Structural Convergence Improvement Schemes on Adaptive Control Redesigning a Lyapunov's Function

(Lyapunov 함수를 재설계한 적응제어와의 구조적 수렴향상 방법에 대한 연구)

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要 約

적응제어의 수렴분석 방법이 지난 수십년간 연구되어 왔으나, 적응제어 시스템의 고속수렴에 대한 구조적 중요성은 아직도 논쟁이 되고 있다. 본 논문은 적응제어 error 모델에 대한 지수적 수렴속도의 상대적 개선방법을 고찰하였다. Error시스템의 궤환구조를 변화시킴으로써 수렴속도를 개선하기 위해 Lyapunov의 직접방식을 적응제어 시스템에 적용하였다. 몇가지 simulation 예를 들어, 이 방법의 고속수렴과 robustness를 보였다.

Abstract

The convergence analysis of adaptive control schemes has been studied over the past decades, but the importance of structure to fast convergence of adaptive control systems is still a controversial issue. This paper deals with the relative improvement of the exponential rate of convergence in adaptive error models. The Lyapunov's direct method is applied to adaptive control systems in order to improve the convergence rate by modifying the feedback structure of the error systems. Some simulation examples are illustrated to show fast convergence and robustness of these schemes.

I. Introduction

In the 1970's, global asymptotic stability was the primary research issue in adaptive control. Monopoli [1] improved the model reference adaptive control (MRAC) scheme by introducing the augmented error technique. Landau [2] surveyed the model reference adaptive system (MRAS) techniques. Narendra et al. [3,4] Morse [5], Egardt [6], Goodwin et al. [7], and Landau et al. [8,9] contributed several important papers which address the global asymptotic stability of the MRAC systems using either the Lyapunov's direct method or the Popov's hyperstability approach.

In the early 1980's robustness and convergence

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studies on adaptive control became prevalent because a practical adaptive controller in the presence of uncertainties includes unmodeled high-frequency dynamics, observation noise, high-frequency reference or control inputs, etc. [10]. Despite global stability, the mechanism and the convergent properties of existing adaptive algorithms may be easily affected when exposed to the uncertainties from a practical point of view. An adaptive control algorithm should be tolerant in the context of parameter convergence and robustness both in the steady-state response and in the transient characteristics before it can be implemented to practical applications such as robotic control, aircraft flight control, product flow-control, etc. Various adaptive schemes [3,6,11,12] face instability mechanisms inherent in the adaptive error systems such as linear instability, fast adaptation instability, high frequency instability, and parameter drift [13,14,15]. In the MRAC scheme, high frequency modes are anticipated if the dynamics of the reference model are too fast or if the adaptation gain is large.

The objective of this paper is to study the convergent behavior of adaptive control systems with the synthesis of a feedback structure. A refinement of the error model, or a modification of the feedback block can improve the rate of convergence considering a tradeoff between robustness and fast convergence. In Section 2, a typical error model of adaptive control systems by Narendra et al. [3,4] is shown. New results on designing the feedback structure by the Lyapunov's method appear in Section 3 where we introduce a generalized design tool for adaptive control algorithms which improves the rate of convergence. Finally, Section 4 deals with simulation examples, and conclusions.

II. The Error Model of Adaptive Control Systems

A generalized form of adaptive control algorithms can be represented by the following bilinear differential equations [10]:

$$\dot{x} = Ax + f_1(r;\xi,\phi) \tag{1}$$

$$\dot{e} = A_C e + f_2(\xi,\phi,e) \tag{2}$$

$$\dot{\phi} = f_3(\xi,\phi,e) \tag{3}$$

where A, A_c are stable matrices and $f_1(.,..,.)$, $f_2(.,.,.)$, $f_3(.,.,.)$ are bilinear functions. Equation (1) shows the state representation of the controlled process while (2) and (3) can be thought of as the dynamics of the state errors and the parameter errors, respectively.

