Estimation of Frequencies of Multiple Sinusoids by the Modified ESPRIT Method

(수정된 ESPRIT 방법을 이용한 다단 정현파의 주파수 추정)

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要 約

조화 신호의 회복을 위한 수정된 ESPRIT(MESPRIT) 방법이 제시되고 분석된다. 백색 및 칼라 잡음으로 오염된 정현 신호의 주파수 추정을 제시된 방법으로 연구한다. MESPRIT 및 TK 방법으로 Monte-carlo 시뮬레이션하여 평균값, 실효값 및 상대 바이어스를 비교한다.

Abstract

The modified ESPRIT (MESPRIT) method for harmonic retrieval is presented and analyzed. The estimation of frequencies of sinusoidal signals corrupted by white or colored measurement noise is considered for the MESPRIT method. Monte-carlo simulations are conducted for the comparison of MESPRIT method with TK method in terms of sampled mean, root mean square and relative bias.

I. Introduction

Estimation of spectrum from finite noisy measurements is a very interesting and practical problem. It has been studied and used in many fields. With the rapid development of modern technology, the need for estimation of spectra becomes ever increasing, and therefore motivates more and more researchers on this issue.

We are mainly interested in estimation of multiple sinusoidal frequencies, or narrow-banded spectral estimation in its broad sense, from finite noisy data. As is well known, estimation of frequencies is closely related to spectral estimation[9]. As a special important case in spectral estimation, frequency estimation has recieved much attention in the last two decades[6-8][11][13], when researchers have concentrated themselves on improving resolution. As a result, a number of new ideas have been proposed[1][2][5][21][22].

The high resolution methods that have been proposed to solve the problem vary from linear

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prediction methods (Linear Predicition (LP) [10][18] and Forward-Backward LP (FBLP) [6][11][24]), Yule-Walker methods (Overdetermined Yule-Walker (OYW)[14] and High-Order Yule-Walker (HOYW)[15][16][19]) and subspace methods (Pisarenko Harmonic Decomposition (PHD)[22], MUltiple SIgnal Classification (MUSIC) [21] and EigenVector (EV)[23]) to Tufts-Kumaresan (TK)[3] method, Maximum Likelihood (ML)[4][17] method and Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT)[1][2] method. It is known so far that the ML method has the best performance and can be used to find the statistically efficient estimators[4] . However the ML function is highly non-linear in its parameters, only performed iteratively and therefore expensive to implement. On the other hand, the non-iterative TK method has been illustrated to have the second best performance.

In this paper, we present a modified ESPRIT (MESPRIT) method through an approach similar to ESPRIT. Using the MESPRIT method, we are to estimate the frequencies of multiple sinusoids from measurement data corrupted by white or colored noise in various examples. Monte-carlo simulations are conducted for MESPRIT and TK methods from a viewpoint of sampled mean, root mean square and relative bias. And then the performances of MESPRIT method are compared with those of TK [3][5] method.

II. Problem Formulation

Consider the following sinusoidal signal

$$x(t) = \sum_{i=1}^{m} \mathfrak{L}_i \sin(\omega_i t + \zeta_i) \qquad (1)$$

where $\mathfrak{L}_i, \zeta_i \in R$, $\omega_i \in (o, \pi)$ and $\omega_i \neq \omega_j$ for $i \neq j$.

Let y(t) denote the noise-corrupted measurements of x(t)

$$y(t) = x(t) + e(t) \qquad (2)$$

where e(t) is a sequence of independent and identically distributed random variable of zero mean and variance σ^2. It is assumed that x(t) and e(t) are uncorrelated for any t.

As is known, x(t) obeys the following autoregressive (AR) process[6][12]

$$A(q^{-1})x(t) = 0 \qquad (3)$$

where q^{-1} denotes the unit delay operator and $A(q^{-1})$ is a polynomial of of degree of 2m defined by

$$A(q^{-1})y(t) = 1 + a_1 q^{-1} + \cdots + a_{2m} q^{-2m} = \prod_{i=1}^{m} (1 + 2\cos\omega_i q^{-1} + q^{-2}) \qquad (4)$$

It follows from (2)-(4) that y(t) obeys the following degenerate autoregressive moving-average (ARMA) process[12]

$$A(q^{-1})y(t) = A(q^{-1})e(t) \qquad (5)$$

It is easy to show that the roots of A(z) appear on the unit circle at $\exp(\pm j\omega_i)$, i=1, 2, ..., m.

Next multiplying both sides of (5) by a nonzero polynomial in q^{-1}, say $B(q^{-1})$, we obtain

$$C(q^{-1})y(t) = C(q^{-1})e(t) \qquad (6)$$

where

$$C(q^{-1}) = B(q^{-1})\ A(q^{-1}) \qquad (7)$$

Throughout the paper it will be assumed that $C(q^{-1})$ is a polynomial degree L ($L > 2m$) given by

$$C(q^{-1}) = c_0 + c_1 q^{-1} + \cdots + c_L q^{-L} \qquad (8)$$

The problem is to estimate the angular frequencies $\{\omega_i\}$ from the available data y(1),...., y(N).

The frequency estimates are usually obtained from the following two-step procedure:

(i) Estimate the coefficients $\{c_i\}$

(ii) Find the frequency estimates $\{\omega_i\}$ either from the angular positions of 2m largest-modulus roots, i.e., $\varphi_i \exp(\pm j\hat{\omega}_i)$, i=1,..., m, of C(z) or from those values at the spectrum $1/\|\hat{C}(\exp(j\omega))\|^2$ reaches its largest peaks.

In subspace methods, specifically in the MUSIC, EV, ESPRIT and MESPRIT methods, the first step is to find the invariant subspace of the covariance matrix. The frequencies are then estimated from a spectrum-like function (or its reciprocal) through minimization (or maximization).

III. Modified ESPRIT method

In this section we will consider a modified ESPRIT (MESPRIT) method for estimation of frequencies which is based on the concepts of signal subspace and closely related to singular value decomposition (SVD) [20] [24] in on way.

The orignal idea of MESPRIT method was drawn from the technique[1][2] that was discussed in the direction of arrival (DOA) estimation and the estimation of parameters of complex sinusoids. The technique[1][2] called ESPRIT used the generalized eigenvalue problem and matrix pencil that was constructed by auto- and crosscovariance matrix.

A modified ESPRIT (MESPRIT) method, which uses the measured data directly, a simple eigenvalue principle and the truncated SVD[5][20] and avoids the use of the matrix pencil, will be presented. In the following we will discuss this MESPRIT method.

For convenience, first we will consider the case of complex sinusoids (note that real sinusoids can be expressed into complex sinusoids).

Let

$$x(t)=\sum_{i=1}^{m} \mathcal{L}_i \exp(j\,\omega_i\,t) \tag{9}$$

$$y(t)=x(t)+e(t), \quad t=1,\cdots,N \tag{10}$$

where $\omega_i \epsilon$ $(-\pi,\ \pi)$ and $\mathcal{L}_i$ are the normalized frequency and complex amplitude of the i:th complex sinusoid, respectively, and e(t) is white Gaussian noise of zero mean and variance σ^2. It is assumed that $\omega_i \neq \omega_j$ for $i \neq j$.

Assume that there is no noise in the measurement, i.e. y(t)=x(t) in (10). Next we write $\mathcal{L}_i$, cf (9), into the following form

$$\mathcal{L}_i=|\mathcal{L}_i|\exp(j\phi_i) \tag{11}$$

Instead of constructing auto- and crossovariance matrices, define two 2(N-L) x L matrices $\mathbf{X}_1$ and $\mathbf{X}_2$ as follows;

$$\mathbf{X}_1=\begin{pmatrix} x_1 & x_2\cdots\cdots & x_L \\ \vdots & \vdots & \vdots \\ x_{N-L} & x_{N-L+1}\cdots & x_{N-1} \\ x^*_{L+1} & x^*_{L}\cdots\cdots & x^*_2 \\ \vdots & \vdots & \vdots \\ x^*_N & x^*_{N-1} & x^*_{N-L+1} \end{pmatrix},\quad \mathbf{X}_2=\begin{pmatrix} x_2 & x_3\cdots\cdots & x_{L+1} \\ \vdots & \vdots & \vdots \\ x_{N-L+1} & x_{N-L+2}\cdots & x_N \\ x^*_{L} & x^*_{L-1}\cdots\cdots & x^*_1 \\ \vdots & \vdots & \vdots \\ x^*_{N-1} & x^*_{N-2}\cdots & x^*_{N-L} \end{pmatrix} \tag{12}$$

where "*" denotes complex conjugate. To assure that $\mathbf{X}_1$ and $\mathbf{X}_2$ have rank m, L should be chosen between 2m and N-2m[20].

It is straightforward to show that

$$\mathbf{X}_1=\mathbf{E}_1\mathbf{A}\ \mathbf{E}_2 \tag{13}$$

$$\mathbf{X}_2=\mathbf{E}_1\mathbf{\Phi}\mathbf{A}\ \mathbf{E}_2 \tag{14}$$

where

$$\mathbf{E}_1=\begin{pmatrix} \exp(j\omega_1+j\phi_1) & \exp(j\omega_2+j\phi_2) & \cdots & \exp(j\omega_m+j\phi_m) \\ \vdots & \vdots & & \vdots \\ \exp(j\omega_1(N-L)+j\phi_1) & \exp(j\omega_2(N-L)+j\phi_2) & & \exp(j\omega_m(N-L)+j\phi_m) \\ \exp(-j\omega_1(L+1)-j\phi_1) & \exp(-j\omega_2(L+1)-j\phi_2) & \cdots & \exp(-j\omega_m(L+1)-j\phi_m) \\ \vdots & \vdots & & \vdots \\ \exp(-j\omega_1 N-j\phi_1) & \exp(-j\omega_2 N-j\phi_2) & \cdots & \exp(-j\omega_m N-j\phi_m) \end{pmatrix} \tag{15}$$

$$\mathbf{E}_2=\begin{pmatrix} 1 & \exp(\omega_1)\cdots\exp(j\omega_1(L-1)) \\ 1 & \exp(j\omega_2)\cdots\exp(j\omega_2(L-1)) \\ \vdots & \vdots \qquad \vdots \\ 1 & \exp(j\omega_m)\cdots\exp(j\omega_m)L-1)) \end{pmatrix} \tag{16}$$

$$\mathbf{\Phi}=\mathrm{diag}\{\exp(j\omega_1),\ \exp(j\omega_2),\cdots,\exp(j\omega_m)\} \tag{17}$$

$$\mathbf{A}=\mathrm{diag}\{|\mathcal{L}_1|,\ |\mathcal{L}_2|,\cdots,\ |\mathcal{L}_m|\} \tag{18}$$

Consdier the following simple eigenvalue problem:

$$\mathbf{X}_2^{+}\mathbf{X}_1\mathbf{z}=\lambda\mathbf{z} \tag{19}$$

where $\mathbf{X}_2^{+}$ is the pseudoinverse of $\mathbf{X}_2$. Since E_1 and E_2 are of full rank, $\mathbf{X}_2$ can be expressed into

$$\mathbf{X}_2^{+}=\mathbf{E}_2^{+}\mathbf{A}^{-1}\mathbf{\Phi}^{-1}\mathbf{E}_1^{+} \tag{20}$$

Then $\mathbf{X}_2^{+}\mathbf{X}_1 = E_2^{+}\Phi^{-1}E_2$. Substituting this to (19), finally we have

$$\mathbf{E}_2^{+}\mathbf{\Phi}^{-1}\mathbf{E}_2\mathbf{z}=\lambda\mathbf{z} \tag{21}$$

Since both E_2 and E_2^{+} are of rank m and Φ^{-1} is nonsingular, $E_2\Phi^{-1}E_2$ is also of rank m. Then it is easy to show that the eigen-problem has m eigenvalues $\lambda_i = \exp(-j\omega_i)$, i=1, ..., m, with the corresponding eigenvectors z_i being the i:th column of $'E_2$, and L-m zero eigenvalues with the corresponding eigenvectors z_i in the null space of E_2. The nonzero eigenvalues are hence located on the unit circle at $\exp(-j\omega_i)$, i=1, ..., m.

Thus the true frequencies can be obtained from the unit-modulus eigenvalue of $\mathbf{X}_2^{+}\mathbf{X}_1$ in the noiseless case. For noisy data, the eigenvalues will be perturbed. However, it is known that the distinct eigenvalues are not so sensitive to noise perturbation[24]. Hence the frequency estimates can be obtained from the m largest eigenvalues. In order to suppress noise, the signal subspace concept that uses the truncated SVD to filter out some noise effect, can be used to estimate the "signal matrices" of the noisy matrices $\hat{X}_2$ (or $\hat{X}_2^+$) and $\hat{X}_1$.

In the case of m real sinusoids, some changes should be made in the analysis. Since there are 2m complex sinusoids in the signal, L should be chosen between 2m and N-2m. The frequency estimates can be obtained from the 2m eigenvalues of largest modulus which, in the noiseless case, would appear in complex conjugate paris at $\exp(\pm j\omega_i)$, i=1, ...,m. For noisy measurements, the eigenvalues will be purturbed by the noise. Using the tructated SVD, however, the signal matrix can be estimated from the noisy matrix. The frequency estimates can be obtained from the signal matrix which would have the 2m eigenvalues in complex conjugated pairs at exp $(\pm j\hat{\omega}_i)$, i=1,...,m.

IV. Simulation Examples

In this section, simulation examples are given for frequency estimation performances of sinusoids-in-noise process, using the method developed in the previous sections. In all of the examples, the signal was assumed to consist of two sinusoids. Various cases were investigated.

The algorithm estimating the frequencies can be summarized in the following.

(i) Reduce the noise effect on the noisy matrices $\hat{X}_1$ and $\hat{X}_2$ by truncated SVD.

(ii) Compute the truncated pseudoinverse of $\hat{X}_2$.

(iii) Find the m largest eigenvalue $\hat{\lambda}_i$'s of $\hat{X}_{2,T}$ $\hat{X}_{1,T}$.

(iv) From $\hat{\lambda}_i$, e estimates $\hat{\omega}_i$ are obtained by $\hat{\omega}_i = -\mathrm{Im}\,[\ln \hat{\lambda}_i]$.

Example 1. (short data, widely separate frequencies)

The data we simulated is given by

$$y(t)=\sqrt{2}\sin(0.7226t)+\sqrt{2}\sin(1.0367t)+e(t) \tag{22}$$

where e(t) is zero mean white Gaussian process. The data length is N=64, and the SNR is 10 dB, i.e., SNR1= SNR2=10 dB (SNR=10 log $(\mathcal{L}_i^2/2\sigma^2)$ i=1, 2, cf. (1) and (2)). We have used 50 different noise realization and computed the frequency estimates for a number of "design parameters" like L.

The following quantities have been evaluated (for i=1,2) in all the examples:

$$\bar{\omega}_i=\frac{1}{50}\sum_{j=1}^{50}\hat{\omega}_i^j \qquad \text{(sampled mean value of } \hat{\omega}_i\text{)}$$

$$\mathrm{rms}(\hat{\omega}_i)=\sqrt{\mathrm{mse}(\hat{\omega}_i)} \qquad \text{(root mean square of } \hat{\omega}_i\text{)}$$

where

$$\mathrm{mse}(\hat{\omega}_i)=\frac{1}{50}\sum_{j=1}^{50}(\hat{\omega}_i^j-\omega_i)^2 \qquad \text{(mean square error of } \hat{\omega}_i\text{)}$$

$$\delta(\hat{\omega}_i)=\frac{|\bar{\omega}_i-\omega_i|}{\hat{\omega}_i} \qquad \text{(percentage bias of } \hat{\omega}_i\text{)}$$

Results for the approximately "best " design parameters are given in Table 1. For each method, the spectrum for one realization is show in Figure 1. The angular positions of frequency estimates for 50 realizations illustrated in Figure 2.

It is easy to see from this example that the MESPRIT and TK methods provide about the same accuracy, which is quite satisfactory.

Example 2. (short data, closely spaced frequencies)

In this example the data is taken from the following process:

$$y(t)=\sqrt{2}\sin(0.7226t)+\sqrt{2}\sin(0.8168t)+e(t) \tag{23}$$

where e(t) is a Gaussian white noise of zero mean and variance $\sigma^2=0.01$ (SNR=20 dB), the data length is 64, and 50 noise realizations are used. The procedures in Example 1 are repeated. The performances are summarized in Table 2.

It can be seen from this example that the performances provided by the MESPRIT method are comparable to those in TK method.

Example 3. (longer data, widely separate frequencies)

The process in this example satisfies (22) but we have taken 200 data and simulated for 25

Table 1. Estimation performances of Example 1.

method	$\bar{\omega}_1$	$\bar{\omega}_2$	rms $(\hat{\omega}_1)$	rms $(\hat{\omega}_2)$	$\delta(\hat{\omega}_1)$	$\delta(\hat{\omega}_2)$	design parameter
TK	0.7227	1.0367	0.0020	0.0025	0.00019	0.00010	L=35
MESPRIT	0.7227	1.0363	0.0020	0.0024	0.00012	0.00040	L=40

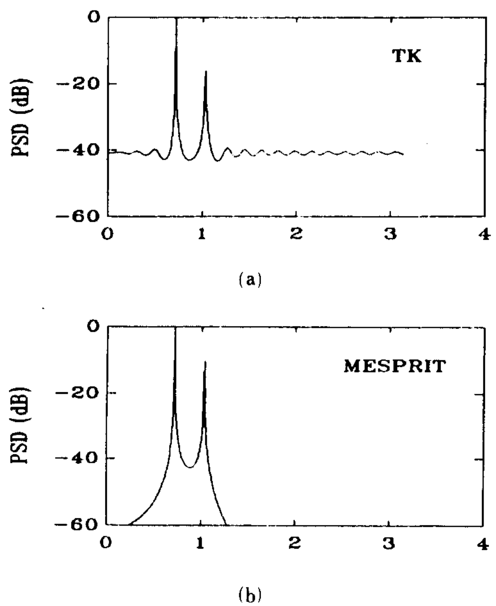


Fig.1. Normalized spectral densities of Example 1. The angular frequency ω is in radian/sec.

realizations. The SNR is 4 dB. Results are presented in Table 3.

Example 4. (longer data, closely space frequencies)

This example is the same as Example 2 except that N=200, SNR=12 dB, and that 25 noise

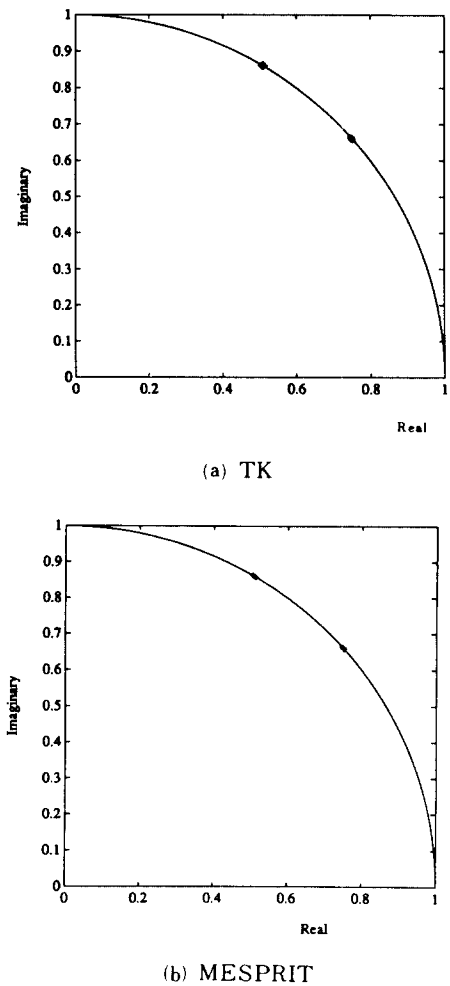


Fig.2. The angular positions of frequency estimates for 50 realizations of Example 1.

Table 2. Estimation performances of Example 2.

method	$\bar{\omega}_1$	$\bar{\omega}_2$	rms $(\hat{\omega}_1)$	rms $(\hat{\omega}_2)$	$\delta(\hat{\omega}_1)$	$\delta(\hat{\omega}_2)$	design parameter
TK	0.7227	0.8169	0.0020	0.0019	0.00013	0.00008	L=40
MESPRIT	0.7227	0.8167	0.0021	0.0020	0.00023	0.00016	L=40

Table 3. Estimation performances of Example 3.

method	$\bar{\omega}_1$	$\bar{\omega}_2$	rms $(\hat{\omega}_1)$	rms $(\hat{\omega}_2)$	$\delta(\hat{\omega}_1)$	$\delta(\hat{\omega}_2)$	design parameter
TK	0.7226	1.0368	0.0007	0.0007	0.00001	0.00005	L=50
MESPRIT	0.7226	1.0368	0.0007	0.0007	0.00007	0.00009	L=60

Table 4. Estimation performaces of Example 4.

method	$\bar{\omega}_1$	$\bar{\omega}_2$	rms $(\hat{\omega}_1)$	rms $(\hat{\omega}_2)$	$\delta(\hat{\omega}_1)$	$\delta(\hat{\omega}_2)$	design parameter
TK	0.7224	0.8170	0.0005	0.0003	0.00020	0.00019	L=60
MESPRIT	0.7225	0.8169	0.0005	0.0003	0.00014	0.00013	L=60

realizations are run. Estimation performances are listed in Table 4.

The following comments are made from the last two examples:

(i) The MESPRIT and TK methods can provide better estimates for longer data even if the SNR is lower, cf Example 1 and 2. This coincides with the theoretical conclusion that for large ssamples (i.e., $N \rightarrow \infty$) they can give consistent frequency estimates.

(ii) Although the SNR are low, the MESPRIT method still gives accurate results, especially when the frequency difference is small.

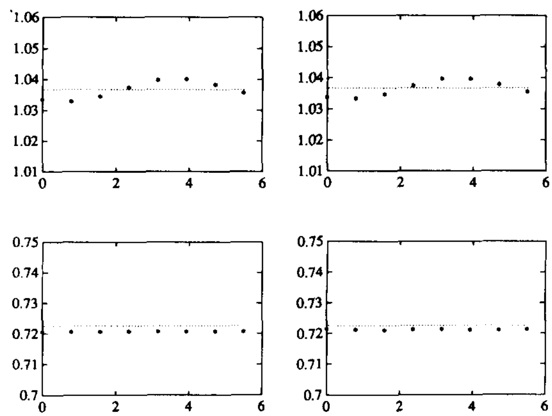
In all the examples above, we have set the initial phases to zeros, which is not necessarily true in the practical situations. Although the initial phases can be estimated by some techniques, they may affect the estimation accuracy. In the following, we give a example.

Example 5. (short data, initial phase sensitivity)

The tested process is now given by the following:

$$y(t) = \sqrt{2}\sin(0.7226t) + \sqrt{2}\sin(1.0367t+\zeta) + e(t) \quad (24)$$

We have taken one realization from Example 1 and estimated the frequencies using the "best" design parameters in Table 1 for a number of initial phase values: $\zeta=[0,\pi/4,\pi/2,3\pi/4,\pi,5\pi/4,3\pi/2,7\pi/4]$. Estimation results are shown in Figure 3.



(a) TK.   (b) MESPRIT

Fig.3. Frequency estimates with diffrerent initial phase $(0,\pi/4,\pi/2,3\pi/4,\pi,5\pi/4,3\pi/2,7\pi/4)$ of Example 5.

We can see from Figure 3 that the estimates from the MESPRIT and TK methods are not so sensitive to the initial phases, no matter whether the number of data is large or small. This is well understood, in MESPRIT and TK methods for instance, since (5) and (6) carries no information about the initial phases. Though we have used one noise realization in simulation, the conclusion should hold for many realizations.

Example 6. (short data, sensitivity to noise)

The process in this example satisfies Example 1 except that SNR is varied. We have analyzed the sensitivity to noise and estimated the frequencies using the "best" design parameters in Table 1 from a number of SNR's, namely, [0, 5, 10, 20, 40 (dB)]. The performances are summarized in Table 5, Table 6 and Table 7.

The following comments are made from Example 6.

(i) The MESPRIT method is shown to be as good as TK method in terms of Sampled means root mean squares and relative biases for high SNR.

(i) For the MESPRIT method, the lower SNR is, the smaller eigenvalue difference σ_{2m}- σ_{2m+1} is.

In the previous sections we concetrated ourselves only on the white measurement noise, the colored case was not studied. Below we give an example in which the noise is a moving-average process.

Example 7. (short data, colored noise, robustness).

This example is the same as Example 1 except that e(t) is a MA process of order 3:

$$e(t) = \varepsilon(t) - 0.823\,\varepsilon(t-1) + 1.151\,\varepsilon(t-2) - 0.305\,\varepsilon(t-3) \tag{25}$$

where $\epsilon(t)$ is an uncorrelated random sequence of zero mean and variance 0.0251. This corresponds

Table 5. Estimated sampled means of Example 6.

SNR		0	5	10	20	40	design parameter
TK	$\bar{\omega}_1$	0.7227	0.7227	0.7227	0.7226	0.7226	L=35
	$\bar{\omega}_2$	1.0371	1.0367	1.0367	1.0367	1.0367	
MESPRIT	$\bar{\omega}_1$	0.7230	0.7227	0.7227	0.7226	0.7226	L=40
	$\bar{\omega}_2$	1.0362	1.0362	1.0363	1.0366	1.0367	

Table 6. Estimated rms's of Example 6.

SNR		0	5	10	20	40	design parameter
TK	rms ($\hat{\omega}_1$)	0.0067	0.0036	0.0020	0.0006	0.00006	L=35
	rms ($\hat{\omega}_2$)	0.0085	0.0046	0.0025	0.0008	0.00008	
MESPRIT	rms ($\hat{\omega}_1$)	0.0068	0.0037	0.0020	0.0006	0.00008	L=40
	rms ($\hat{\omega}_2$)	0.0081	0.0043	0.0024	0.0008	0.00008	

Table 7. Estimated relative biases of Example 6.

SNR		0	5	10	20	40	design parameter
TK	$\delta(\hat{\omega}_1)$	0.00015	0.00023	0.00019	0.00008	0.000008	L=35
	$\delta(\hat{\omega}_2)$	0.00038	0.00009	0.00010	0.00005	0.000006	
MESPRIT	$\delta(\hat{\omega}_1)$	0.00055	0.00022	0.00012	0.00004	0.000004	L=40
	$\delta(\hat{\omega}_2)$	0.00055	0.00051	0.00040	0.00014	0.000014	

Table 8. Estimation performances of Example 6.

method	$\bar{\omega}_1$	$\bar{\omega}_2$	rms ($\hat{\omega}_1$)	rms ($\hat{\omega}_2$)	$\delta(\hat{\omega}_1)$	$\delta(\hat{\omega}_2)$	design parameter
TK	0.7228	1.0369	0.0009	0.0006	0.00032	0.00017	L=25
MESPRIT	0.7228	1.0370	0.0009	0.0006	0.00033	0.00024	L=25

to SNR = 11 dB.

Estimation performances are evaluated using the methods developed for white noise in the previous section. Results are provided in Table 8.

For the MESPRIT and TK methods, the frequency estimates are very accurate, even if the noise level is low. We may say that they have certain robustness. This can heuristically be argued as follows. Due to the low noise level, the eigenvalues and eigenvectors of the MESPRIT matrices of colored noise are not much perturbed from those of white noise. In other words, the signal subspace is less perturbed than the noise subspace in this example, so the MESPRIT method can work to some extent.

V. Conclusions

A modified ESPRIT (MESPRIT) method for harmonic retrieval was presented and studied for the problem of estimating the frequencies of sinusoidal signals corrupted by white or colored measurement noise in this paper. Monte-carlo simulation examples were conducted for the MESPRIT and TK methods. Results of sampled mean, root mean square and relative bias were compared.

The following conclusions are drawn:

(i) When modeling the sinusoids-in-noise process, a high-order model is preferred in order to achieve better estimation accuracy.

(ii) When only short data records are available, it is advantageous to choose the MESPRIT and TK methods. Furthermore the estimates from MESPRIT method may be not sensitive to the initial phases as TK method.

(iii) The MESPRIT method is shown to be as good as TK method in terms of sampled mean, root mean square and relative bias of estimated frequencies for high SNR's.

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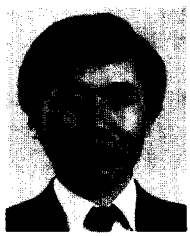

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