in references[6-13]. In the conventional regression analysis, deviations between the observed values and the estimated values are supposed to be due to observation errors. We assume, on the contrary, that these deviations depend on the fuzziness of the system structure in possibilistic regression models. Thus, the parameters in the regression models are assumed to be intervals. Tanaka et al.[14] has proposed the interval linear regression analysis with interval parameters instead of fuzzy parameters[15] and has been researched[16].

Chisman[17] has applied LP to establish the standard time. The objective is to find the standard time which minimizes the sum of the absolute deviation between the actual total time for a job and calculated total time for that job.

In this paper, we propose the regression method for obtaining an interval linear regression model with constraint conditions with respect to interval parameters, which ara a sort of expert knowledge. In actual problems, information about the parameters of the models are often obtained in a vague expression. Thus, this expert knowledge with respect to parameters is transferred to interval constraint conditions within which the estimated parameters will be found as intervals. Then, we try to obtain the interval parameters which are compatible with these constraint conditions. We obtain the upper and lower limits of the actual working time instead of a real value. In order to do this, let us define an interval linear regression model as follows:

$$Y = A_1 x_1 + \cdots + A_n x_n \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots (1)$$

where A_i is an interval parameter representing the actual working time of ith work. The rough knowledge about the actual working time of work are obtained from the factory manager so as to enable us to estimate a reasonable interval. This is used as constraint conditions of the LP problem. Since we introduce the interval constraint conditions proposed by the expert, we obtain a more actual and reasonable analysis than before. Using the estimated actual working time by an established model, we estimate the number of workers being necessary in a week. Also, it is able to deal with the change of the amount of work.

The merits of this method are the following. First, it is easy to obtain the interval linear model by solving a LP problem to which the regression problem is reduced. Second, it is easy to add constraint conditions about interval parameters, which are a sort of expert knowledge.

As an application, within a case which has given certain data, we estimate the actual working time of jobs using the data obtained from the operation of a processing line from January to April 1987 in a heavy industry company.

Furthermore, the number of workers in a future plan are estimated by using the estimated actual working time. All results are obtained as intervals which present our partial ignorance in a vague situation. This method reflects our partial ignorance through intervals and is fitting to analyze vague phenomena like the operation of a processing line[18].

2. Interval Linear Regression Models with Constraint Conditions

Let us define an interval linear regression model

$$Y = A_1 x_1 + \cdots + A_m x_m \qquad \cdots\cdots (2)$$

where A_j is an interval parameter. Using the interval parameter A_j with the center α_j and the spread c_j, denoted as $A_j = (\alpha_j, c_j) = \{a \mid \alpha_j - c_j \leqq a \leqq \alpha_j + c_j\}$, Eq.(2) is transformed from interval arithmetic[19,20] as follows:

$$Y = (\alpha x,\ c \mid x \mid) \qquad \cdots\cdots (3)$$

where $\alpha = (\alpha_1, \cdots, \alpha_n)$, $x = (x_1, \cdots x_n)^t$, $c = (c_1, \cdots, c_n)$, $\mid x \mid = (\mid x_1 \mid, \cdots, \mid x_n \mid)^t$.

Let us define inclusion relation between intervals as follows:

$$A_i \subset A_j$$
$$\uparrow \ \downarrow$$
$$\alpha_j - c_j \leqq \alpha_i - c_i,\ \ \alpha_j + c_j \geqq \alpha_i + c_i \qquad \cdots\cdots (4)$$

Now, we assume that we can obtain observation data, (y_i, x_i), $i = 1, \cdots, n$. An interval linear regression problem data is formulated under the following assumptions.

1) Output value y_i is included in the estimated interval $Y(x_i)$

$$Y(x_i) = A_1 x_{i1} + \cdots + A_n x_{in} \supset y_i,\ \ i = 1, \cdots, n. \qquad \cdots\cdots (5)$$

Using Eq.(4), this condition is transformed as follows:

$$y_i \leqq \alpha x_i + c \mid x_i \mid, \quad i=1,\cdots,n,$$
$$y_i \geqq \alpha x_i - c \mid x_i \mid, \quad i=1,\cdots,n. \quad \cdots\cdots\cdots (6)$$

2) Let us introduce the following performance index

$$J(c)=\sum_i c \mid x_i \mid \quad \cdots\cdots\cdots (7)$$

where $c \mid x_i \mid$ is the spread of the estimated interval $Y(x_i)$. Thus, $J(c)$ is the total amount of the spread of $Y(x_i)$ which corresponds to the total amout of errors in the conventional regression model.

From the above assumptions, the problem is formulated as the following linear programming(LP) problem.

[Problem LP 1]

$$\min_{\alpha,\, c} J(c)=\sum_i c \mid x_i \mid,$$

subject to (6) and $c \geqq 0$.

In many actual cases, we have to obtain beforehand the rough information about parameters form the expert, which serves constraint conditions. This can be transferred into the following inclusion relation:

$$A_j \subset B_j, \quad j=1,\cdots,n \quad \cdots\cdots\cdots (8)$$

where $B_j(\beta_j, d_j)$ is an interval given by the expert. Eq.(8) leads to

$$\beta_j - d_j \leqq \alpha_j - c_j, \quad j=1,\cdots,n$$
$$\beta_j + d_j \geqq \alpha_j + c_j, \quad j=1,\cdots,n \quad \cdots\cdots\cdots (9)$$

where β_j-d_j and β_j+d_j are the lower value and the upper value of the interval B_j, respectively. The linear regression problem with constraint conditions is then reduced to the following LP problem based on the constraint conditions of Eqs.(6) and (9).

[Problem LP 2]

$$\min_{\alpha,\, c} J(c)=\sum_i c \mid x_i \mid,$$

subject to (6), (9) and $c \geqq 0$.

3. Application to Measurement of Processing Work

The object of this section is to analyze the actual working time by the proposed interval regression model.

3.1 The Actual Working Time

Let us estimate the actual working time of the processing work. The input-output data are shown in Table 1. Table 2 represents the products, amout of products, the

Table 1. Input-output data

Output y_i	Input x_{i1} ·· ·· ·· ·· ·· ·· ·· ·· x_{in}
y_1	x_{11} ·· ·· ·· ·· ·· ·· ·· ·· x_{1n}
·	· ·
·	· ·
·	· ·
y_N	x_{N1} ·· ·· ·· ·· ·· ·· ·· ·· x_{Nn}

Table 2. Input-output data of processing work

unit: /week

Week no.	Working day	Total working time (y)	Workers	Products A x_1	B x_2	C x_3	D x_4	E x_5	F x_6
1	6	11514	193	1.3	2.9	2.6	2.3	1.8	2.1
2	6	11765	189	1.2	3.0	2.9	2.4	1.6	2.4
3	6	11434	184	1.3	2.7	2.3	2.2	1.9	1.8
4	5	9931	189	1.2	2.4	2.2	2.1	1.7	1.7
5	6	11493	189	4.4	3.1	3.2	2.5	2.3	2.2
6	6	11029	189	4.2	2.7	3.4	2.7	2.1	2.4
7	6	11546	191	4.3	3.0	2.9	2.1	2.1	2.3
8	6	11462	190	4.6	3.1	4.0	2.7	2.8	2.1
9	6	12395	197	3.7	3.7	2.6	3.2	3.3	2.6
10	5	10572	199	2.9	2.9	2.4	2.7	2.5	2.2
11	6	12168	199	3.3	3.6	2.9	3.5	3.6	2.3
12	6	12279	197	3.6	3.9	2.3	3.1	3.0	2.9
13	6	11462	196	3.2	3.8	3.1	3.1	2.8	2.5
14	6	11450	198	3.1	3.6	3.1	3.2	2.7	2.7
15	6	11718	198	3.4	4.1	3.3	2.9	3.0	2.3
16	6	11786	199	3.3	3.8	3.2	2.9	2.7	2.5
17	6	9921	199	1.9	3.0	2.5	3.3	4.0	2.7
18	6	10083	200	2.1	3.1	2.2	3.3	3.7	2.7
19	6	10029	200	1.9	3.1	2.6	3.2	4.1	2.8
20	6	9931	200	3.1	2.8	2.8	3.7	4.5	2.7
21	6	11369	200	2.0	3.7	3.1	3.0	3.9	2.5

number of workers, total working time and total working days per week by a processing work team in 21 weeks ranging from January to April 1987 in a heavy industry. The data from the first week to the 16th week are used to obtain the model, five weeks' data ranging from the 17th week to the 21th week are used as data to check the efficiency of the model.

The total actual working time $Y(x_i)$ is estimated by

$$Y(x_i) = A_1 x_{i1} +, \cdots, + A_6 x_{i6} \quad \cdots\cdots\cdots\cdots (10)$$

where the amount of production of products A, B, C, D, E and F are denoted by x_1, x_2, x_3, x_4, x_5 and x_6, respectively. The information for setting the interval paramter B_j from the factory manager is given in Table 3.

Next, the estimate of the actual working time is obtained by solving LP 2, using

Table 3. Constraint conditions of interval parameter

unit: hours

Products	Lower value	Upper value
A	$\alpha_1 - c_1 \geq 443$	$\alpha_1 + c_1 \leq 1700$
B	$\alpha_2 - c_2 \geq 1039$	$\alpha_2 + c_2 \leq 1725$
C	$\alpha_3 - c_3 \geq 799$	$\alpha_3 + c_3 \leq 1554$
D	$\alpha_4 - c_4 \geq 275$	$\alpha_4 + c_4 \leq 400$
E	$\alpha_5 - c_5 \geq 106$	$\alpha_5 + c_5 \leq 337$
F	$\alpha_6 - c_6 \geq 150$	$\alpha_6 + c_6 \leq 354$

input-output data in Table 2 and constraint conditions about the interval parameters in Table 3. Using the estimated interval parameters, Eq.(10) is transformed as follow:

$$\begin{aligned} Y(x_i) &= (623, 180)x_{i1} + (1429, 296)x_{i2} + (1177, 378)x_{i3} + (400, 0)x_{i4} + (328, 9)x_{i5} + (354, 0)x_{i6} \\ &= [443, 803]x_{i1} + [1133, 1725]x_{i2} + [799, 1554]x_{i3} + [400, 400]x_{i4} + [319, 337]x_{i5} + [354, 354]x_{i6} \end{aligned}$$

We try to examine the inclusion relation between these estimated interval parameters and constraint conditions about the interval parameter in Table 3. The actual working time appears in the left side of the constraint conditions. Thus, we can see that product A is located to the left side, but the allowance of spread is largely to the right side. Same as above, product B is placed to the right side, product C will become completely coincide with the constraint conditions, and products D, E and F are shown in the right side. In particular, the spread is 0 in products D and F. It doesn't mean that the spread of products D and F is not needed at all, but that the relation between products D and F and the total actual working time is crisply approximated. By Eq.(11), the estimated interval value of processing work is denoted in Fig. 1. The checking data, not used in establishing the model, are from the 17th week to the 21th week. From this figure, we can see that the total actual working time and estimated actual working time are very reasonable. Only in the estimated working time of 20th week, the estimated actual working time is not included in the constraint conditions.

3.2 The Estimation of the Number of Workers

From establishing the actual working time of a work, the number of workers, that is, the fixed personnel, is determined. The fixed personnel means the reasonable personnel. The determination of the fixed personnel means to determine the necessary personnel in each work's accomplishment based on constant conditions[21]. The determination of the number of workers is done. That is, when the number of products is given as $x_1, \cdots, x_n$ produced by a work team, and the interval actual working times $A_1, \cdots, A_n$ are already obtained,

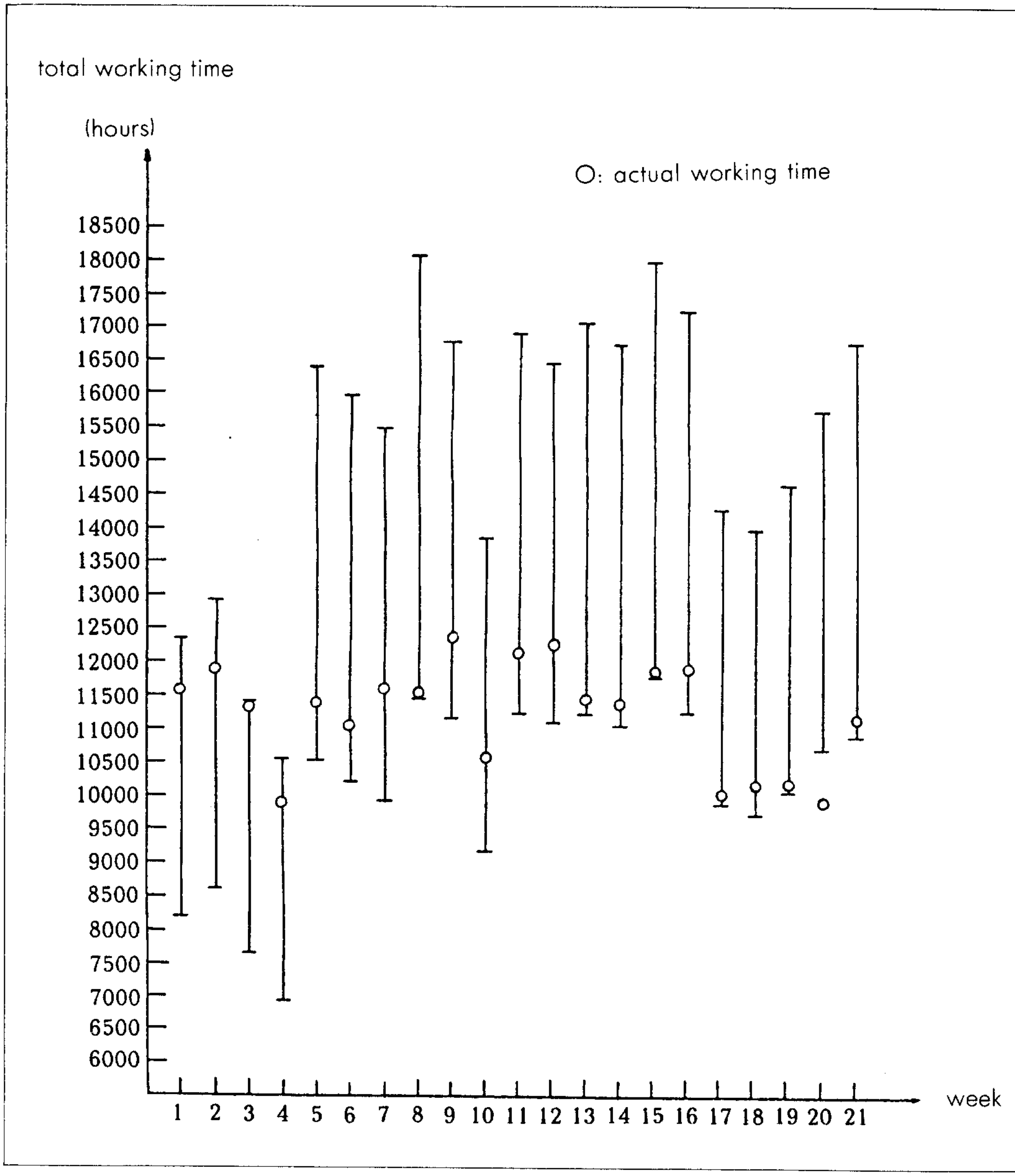

Fig. 1 The results of a model in a processing work.

then the number of necessary workers is determined as follows:

$$N=(\sum_j A_j x_j)/T \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (12)$$

where T denotes the total actual working time per worker within a fixed period. In many cases, T should be represented by an interval because of its fuzziness. Therefore, the number of workers is also obtained as an interval with a spread, because Eq. (10) is given by the interval actual working time and the interval working time. Eq.(12) is computed using the following interval arithmetic. That is, in general, when $0 \notin V$, it is expressed as follows:

$$\frac{U}{V}=\frac{(u, d)}{(v, k)}=\frac{[u-d,\ u+d]}{[v-k,\ v+k]}=\left[\frac{u-d}{v+k}, \frac{u+d}{v-k}\right] \quad \cdots\cdots (13)$$

From the result of the computation of Eq. (12) and Table 4, we can obtain the number of necessary workers $N(x_i)$ in the ith week.

$$\begin{aligned} N(x_i) &= \{(623, 180)x_{i1}+(1429, 296)x_{i2}+ (1177, 378)x_{i3}+(400, 0)x_{i4} + (328, 9)x_{i5}+(354, 0)x_{i6}\}/T_i \\ &= \{[443, 803]x_{i1}+[1133, 1725]x_{i2}x+ [799, 1554]x_{i3}+[400, 400]x_{i4}+ [319, 337]x_{i5}+[354, 354]x_{i6}\}/T_i \end{aligned} \quad (14)$$

Table 4. Estimated interval paramter

unit: hours

Products	Inverval parameter	Center	Spread
A	A_1	623	180
B	A_2	1429	296
C	A_3	1177	378
D	A_4	400	0
E	A_5	328	9
F	A_6	354	0

where T_i denotes the total actual working time for the ith week per worker. Fig. 2 represents the relation between the number of workers obtained by Eq.(14) and the number of actual workers. From this figure, we can see that the model of Eq.(14) adopts the checking data(from the week 17 to week 21). The number of estimated workers of 20th week does not include the number of actual workers. The number of

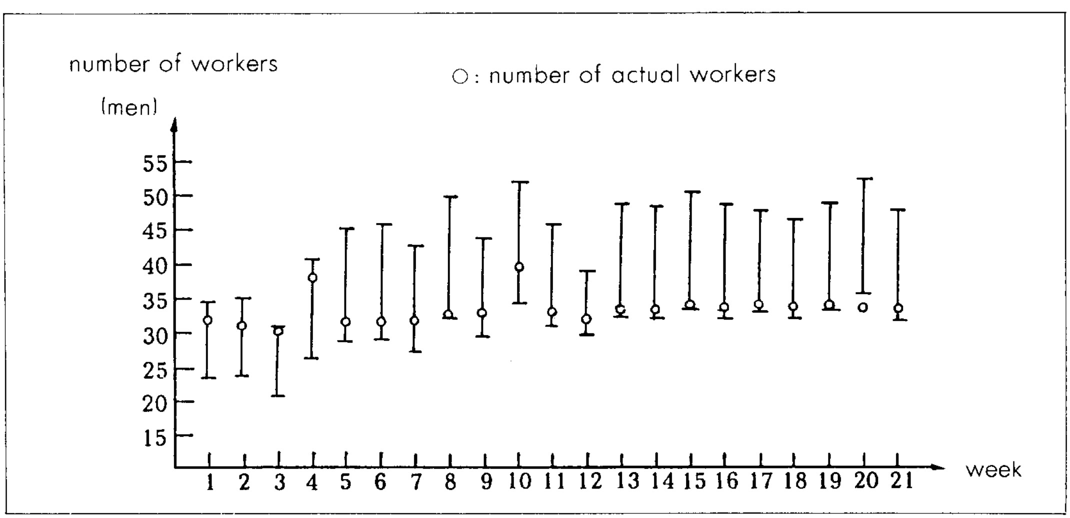

Fig. 2 The number of estimated workers in a processing work team

workers in this figure shows the average, which is the number of workers of a daily processing team. This value is obtained through dividing the number of total actual workers by the number of working days in a week. From estimation of the number of workers, which has been done here, we can make an evaluation of the number of present workers such as the following.

The number of workers is given by the interval shown in Fig. 2. It denotes the number of possible workers. If the number of workers is placed in the left side of the interval, the work is done by a comparatively few number of workers. Then we can think of a small allownace of work. If the number of workers is shown to right side of the interval, we can think that there is a certain allowance of work, because the work is done by a comparatively large number of workers. The number of necessary workers for the amount of future work can be estimated from Eq.(14). For example, when values such as $x_{i1}=1.0$, $x_{i2}=3.0$, $x_{i3}=2.0$, $x_{i4}=1.5$, $x_{i5}=1.8$, $x_{i6}=2.2$, $T_I=50$ are given, and the working days are six, we can estimate the number of workers in one day as the interval[24.6, 36.9].

4. Conclusions

In this paper, we estimated the actual working time by using an interval linear regression anlaysis with constraint conditions. Also, the number of workers in a future plan is estimated by using the estimated actual working time. The interval linear regression analysis with constraint conditions is the method in which the information of the expert is accepted. The estimated value is denoted by an interval. In actual situations, we must consider not only the information which is obtained about parameters, but also parameters which are obtained by the result of regression analysis. We can obtain parameters for actual application, by means of adding constraint conditions into parameters.

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