The adjustable controller establishes 2n parameters from the relation of the controlled plant and the model equations in the linear time-invariant (LTI) single-input single-output (SISO) case [3,4], and the error equation can be expressed as

$$\dot{e}(t) = A_C e(t) + b_C v(t) \tag{4}$$

$$e_1(t) = c_1 e(t) \tag{5}$$

Here, $A_c \in R^{n1xn1}$; e(t), b_c, $C_1^T \in R^{n1}$; $v(t) = \phi^T(t)\, g(t) \in R^{n1}$ where $g(t) = g(\xi'\ \phi, e)$ and the tracking error is defined by $e(t) = x_{mc}(t) - x_c(t)$ where $x_{mc}(t)$ and $x_c(t)$ represent the n_1 x1 state vector of the reference model and that of the controlled system, respectively. $\phi(t)$ and $\xi(t)$ are the m_1 x1 vectors of the parameter errors and the observation variables, respectively.

If the Lyapunov function candidate is defined as

$$V(e,\phi,t) = e^T(t)\,Pe(t) + \phi^T(t)\,\Gamma^{-1}\phi(t) \tag{6}$$

then the condition for global asymptotic stability of the overall system is derived by the MKY lemma and the following lemma,

Lemma 2.1 [3, Narendra et al.] Given a stable $n_1 x n_1$ matrix A_c, a symmetric $m_1 x m_1$ matrix Γ, vectors c^T, $b_c \in R^{n1}$, and $\xi(t) \in R^{m1}$ whose elements $\xi_i(t)$'s are bounded and piecewise continuous, the equilibrium state of $(n_1 + m_1)$ differential equations, (4) and

$$\dot{\phi} = -\Gamma e_1 g(\xi) \tag{7}$$

are stable and $\lim_{t\to\infty} e_1(t) = 0$ if the transfer function $c_1(sI - Ac)^{-1}b_c$ is stricIty positive real (SPR). Moreover, if $g(\xi(t))$ is sufficiently rich, then

$$\lim_{t\to\infty} \|\phi(t)\| = 0 \tag{8}$$

For example, $c_1 = [10\ ...\ 0] \in R^{n1}$, $n_1 = 3n-2$, and $m_1 = 2n$ when the relative degree is one.

The condition of sufficient richness is necessary and sufficient for uniformly asymptotic stability

(USA). The following theorem shows a richness condition for $b_c g^t(\xi(t))$.

Theorem 2.1 [16, Morgan] Let A_c be a stable $n_1 \times n_1$ matrix of bounded piecewise continuous functions. Let P(t) be a symmetric positive definite matrix of bounded continuous functions such that $\dot{P} + A_c^T P + PA_c$ is negative definite. Let $b_c g(\xi^T(t))$ be an $n_1 \times m_1$ matrix of bounded piecewise continuous functions. Assume that there exist positive numbers $T_0, \epsilon_0, \delta_0$ such that, given $t_1 \geqslant 0$ and a unit vector $w \in r^{m_1}$, there is a $t_2 \in [t_1, t_1 + T_0]$ such that

$$\left| \int_{t_2}^{t_2+T_0} b_c g^T(\xi(\gamma)) w dr \right| \geq \epsilon_0 \tag{9}$$

Then the system

$$\dot{z}(t) = \begin{bmatrix} A_c & b_c g^T(\xi) \\ -\Gamma g(\xi) b_c^T P & 0 \end{bmatrix} z(t) \tag{10}$$

is USA where $z(t) \triangleq [e^T(t); \phi^T(t)]^T$.

Note that, if the P matrix is constant, the transfer function of this error model is SPR satisfying the following positive real conditions,

$$A_c^T P + PA_c = -Q \tag{11}$$

$$b_c^T P = c_1 \tag{12}$$

where $P = P^T > 0$ and $Q = Q^T > 0$. The block diagram of the error system is shown in Fig.1. The system consists of two parts — a SPR LTI feedforward block and a nonlinear time-varying (NLTV) feedback block — and therefore, Popov's hyperstability theory can also be applied to the above error model. Note that Popov's hyperstability approach is more flexible than Lyapunov's direct method since the conditions for the closed-loop error system are general and simple at the cost of convergence. Different configurations of the error models can be achieved by using various controller structures for adaptive systems.

III. Structural Modification to Improve The Rate of Convergence

1. *Rate of Convergence*

It is known that the overall error dynamics are uniformly asymptotically stable if and only if the observation vector $\xi(t)$ is sufficiently rich and if this is true then an upper bound on the rate of

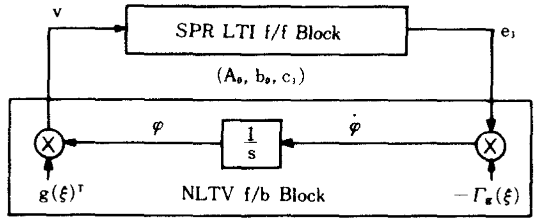

Fig.1. A typical error model.

convergence is available [17]. Usually, the rate of convergence means the minimum amount of time t_r so that solutions with unit initial condition at time t_0 will have length less than or equal to 1/2 (or other fractional numbers) for all times greater than $t_0 + t_r$. Therefore, the global speed of convergence of the solution to (10) is completely characterized by the rate of convergence t_r. Little is known about the rate of convergence in connection with system properties such as transient response, tracking performance, robustness, etc. Relative improvement on the transient characteristics such as the rate of convergence can be obtained by modifying the NLTV feedback structure of the error system with the SPR LTI feedforward system. We propose two main theorems which generalize a refinement of the feedback structure of the error system to obtain improved exponential convergence. A condition of the richness of the input is important in the context of our results.

2. *Main Results; Synthesis of Feedback Structures That Improve The Rate of Convergence*

The transient performance may be improved by forcing the resultant time-derivative of the Lyapunov function candidate into a more negative value given the same form of the chosen Lyapunov function. Here, we use this fact to modify the control structure and thus improve the rate of convergence.

Let the Lyapunov function candidate be defined as

$$V(e, \phi, t) = \| e \|_P^2 + \| \phi \|_{\Gamma^{-1}}^2 = z(t)^T \Sigma z(t) \tag{13}$$

where $\Sigma = \begin{bmatrix} P & 0 \\ 0 & \Gamma^{-1} \end{bmatrix}$

Using the generalized adaptive algorithm in the bilinear form of (2) and (3), and differentiating $V(e, \phi, t)$' we obtain

$$\dot{V} = -\|e\|_Q^2 + 2[e^T P f_2(\xi, \phi, e) + \phi^T \Gamma^{-1} \dot{\phi}] < 0 \quad (14)$$

or

$$\dot{V} = -\|e\|_Q^2 + 2[e_1(t)v(t) + \phi^T \Gamma^{-1} \dot{\phi}] < 0 \quad (15)$$

where $A_c^T P + PA_c = -Q$, $b_c^T P = c_1$, $c_1 e = e_1$ and $f_2(\xi, \phi, e) = b_c v$ with $P = P$, $Q = Q^T > 0$. $v(t)$ was defined in (4).

Consider the error equation of the adaptive system for both the tracking error and the parameter error in (10) and the Lyapunov function defined as in (13). The derivative of the Lyapunov function plays an important role in improving the rate of convergence because a more negative value of $V(t)$ results in a better performance. Let $\dot{\phi} = -\Gamma g(\xi)e_1$ and $v(t) = \phi^T(g(\xi) - \phi e_1 \xi^T \Lambda\xi)$ in (15), then,

$$\dot{V}(t) = -z(t)^T \Omega z(t) < 0 \quad (16)$$

where $\Omega = \begin{bmatrix} Q & 0 \\ 0 & e_1^2 \xi^T \Lambda \xi I \end{bmatrix}$ and $Q = -(A_c^T P + P A_c)$

Subtracting $V(t)$ from $V(t + T)$, we obtain

$$V(t+T) - V(t) = \int_t^{t+T} \dot{V}(\tau)\,d\tau = -\int_t^{t+T} z^T(\tau)\Omega z(\tau)\,d\tau$$
$$= -z^T(t)\left[\int_t^{t+T} \Phi^T(\tau, t)\Omega\Phi(\tau, t)\,d\tau\right]z(t) \quad (17)$$

where $\Phi(\tau, t)$ is the transition matrix defined by $\Phi(\tau, t)z(t) = z(\tau)$. If we assume that the persistency of excitation holds

$$\beta I \geq \int_t^{t+T} \Phi^T(\tau, t)\Omega\Phi(\tau, t)\,d\tau \geq \alpha I \quad (18)$$

where $\beta > \alpha > 0$, then we can conclude that

$$V(t+T) - V(t) \leq -\frac{\alpha}{\lambda_{max}(\Sigma)} V(t) \quad (19)$$

Rearranging the above equation, the exponential convergence of the Lyapunov function is obtained as

$$V(t+\tau) \leq (1-c)\,V(t) \quad (20)$$

$$V(t+\tau) \leq e^{-c} V(t) \quad (21)$$

$$V(tN) \leq e^{-cN} V(0) \quad (22)$$

where $1 > c = \frac{\alpha}{\lambda_{max}(\Sigma)} > 0$. Thus, $z(t)$ converges to zero exponentially fast from the definition of $V(\cdot)$. Proof of the exponential convergence of the discrete-time version is shown in [18] and [19] . Here, it is worth mentioning that the exponential rate will be increased for large values of α and for small $\lambda_{max}(\Sigma)$. The former implies sufficient excitation of input signals or a larger positive definite Ω matrix-structural modification of the NLTV block of the error system in Fig.2-while the latter implies large adaptation gains. This leads to limitations on the adaptation gain, Γ, considering robustness. In the next theorem, we examine the convergence properties for two choices of $v(t)$ in the control law, e.g. in (15).

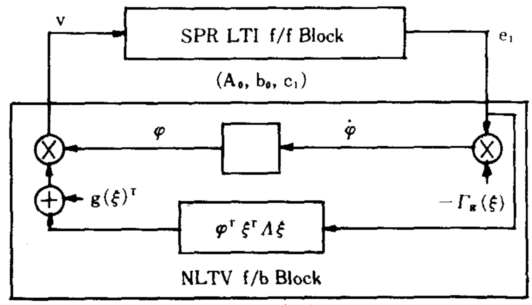

Fig.2. An example of improved schemes.

Theorem 3.1 (Fast Convergence) Let the Lyapunov function be defined as in (13). Consider two choices for $v(t)$ in (4), or in (15). First $v(t) = v_1(t) \triangleq \phi^T g(\xi, \phi, e)$, and second $v(t) = v_2(t) \triangleq \phi^T[g(\xi, \phi, e) + h(\xi, \phi, e)]$ where $g(\xi, \phi, e)$ and $h(\xi, \phi, e)$ are any NLTV functions in R^{m_1} guaranteeing that $\dot{V}(t) \leqslant 0$ using $v_2(t)$, that is,

$$\phi^T[(g(\xi, \phi, e) + h(\xi, \phi, e))e_1 + \Gamma^{-1}\dot{\phi}] \leq 0 \quad (23)$$

and $\lim_{t\to\infty} |\dot{V}(t)| = 0$. For example, $g(\xi, \phi, e) = \frac{\xi}{\lambda_1 + \lambda_2 \xi^T \xi}$, $h(\xi, \phi, e) = -\phi e_1 \xi^T \Lambda\xi$ with positive definite Λ and $\lambda_1, \lambda_2 > 0$. Let the adaptive algorithm be

$$\dot{\phi} = -\Gamma g(\xi, \phi, e)\, e_1, \ \Gamma > 0 \tag{24}$$

Then, the exponential rate of convergence is relatively improved using $v_2(t)$ compared with that using $v_1(t)$ at time $t_1 = t_o + \epsilon$, $\epsilon > 0$ where t_o is the initial reference time.

Proof: The time derivatives of V (e, ϕ, t) for $v_1(t)'$ and $v_2(t)$ are $\dot{V}_1(t) = -\delta_1$ and $\dot{V}_2(t) = -\delta_1 - \delta_2$, respectively, where $\delta_1 = \| e \|_Q^2 \geqslant 0$ and $\delta_2 = 2e_1^2 \phi^T \phi \xi^T \Lambda \xi \geqslant 0$. Therefore, there exists $\epsilon > 0$ such that $|\dot{V}_1(t_1)| \leqslant |\dot{V}_2(t_1)|$ holds for all t_1, $t_1 = t_o + \epsilon$. Since $V_1(t_o) = V_2(t_o)$ is a fixed initial deviation and V(t) is monotonically decreasing,

$$V_1(t_1) \geq V_2(t_1) \tag{25}$$

for all $t_1 = t_o + \epsilon$, $\epsilon > 0$. From the exponential convergence, the rate of convergence increases with $v_2(t)$ since $\Omega_1 \leqslant \Omega_2$ in (18) and thus,

$$\sup_\epsilon [\alpha_1] \leq \sup_\epsilon [\alpha_2] \tag{26}$$

in (19) where the subscripts 1,2 correspond to the feedback structures for $v_1(t)$, $v_2(t)$, respectively. Therefore, from (22), the exponential rate of convergence using $v_2(t)$ is relatively fast compared with that using $v_1(t)$. Moreover, the condition $\lim_{t\to\infty} |\dot{V}(t)| = 0$ assures parameter convergence in the steady-state.

Note that $v_1(t) = \frac{\phi^T \xi}{\lambda_1 + \lambda_2 \xi^T \xi}$ and $v_2(t) = \frac{\phi^T \xi}{\lambda_1 + \lambda_2 \xi^T \xi} - \phi^T \phi e_1 \xi^T \Lambda \xi$. $v_1(t)$ and $v_2(t)$ are the associated inputs to the feedforward SPR block of the error model with different feedback structures. The above theorem shows how some structural modfication inproves the transient response of adaptive control systems. Fig.2 shows the modification of the feedback block in the error model for increasing the rate of convergence. This theorem provides a very general result and some adaptation laws in [20] can be shown to be special cases of it. The next theorem shows how proportional adaptation, as defined in [12], can also be modifed to improve the rate of convergence.

Theorem 3.2 (Proportional Adaptation) For the Lyapunov function in (13), Consider two choices for v(t) in (4), or in (15). First $v(t) = v_1(t) \triangleq \phi^T g(\xi, \phi, e)$, and second $v(t) = v_2(t) \triangleq (\phi + \phi P)^T [g(\xi, \phi, e) + h(\xi, \phi, e)]$ where $g(\xi, \phi, e)$ and $h(\xi, \phi, e)$ are any NLTV functions in R^{m1} guaranteeing $\dot{V}(t) \leqslant 0$ using $v_2(t)$, that is,

$$(\phi + \phi_P)^T [g(\xi, \phi, e) + h(\xi, \phi, e)]\, e_1 + \phi^T \Gamma^{-1} \dot{\phi} \leq 0 \tag{27}$$

and $\lim_{t\to\infty^+} |\dot{V}(t)| = 0$. For example, $g(\xi, \phi, e) = \frac{\xi}{\lambda_1 + \lambda_2 \xi^T \xi}$ and $h(\xi, \phi, e) = -(\phi + \phi P)\, e_1 \xi^T \Lambda \xi$ with positive definite Λ and $\lambda_1, \lambda_2 > 0$. Let the integral and proportional adaptation be

$$\dot{\phi} = -\Gamma g(\xi, \phi, e) e_1, \ \Gamma > 0 \tag{28}$$

$$\phi_P = -\Gamma_P g(\xi, \phi, e) e_1, \ \Gamma_P > 0 \tag{29}$$

Then, the exponential rate of convergence is relatively improved by using $v_2(t)$ compared with that using $v_1(t)$ at time $t_1 = t_0 + \epsilon$, $\epsilon > 0$ where t_0 is the initial reference time.

Proof: The proof is similar to the previous theorem, but, $\delta_1 = \| e \|_Q^2 \geqslant 0$ and $\delta_2 = 2e^2 g(\xi, \phi, e)^T \Gamma_{pg}(\xi, \phi, e) + 2e_1^2 (\phi + \phi_P)^T (\phi + \phi_P) \xi^T \Lambda \xi \geqslant 0$ in this case.

Note that $v_1(t) = \frac{\phi^T \xi}{\lambda_1 + \lambda_2 \xi^T \xi}$ and $v_2(t) = \frac{\phi^T \xi}{\lambda_1 + \lambda_2 \xi^T \xi} - e_1 g^T(\xi, \phi, e)\, \Gamma_{pg}(\xi, \phi, e) - e_1 (\phi + \phi P)^T (\phi + \phi P) \xi^T \Lambda \xi$. A standard proportional adaptive algorithm results if we select $h(\xi, \phi, e) = 0$.

Note that the control input u(t) can be selected from a wide variety of choices of v(t). The proportional adaptation scheme in [9] is a special case of the theorem. The robust adaptive control scheme may be combined with proportional adaptation to obtain fast convergence. But, as mentioned in the previous section, there is some trade-off between fast convergence and robustness because the adaptation gain has an upper bound.

IV. Simulation Results and Conclusions

1. *Simulation Results*

An example similar to Rohrs' was tested to prove the relative improvement of an adaptive control algorithm by structural refinement. Fig.3 and Fig.5 show simulation results of the signal independent adaptation schemes and the structural refinement by proportional adaptation, respectively. The reference model $H_m(s)$ and the plant $H_p(s)$ are described as follows:

$$H_m(s) = \frac{3}{s+3} \tag{30}$$

$$H_P(s) = \frac{30}{(s+2)(s+15)} \tag{31}$$

The reference input was a staircase, 10 to 20 at 7 seconds. In Fig.3, the plant output shows some oscillatory transient response both in the tracking error and in the parameter error. In Fig.5, the rate of convergence in the transient response is improved evidently by using the proportional adaptation scheme under the same conditions. Fig.4 and Fig.6 represent the parameter error vs. the tracking error of Fig.3 and Fig.5, respectively, in the phase space. The adaptation gain was chosen to be 20, and the proportional adaptation gain was also set equal to 20 in Fig.5. In Fig.7 and Fig.8, the adaptation gains were 70 and 5, respec tively, representing the cases for high and low gains. As the adaptation gains are increased, the poles of the error system moves into the ocill-

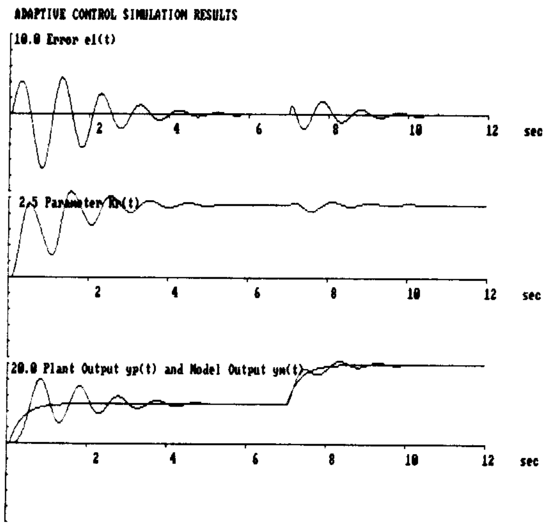

Fig.3. Convergence with robustness (fast adaptation; all gains =20).

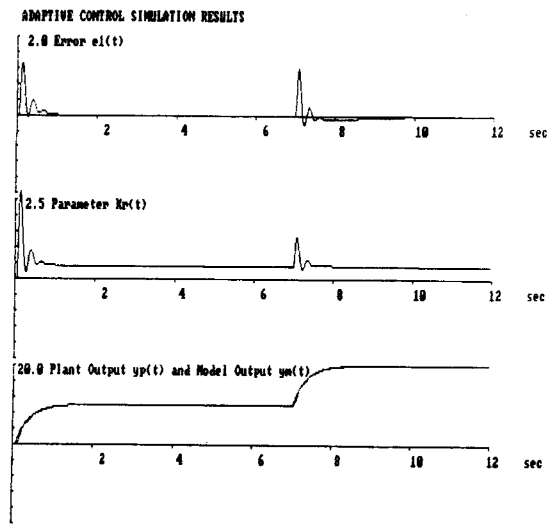

Fig.5. Convergence with robustness and proportional adaptation (all gains=20).

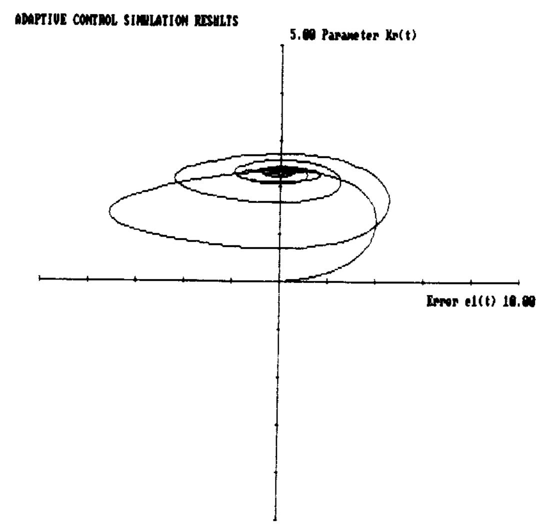

Fig.4. Parameter Kr(t) vs. error el (t) in Fig.4.

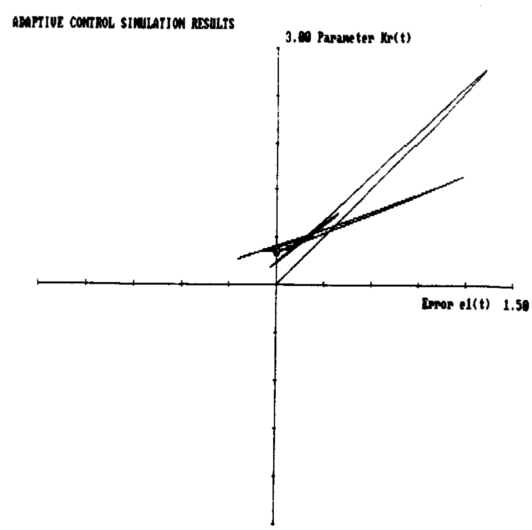

Fig.6. Parameter Kr(t) vs. error el(t) in Fig.5.

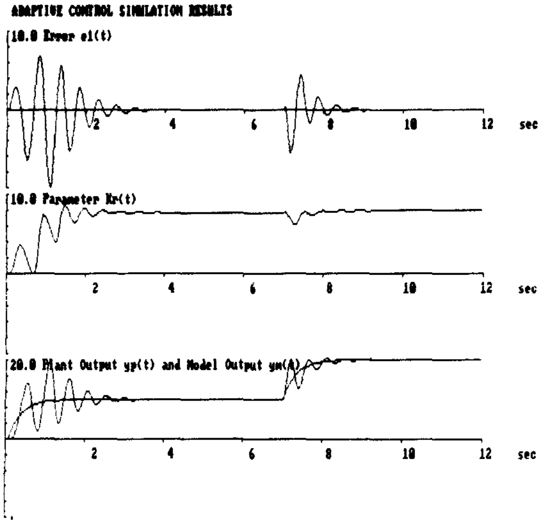

Fig.7 Robust convergence with high gains (all gains=70).

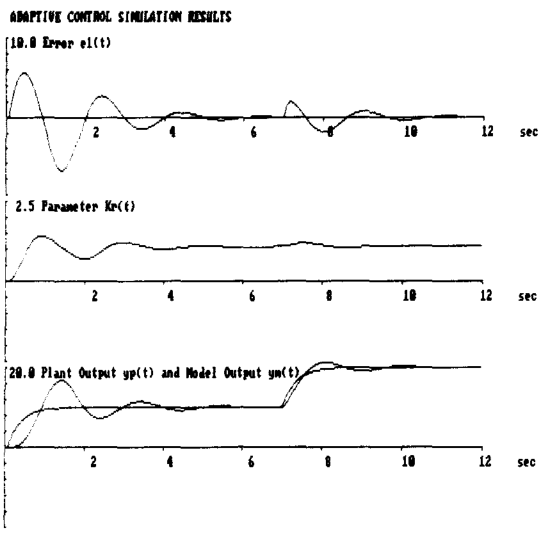

Fig.8. Robust convergence with low gains (all gains=5).

atory regions and even into the unstable regions for higher gains. If the plain adaptive algorithm is used in the above case, highly oscillatory response is obtained with this reference input due to fast asaptation instability. Therefore, the adaptation gains are selected according to the operating range and the reference signals by considering the final approach analysis and infinite gain operators.

2. Conclusions and Discussion

These approaches suggest that the rate of convergence increases exponentially by modifying feedback structures of the error systems. The proposed theorems also generalize the strategy in choosing nonlinear control inputs in adaptive control.

An important point of view in parameter error convergence versus tracking error convergence is that the tracking error or the identification error converges to zero even though the parameter error does not go to zero. The reasons for this are due to (i) lack of excitation [21], (ii) unmodeled dynamics and disturbances [15]. The order of excitation of the observation vector or the reference input should be greater than or equal to the number of parameters in the adjustable system or controller for the parameter convergence [22].

It is emphasized that adaptive control systems should be robust under unstructured uncertainties. Thus, the algorithms need modification to guarantee robustness. Åström [21] analyzed these instability mechanisms to show that the persistency of excitation plays a major role in the adaptation problem in the presence of unmodeled dynamics and disturbances. With bounded noise, parameter convergence can be achieved with the deadzone concept [23]. If the noise is modeled as a stochastic process, parameter convergence is related to boundedness of system variables. A positive real condition arises for certain transfer functions and the martingale convergence theorem may be substituted for the Lyapunov type analysis of global convergence [24]. The Popov's hyperstability approach is similar to the Lyapunov's direct method, but the convergence rate is relatively slow due to the coupling terms in the performance indexing function.

It may be difficult to analyze how unmodeled uncertainties affect the convergence rate of the modified schemes. Best performance might be obtained by forcing the modes of the error system to be very fast considering robustness and by updating the parameters with persistently exciting signals.

References

[1] R.V. Monopoli, "Model reference adaptive

control with an augmented error signals," *IEEE Trans. Automat. Contr.*, vol. AC-19, pp. 474-484, Oct. 1974.

[2] I.D. Landau, "A survey of model reference adaptive techniques-theory and applications," *Automatica*, vol. 10, pp. 353-379, 1974.

[3] K.S. