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A Design of Stable Continuous-time Model Reference Adaptive Controllers by a Hyperstability Method

(超安定度 方法에 依한 安定한 時連續 基準모델 適應制御器의 設計)

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要 約

本 論文에서는 線形 時不變 最小位相, 그리고 任意의 相對次數를 갖는 時連續 工程에 대한 基準모델 適應制御에 適用可能한, 媒介變數 推定法則에 特殊한 有理函數 形態의 線形演算子를 使用하고 制御入力成分의 附加信號項을 除去하는 새로운 形態의 適應制御方式을 提案하였다.

이 方式은 相對次數가 2以上이면 同一한 制御構造로 適應制御에 適用할 수 있으며, 또한 모델이 零點을 가지지 않는 境遇에는 附加信號가 전혀 없는 制御시스템을 構成할 수 있다. 이 方式을 適用한 適應시스템의 漸近的 安定度는 超安定度 方法에 의해 보였다.

Abstract

In this paper, a new adaptive control scheme is proposed that uses a special form of rational function-type linear operator in the parameter adaptation and that removes the augmenting signal terms of the control input components. This adaptation scheme is applied to the MRAC of continuous-time, linear time-invariant, minimum-phase plants whose relative degrees are arbitrary.

This scheme can be applied without any change of the controller structure to the adaptive systems regardless of the relative degree if it is greater than 1. And this scheme does not require any signal augmentation for arbitrary relative-degree plants if the reference model has no zeros. The asymptotic stability of the adaptive systems controlled by this scheme is shown by a hyperstability method.

I. Introduction

In adaptive control systems, the strictly positive realness plays an important role in the stability analysis methods. In general, in the MRAC, the model cannot have an arbitrary transfer function irrespectively of the plant because the transfer function of the plant together with the controller must match the model asymptotically. Thus we should take into account the notion of the relative

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degree which is denoted by $n^* \triangleq n\text{-}m$(the plant has 'n' poles and 'm' zeros).

In the case that n^* is equal to one, the model can be chosen to be SPR. Thus a stable adaptive control objective can be achieved with a basic MRAC structure. Hence one can readily apply a hyperstability method to design a stable adaptive law that is more flexible than the conventional simple gradient-type one. However, when n^* is greater than one, the model can be no longer chosen to be strictly positive real (SPR). Therefore a globally stable adaptive system cannot be obtained with a direct extension of the basic MRAC scheme. However Narendra et. al.[6] introduced a functional operator into the controller structure which equivalently has an effect of adding (n^*-1) stable zeros to the error transfer function to make it be SPR. Though this operator requires a differentiation of the control input signals, when a simple gradient-type adaptive law is used, a differentiator-free adaptive controller can be obtained in spite of this operator only when the relative degree is two. On the contrary, in case of $n^* \geqslant 3$, since higher derivative terms of the parameter vector cannot be obtained directly from known signals from the gradient-type adaptive law, a straightfoward extension of the control structure used for the case $n^*=2$ is not possible. For this reason the augmentation technique of Monopoli[3] was brought to the design of MRAC[6][9] in case of $n^* \geqslant 3$. By this technique it was shown that the gradient-type algorithm can be still applicable.

In this scheme, however, the control input did not consist of all the components filtered by the functional operator but of their steady state equivalences. Thus the effect of the operator, which makes the transfer function of the output error to be SPR, was compensated by augmenting equivalently filtered signals to the output error. In other words, due to the gradient-type algorithm, all the filtered control input by the operator cannot be realized directly and hence the structurely complete control input cannot be fed into the plant. Therefore the initial behavior of the plant may be unsatisfactory and even the system may be unstable. Moreover if the high frequency gain of the plant is unknown, additional parameter adaptation and additional control input augmentation are necessary. Moreover since an extra parameter should be incorporated into the augmentation structure, this results in a more complex control structure.

Based on the idea of the injection method of all the filtered control input to the plant, we suggest an alternative MRAC scheme that uses a generalized adaptation algorithm in the sense that this employs a wider class of linear operators in it, and that has a unified control scheme regardless of both the knowledge of the high frequency gain and the relative degree if this is greater than one.

II. Hyperstability in MRAC Systems

Consider a system B_1 that is linear time invariant, completely controllable, and completely observable. Let us assume that the system can be described by single input u(t) and single output y(t) as

$$\dot{x}(t) = Ax(t) + bu(t)$$
$$y(t) = c^T x(t) + du(t) \quad (1)$$

where $x(t) \in R^n$ is the state vector of the system and $A \in R^{nxn}$, $b,c \in R^n$, and d is a scalar. And the system B_1 under consideration is to be placed in a negative feedback configuration, as shown in Fig. 1, by a nonlinear time varying system B_2 which has an input y(t) and an output -u(t) with the inequality (this corresponds to a passivity condition)

$$\int_0^T u(t)y(t)\,dt \geq \delta^2 \quad (2)$$

where δ is an arbitrary positive constant independent of T for $\forall\, T \geqslant 0$.

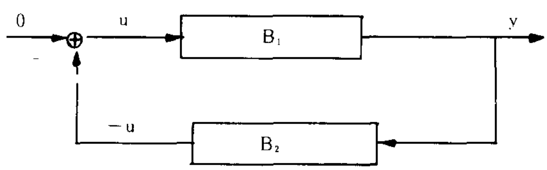


Fig.1. Negative feedback connection of two blocks.

Then the hyperstability of B_1 is defined by the property that the state x(t) is bounded for the input u(t) satisfying the Popov inequality (2)[10]

Let us abuse the notation of the differential operator $s \triangleq d/dt$. Thus a linear function of s, for example H(s), may represent a linear differential operator acting on the operands or a transfer function according to the context. And let a matrix I of an appropriate dimension be used to denote an identity transfer matrix or a numerical unity matrix.

Theorem 1

Suppose that a nonlinear time-varying block B_2 in Fig. 1 be described by the following input-output relation for a continuous vector $\xi(t) \in R^n$ for all $t \geqslant 0$

$$u(t) = -\xi^T(t) H(s) I[\xi(t) y(t)] \quad (3)$$

where H(s) is a positive real (PR) transfer function in the block B_2 that has a simple pole at s=0.

Then the closed loop system is hyperstable if the transfer function of B_1 is SPR. Furthermore if ξ (t) is bounded, the block B_1 is asymptotically hyperstable.

(Proof) Since H(s) is PR, there exist[2] a positive definite matrix kernel $K(t, \tau)=K(t-\tau)$ and a positive constant δ such that for all $T \geqslant 0$

$$\int_0^T u(t)y(t)dt = -\int_0^T [y(t)\xi(t)]^T H(s) I \cdot [\xi(t)y(t)]dt$$

$$= -\int_0^T [y(t)\xi(t)]^T \left[\int_0^t K(t-\tau)[\xi(\tau)y(\tau)]d\tau\right]dt$$

$$\leq \delta^2 \quad (4)$$

Thus we can easily see that the block B_1 is hyperstable. Next, By applying the Kalman-Yacubovich lemma to B_1, we have the inequality (2) again. Thus we conclude that B_2 is also hyperstable. If $\xi(t)$ is bounded, from Barbalat's lemma[8], B_1 is asymptotically stable. □

Since the hyperstability merely requires a PR operator in the feedback path for the SPR B_1, we can readily design a more flexible adaptive law by satisfying the inequality (2). To introduce a special form of PR operators into the adaptive laws in the feedback path, we need a theorem for a condition on PR transfer functions.

Theorem 2

Let P(s) be a monic Hurwitz polynomial of s and F(s) be a rational function of s such that F(s)= 1/[P(s)-P(0)]. Then there exists a finite positive constant μ such that $G(s) \triangleq \mu + F(s)$ is PR.

(Proof) Since F(s) is real when s is real and P(s)-P(0) is a modified Hurwitz(see Remark 1), it is sufficient to show that $Re[G(jw)] \geqslant 0$ for all $w \in R$ for the positive realness of G(s). Remainings are omitted .□

Remark 1

$P_1(s)$ is another Hurwitz polynomial if P(s) is a Hurwitz one. This fact can be seen by checking the Hurwitz determinants of $P_1(s)$. Since P(s) is a Hurwitz, all its Hurwitz determinants are positive. Forming $P_1(s)$ from P(s) only needs removing the constant term and lowering the order. Hence this operation only affects the last Hurwitz determinant of P(s). Since the Hurwitz determinants of $P_1(s)$ are equal to those of P(s) except the last one, $P_1(s)$ is also a Hurwitz. Hence P(s)-P(0) is a modified Hurwitz. Refer to [1].

III. Hyperstable Model Reference Adaptive Controllers

A single-input single-output continuous linear time-invariant minimu-phase plant may be represented by the transfer function

$$G_P(s) = c^T(sI - A_P)^{-1} b_P = k_P \frac{N_P(s)}{D_P(s)} \quad (5)$$

where $G_p(s)$ is strictly proper with monic polynomials $N_p(s)$ and $D_p(s)$ of degrees m(<n) and n respectively with a constant gain parameter k_p. Assume that m,n, and the sign of k_p are known. Thus the sign of k_p can be assumed positive.

A model that represents the desired behavior which the controlled plant follows is supposed to be described by the transfer function

$$G_m(s) = c^T(sI - A_m)^{-1} b_m = k_m \frac{N_m(s)}{D_m(s)} \quad (6)$$

where $N_m(s)$ and $D_m(s)$ are monic Hurwitz polynomials whose degrees are p($\leqslant$ m) and n respectively with a constant gain k_m. Then the adaptive control problem is to design a controller for the plant that has the properties asymptotically

$$\lim_{t\to\infty} e_1(t) \triangleq \lim_{t\to\infty} (y_P(t) - y_m(t)) = 0 \quad (7)$$

where $y_p(t)$ is the output of the plant and $y_m(t)$ is that of the model.

1. Case When $n^*=1$

The adaptive controller may be constructed in a basic structure as in [6]. Two auxiliary signal generators S_1, S_2 are employed to generate filtered signals. S_1 contains an (n-1)-dimensional vector $S_1(t)$ and S_2 contains an (n-1)-dimensional vector $S_2(t)$. S_1 and S_2 are described by the differential equations

$$\dot{s}_1(t) = Fs_1(t) + bu(t) \qquad (8)$$

$$\dot{s}_2(t) = Fs_2(t) + by_p(t) \qquad (9)$$

where F is an (n-1)x(n-1) stable matrix and b is a (n-1) vector. Let $\theta(t)$ represent a 2n-dimensional adjustable parameter vector.

$$\theta^T(t) = |f_0(t),\ f^T(t),\ g_0(t),\ g^T(t)|, \qquad (10)$$

and $\xi(t)$ denote a filtered signal vector whose dimension is 2n.

$$\xi^T(t) = [r(t),\ s_1(t),\ y_p(t),\ s_2(t)]. \qquad (11)$$

If we choose the adaptive law as, for a Hurwitz polynomial P(s) of degree q($\geqslant$1) with the μ chosen to make $\mu + 1/[P(s)-P(0)]$ be PR,

$$\theta_c(t) = -\left[\mu + \frac{1}{P(s)-P(0)}\right]\Gamma\xi(t)e_1(t)$$

$$= \theta(t) - \mu\Gamma\xi(t)e_1(t) \qquad (12)$$

where $e_1(t)$ is the output error and $\Gamma=\Gamma^T$ is a positive definite matrix. If the control input to the plant is given by

$$u(t) = \theta_c^T(t)\xi(t), \qquad (13)$$

then we have an error model which gives $e_1(t)$ as

$$e_1(t) = \frac{k_p}{k_m} G_m(s)\,[\psi^T(t)\xi(t)$$

$$-\mu e_1(t)\xi^T(t)\Gamma\xi(t)] \qquad (14)$$

where $\psi(t) \triangleq \theta(t)-\theta^*$ is the parameter error vector. It can be shown[10] that by this control all the signals in the system are uniformly bounded and $e_1(t)$ tends to zero as $t \to \infty$. If k_p is known (or equivalently if $k_p=k_m$) $f_o(t)$ can be fixed to 1. Then the parameter vector may be of dimension (2n-1) by excluding $f_o(t)$ from $\theta(t)$.

2. Case When $n^* \geqslant 2$

When $n^* \geqslant 2$, a monic Hurwitz polynomial P(s) of degree q may be incorporated into the control structure so that the output error transfer function $k_pG_m(s)P(s)/k_m$ is SPR. Since P(s) is multiplied to the plant as a factor of the numerator polynomial, it is required to generate the control input u(t) by passing signals through a lead compensator P(s). The controller is constructed after a regulator structure as shown in Fig. 2.

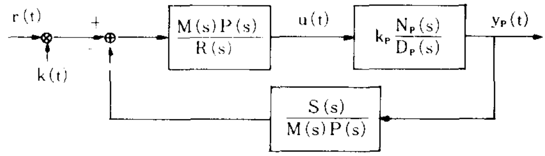


Fig.2. An adaptive controller structure with a compensator P(s).

R(s) and S(s) are polynomials whose coefficients are adjusted adaptively. The degree of R(s) is n-1 and that of S(s) is n-1 or n according to the choice of P(s). M(s) is a monic Hurwitz polynomial such that $M(s)=T(s)N_m(s)$ where T(s) is an observer polynomial of the degree greater than (n-p-1). For $k(t)=k_* \triangleq k_p/k_m$, if $D_p(s)$ and $N_p(s)$ are relatively prime, there exist unique R(s) and S(s) such that

$$R(s)D_p(s) + k_pS(s)N_p(s) = T(s)D_m(s)N_p(s) \qquad (15)$$

for the minimum-phase plants. Then the output error $e_1(t)$ may be represented as

$$e_1(t) = \frac{k_pM(s)P(s)}{T(s)D_m(s)}\left[\frac{R(s)}{P(s)M(s)}u(t) + \right.$$

$$\left. + \frac{S(s)}{P(s)M(s)}y_p(t) - \frac{k^*}{P(s)}r(t)\right] \qquad (16)$$

where $k_pM(s)P(s)/T(s)D_m(s)$ is SPR. If R(s) is realized as R(S)=M(s)+X(s) where the degree of X(s) is lower than that of M(s) by 1 and if the control input u(t) is given by

$$u(t)=P(s)\left|\, k(t)\,[\frac{1}{P(s)}\,r(t)\,]\right.$$

$$\left.-X(s)\cdot[\frac{1}{P(s)M(s)}\,u(t)\,]-S(s)\,[\frac{1}{P(s)M(s)}\,y_P(t)\,]\right| \quad (17)$$

then $e_1(t)$ can be zero asymptotically and hence the closed loop transfer function can evolve to the model. If k_p is known, it can be assumed k_m. Then k(t) can be fixed to 1 and u(t) can be expressed as

$$u(t)=r(t)-P(s)\left|\, X(s)\,[\frac{1}{P(s)M(s)}\,u(t)\,]\right.$$

$$\left.+S(s)\,[\frac{1}{P(s)M(s)}\,y_P(t)\,]\right| \quad (18)$$

However u(t) contains the derivatives of the filtered input/output signals and of the parameter vector. Hence the direct realization of (17) or (18) is not possible. To express (17) or (18) in terms of the parameter vector in a linear form, we introduce the filtered signal vectors as in the relative-degree-1 case. Now let us assume that k_p is unknown. If we employ the filters S_1 and S_2 as in (8) and (9) such that M(s) = det(sI-F)=T(s) $N_m(s)$ and (F,b) is a completely controllable pair, and if the coefficients of X(s) and S(s) are formed as the parameter vectors f(t) and g(t) with a scalar $g_o(t)$, we can define $\xi(t)$ and $\theta(t)$ as in (10), (11) with k(t)= $f_o(t)$. Furthermore if

$$\nu(t)=\frac{1}{P(s)}\,\xi(t) \quad (19)$$

then u(t) can be represented in a linear form in $\theta(t)$ as

$$u(t)=P(s)\,[\,\theta^T(t)\,\nu(t)\,] \quad (20)$$

To get the functional equivalence of (20), the direct expansion of $P(s)\,[\theta^T(t)\ \nu(t)]$ may be checked. Thus we can obtain the equivalence after some manipulations with the relation (9)

$$P(s)\,[\,\theta^T(t)\,\nu(t)\,]=\left[\,[\,P(s)-P(0)\,I\,\theta(t)\right]^T\nu(t)$$

$$+\theta^T(t)\,\xi(t)+f(\theta(t),\ \nu(t)) \quad (21)$$

where $P(s)=s^q+P_{q-1}\,s^{q-1}+\ldots+P_1\,s+P_o$ $(q\geqslant 1)$ and a functional operator f is given by

$$f(\theta(t),\nu(t))=\sum_{K=1}^{q-1}\left[\,[\frac{1}{k!}\,\frac{d^kP(s)}{ds^k}-p_k]\,I\,\theta(t)\right]^T\nu^{(k)}(t) \quad (22)$$

Here $\nu^{(k)}(t)$ represents the k-th derivative of $\nu(t)$. If we choose the adaptive law as

$$[\,P(s)-P(0)\,]\,I\,\theta(t)=-\Gamma\,\nu(t)\,e_1(t) \quad (23)$$

where $\Gamma=\Gamma^T>0$, then control input (20) can be realized as

$$u(t)=-\nu^T(t)\,\Gamma\,\nu(t)\,e_1(t)+\theta^T(t)\,\xi(t)$$

$$+f(\theta(t),\ \nu(t)\,). \quad (24)$$

Since u(t) can be represented equivalently as

$$u(t)=P(s)\,[\,(\theta^*+\psi(t))^T\,\nu(t)\,]$$

$$=\theta^{*T}\xi(t)+P(s)\,[\,\psi(t)^T\,\nu(t)\,] \quad (25)$$

where $\theta(t)\triangleq\psi(t)+\theta^*$ where a constant parameter vector θ^* satisfies (15). A (3n-2) (or 3n) dimensional nonminimal state error differential equation may be written as

$$\dot{e}(t)=A_c\,e(t)+b_cP(s)\,[\,\psi(t)^T\,\nu(t)\,]$$

$$e_1(t)=c^Te(t) \quad (26)$$

where e(t) is the augmented error state vector between the plant and the model and

$$[\,c^T(sI-A_c)^{-1}\,b_c]\,P(s)=\frac{k_PN_m(s)\,P(s)}{D_m(s)} \quad (27)$$

Here the matrix A_c is a stable matrix and b_cC_c are vectors of appropriate dimensions respectively that are determined by θ^*

Since an additional constant feedback gain μ is required for the adaptive law, an additional feedback term due to this is needed. However this requires the derivatives of $e_1(t)$. Thus we suggest a method to get the equivalent effect of this term.

(1) control with augmentation

When the model has finite zeros we use the augmentation technique of the parameter estimate. To do this, let us here define the augmented output error $\epsilon_1(t)$ as

$$\epsilon_1(t)=y_P(t)-y_m(t)-y_a(t)$$

$$=e_1(t)-y_a(t) \quad (28)$$

where $y_a(t)$ is the augmentation signal for the output error. Thus if we generate the control input as (24) and do $y_a(t)$ as

$$y_a(t) = \frac{k_m N_m(s) P(s)}{D_m(s)} [\mu_1 \nu^T(t) \Gamma \nu(t) \varepsilon_1(t)] \tag{29}$$

where $\mu_1=(k_{pm}/k_m)\,\mu$, k_{pm} is the upper bound of k_p, and P(s) is of order q=n-p-1 (q $\geqslant$ 1). If the adaptive law is chosen as

$$[P(s)-P(0)] I \theta(t) = -\Gamma \nu(t) \varepsilon_1(t) \tag{30}$$

then we may write the equivalent error equation

$$\dot{\varepsilon}(t) = A_c \varepsilon(t) + b_c P(s) [\psi^T(t) \nu(t) - \mu_{1m} \nu^T(t) \Gamma \nu(t) \varepsilon_1(t))$$

$$\varepsilon_1(t) = h_c^T \varepsilon(t) \tag{31}$$

where $\mu_{1m} = (k_{pm}/kp) = \geqslant \mu$. Hence we have an error model which gives the output $\epsilon_1(t)$

$$\varepsilon_1(t) = \frac{k_p N_m(s) P(s)}{D_m(s)} [\psi^T(t) \nu(t) - \mu_{1m} \nu^T(t) \Gamma \nu(t) \varepsilon_1(t)] \tag{32}$$

Remark 2

Note that even though the adaptation algorithm is different from that of Narendra and Valavani[5], the same form of error equations are obtained. However, in contrast to theirs, the knowledge of k_p doesn't make an change on the controller structure because no component of the control input is used for signal augmentation. Furthermore this scheme can be applied in a unified manner to any relative degree case if it is greater than one. When n*=2, P(s) can be chosen (s+a) and hence our adaptive law becomes the conventional PI-type one.

Remark 3

The assumption on the upper bound of k_p can be also found in the input error scheme of Sastry[9]. This assumption is not so restrictive because the upper bound is always multiplied to μ. Compared with that of Narendra and Valavani[5], this approach costs another assumption for the unification of the control structure and a simpler structure.

(2) control without augmentation

As stated earlier, if the model has no finite zeros, i.e., $N_m(s)=k_m$. no augmentation is needed. If we choose P(s) to be $D_m(s)$, the feedforward block has a SPR transfer function obviously. In this case one should choose P(s) to be of degree of n-p. Then the augmenting signal may be changed to

$$y_a(t) = k_m \mu_1 \nu^T(t) \Gamma \nu(t) \varepsilon_1(t) \tag{33}$$

Then from (32) ϵ (t) can be expressed as

$$\varepsilon_1(t) = \frac{e_1(t)}{1 + k_m \mu_1 \nu^T(t) \Gamma \nu(t)} \tag{34}$$

Hence we have a new error model which gives $e_1(t)$ as output

$$e_1(t) = k_p k_m \psi^T(t) \nu(t) \tag{35}$$

IV. Proof of Stability

Alternatively the error model (32) or (34) can be viewed as an equivalent control scheme that uses the adaptive law as.

$$\theta_c(t) = -\left[\mu_{1m} + \frac{1}{P(s)-P(0)}\right] \Gamma \nu(t) \varepsilon_1(t)$$
$$= \theta(t) - \mu_{1m} \Gamma \nu(t) \varepsilon_1(t) \tag{36}$$

Note that then we obtain the same error model as (32). If $\psi_c(t) \triangleq \theta_c(t) - \theta^*$ denotes the parameter error vector, eq.(36) can be rewritten as

$$\psi_c(t) = \psi(t) - \mu_{1m} \Gamma \nu(t) \varepsilon_1(t) \tag{37}$$

This equivalent error model is shown in Fig. 3.

The stability of these control schemes are given as follows:

Theorem 3

Consider the error system described by eq. (26) which comes from (5), (6), (8), (9), (19), and (25). Let P(s) represent a monic Hurwitz polynomial of s of degree (n-p-1) such that Gm(s)P(s)

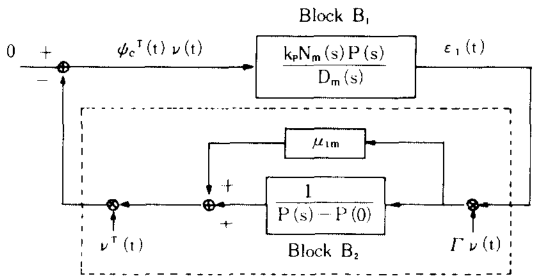


Fig.3. The equivalent error model with a PR operator in the parameter adaptation.

is SPR and strictly proper. Suppose that the upper bound of the high frequency gain of the plant is Known. If the adaptive law (30) and augmentation (28), (29) are employed with $\mu_{1m} \geqslant \mu$ satisfying the condition of Theorem 2 for given P(s), the state error e(t) and parameter error δ (t) are uniformly bounded. Furthermore, all the signals in the control loop are bounded and the output error $e_1(t) \to 0$ as $t \to \infty$

(Proof) The error system can be equivalently represented by (31), we prove for the system (31) and (32). As in Fig. 3., since the transfer function of B_1 is SPR and B2 has a PR operator for the chosen μ_{1m}, the overall system is hyperstable from Theorem 1. This implies that $\epsilon(t)$ and $\psi(t)$ are uniformly bounded. Thus it follows that $\epsilon_1(t)$ is uniformly bounded.

Assume that only the sign of k_p is known. If we define $\chi(t)$ as

$$\chi(t) = \psi^T(t)\,\nu(t) - \mu_{1m}\,\nu^T(t)\,\Gamma\,\nu(t)\,\epsilon_1(t). \quad (38)$$

Then by Kalman-Yacubovich lemma it can be shown that $\epsilon(t) \in L_1 \cap L_\infty$ by checking the inequality (2) with (38). Thus $\epsilon_1(t) \in L_2 \cap L_\infty$ follows. A minimal, phase-variable controllable canonical realization of H(s), $H(s) \triangleq \mu_{1m} + 1/[sP_1(s)]$ in block B_2, shows that $\nu(t)\,\epsilon_1(t) \in L_2$ and that $\psi(t)$ and its k-th derivatives belong to L_2 for k=1,2,..., n-p-1. And one can also see that the k-th derivatives of $\psi(t)$ belong to L_∞ for k=1,2,..., n-p-2 and in turn that $\nu(t)\,\epsilon_1(t) \in L_\infty$.

However, the boundedness of $e_1(t)$ cannot be completely determined[7] by either Lyapunov or hyperstability method. Thus, following Narendra et. al.[4], we proceed to prove the boundedness of the signals by checking the growth rates of them. From (29), we have

$$|y_a(t)| \le \alpha_0(t) \sup_{t \ge \tau} |\nu(\tau)| + \alpha_1(t) \quad (39)$$

where $\alpha_0(t) \in L_2$ and $\alpha_1(t) \in L_1$ or L_2. It can be assumed that $\nu(t)$ belongs to L_{pe} since $\epsilon(t)$ is bounded and so $\chi(t)$ can grow at most exponentially . Hence we obtain

$$|y_p(t)| \le \alpha_0(t) \sup_{t \ge \tau} |\nu(\tau)| + \beta_0 |r(t)| + \alpha_2(t) \quad (40)$$

where $\alpha_2(t) \in L_1$ or L_2 and β_0 is a positive constant. Since $y_p(t)$ can be expressed as

$$y_p(t) = G_m(s)\,r(t) + (k_p/k_m)\,G_m(s)P(s)\,[\psi^T(t)\,\nu(t)] \quad (41)$$

or

$$|y_p(t)| \le \beta_1 \sup_{t \ge \tau} |\nu(\tau)| + \beta_2 |r(t)| \quad (42)$$

for positive constants β_1 and β_2. In the similar way, it can be shown that

$$|\nu(t)| \le \beta_3 \sup_{t \ge \tau} |y_p(\tau)| + \beta_4 |r(t)| + \beta_5 \quad (43)$$

for β_3, β_4, and $\beta_5 > 0$. From (42) and (43) , and noting that all the components of $\nu(t)$ can grow in a similar rate, $\nu(t)$ | and sup $|\nu(\tau)|$, $t \geqslant \tau$, are of same order in magnitude. Thus there exist finite positive constants β_6 and β_7 such that

$$\sup_{t \ge \tau} |\nu(\tau)| \le \beta_6 \sup_{t \ge \tau} |y_p(\tau)| + \beta_7 \quad (44)$$

Therefore we have

$$|y_p(t)| \le \alpha_3(t) \sup_{t \ge \tau} |y_p(\tau)| + \beta_0 |r(t)| + \alpha_4(t) + \beta_8 \quad (45)$$

for $\alpha_3(t) \in L_2$, $\alpha_4(t) \in L_1$ or L_2, and $\beta_8 > 0$. Since r(t) is uniformly bounded, $y_p(t)$ and $\nu(t)$ is uniformly bounded and hence e(t) is uniformly bounded. Since $G_m(s)$ P(s) is SPR and strictly proper, $e_1(t) \to 0$ as $t \to \infty$

And u(t) can be shown to be bounded by the same reasoning. This can be done by checking the relation between u(t) and $s_1(t)$. Thus one can see that $\xi(t)$ is uniformly bounded. It, k_p is known, same conclusion holds obviously. □

Corollary 1

In Theorem 3, if we choose a Hurwitz polynomial P(s) of degree (n-p) such that $G_m(s)\ P(s)= k_m$ and we use the adaptive law (30) with a signal normalization (34), the state error e(t) and parameter error $\psi(t)$ are uniformly bounded. All the signals in the control loop are bounded and the output error $e_1(t)$ tends to 0 as t goes to infinity. And the parameter error vector $\psi(t)$ tends to 0 as t goes to infinity if persistently exciting (PE) condition is satisfied.

(Proof) Since the transfer function of the block B_1 is SPR and the block B_2 has a PR operator for the chosen μ_{1m}, it follows that $\epsilon(t)$ and $\psi(t)$ are uniformly bounded.

By checking inequality (2) it can be shown that $\epsilon_1(t) \in L_2$. In the similar way as in Theorem 3, it can be shown that $\nu(t)\epsilon_1(t) \in L_2$ and that $\psi(t)$ and its k-th derviatives belong to L_2 for k= 1,2, ..., n-p. And one can also see that the k-th derivatives of $\psi(t)$ belong to L_∞ for k=1,2,..., n-p-1 nad in turn that $\nu(t)\epsilon_1(t) \in L_\infty$

Along the proof of the boundedness of $y_p(t)$ in Theorem 3, there used no conditions on Gm(s). Thus as in Theorem 3, we can see that $y_p(t)$ and $\nu(t)$ are uniformly bounded. This implies that $\psi^T(t)\ \nu(t)$, $\epsilon_1(t)$, and $\chi(t)$ are uniformly bounded. Hence it follows that so are e(t) and $e_1(t)$. Therefore from eq. (34) $e_1(t)$ tends to 0 as t goes infinity. Since $\nu(t)\epsilon_1(t) \in L_2$, $\psi(t) \rightarrow 0$ as $t \rightarrow \infty$ if there exist positive constants T, α,β such that for all $t \geqslant 0$

$$\infty > \beta I \geq \int_t^{t+T} \nu(\tau)\,\nu(\tau)^T d\tau \geq \alpha I > 0 \quad (46)$$

Finally it is readily seen that u(t) and $\psi(t)$ are bounded as in Theorem 3. □

V. Computer Simulation Result

In this section we present some simulation results for third order continuous-time linear time-invariant plants of two types. The plants are assumed minimum phase and of relative degree 3.

Augmenting method and nonaugmenting method are applied to those types respectively to show the boundedness of the output of the controlled systems. The plant and the model transfer functions are chosen as shown in Table 1.

Table 1. The transfer functions of the plant and the model

Type	Plant $G_P(s)$	Model $G_m(s)$
(1)	$\frac{1}{s(s+2.5)\ (s+5)}$	$\frac{1}{(s+1)\ (s+2)\ (s+3)}$
(2)	$\frac{1}{s(s-0.5)\ (s+5)}$	

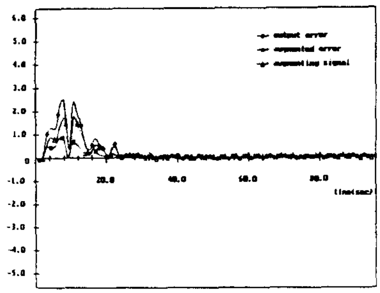
The operators P(s) in the form of a rational function of s used in the adaptive laws and corresponding μ's are tabulated in Table 2. The parmater-estimate-component-augmenting method and the nonaugmenting one ae compared.

Table 2. Operators and corresponding μ's for two methods

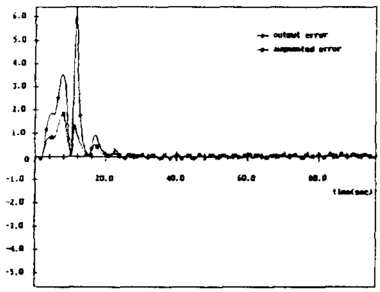
Method	operator P(s)	μ
Augmenting	(s+1) (s+2)	0. 12
Nonaugmenting	(s+1) (s+2) (s+3)	0. 05

Since the model has no zeros, the auxiliary signal generators are used as observers. Thus M(s) can be set arbitrarily in deterministic environments. In this simulation it is set (s+1) (s+2). The reference input r(t) is set to a square wave with the amplitude of 24 units and frequency 0.05 Hz. The constant parameter adaptation gain matrix Γ was chosen a unity matrix.

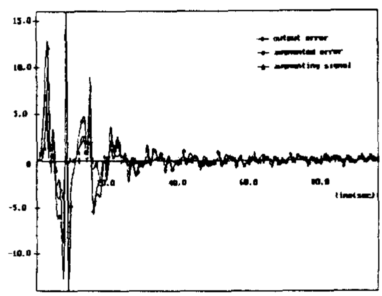
The simulation results for the proposed adaptive schemes show that these laws give bounded output error responses. In Fig. 4., (a) and (b) show the responses of the stable plant and (c) and (d) exhibit those of the unstable plant. In (a) and (b), both methods show bounded output errors which seem to approach to 0 asymptotically. Moreover it is observed that the proposed method gives a better initial transient output error response than the method of Monopoli's control-input augmentation. This can be explained qualitatively: Since in the initial time interval the proposed methods have an effect of adding stable zeros on the plant, a more smooth initial



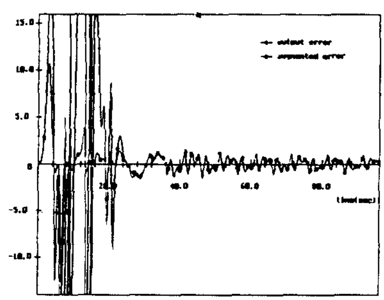
(a)



(b)



(c)



(d)

Fig.4. The output error responses and aygmentation signals for two plants.
(a) Augmenting
(b) Nonaugmenting
(c) Augmenting
(d) Nonaugmenting

output response can be obtained.

Even when the plant is given as type (2), the proposed methods still give good results though some oscillatory initial transients are observed. It is also observed that, however, the Monopoli's method fails to control this plant.

VI. Conclusions

In this paper, a new adaptive control scheme is proposed that uses a special form of rational function-type linear operator in the parameter adaptation and that removes the augmenting signal terms of the control input components. This adaptation scheme is applied to the MRAC of continuous-time, linear time-invariant, minimum-phase plants whose relative degrees are arbitrary. The asymptotic stability of the adaptive systems controlled by this scheme is shown by a hyperstability method. This scheme has the following advantages:

(1) A unified form of the adaptive law can be used for arbitrary relative degree plants.
(2) A wider range of parameter adaptation laws can be designed easily for various MRAC applications.
(3) This only needs the augmentation signal of parameter estimates components and hence the control structure need not be modified even though k_p is unknown.
(4) Augmentation is not required for arbitrary degree plants if the model has no zeros.

The computer simulation results show that the proposed scheme gives good results. The Extension of this scheme to discrete time is straightforward. The analysis of the robustness of this scheme to the unmodeled dynamics is left for future study.

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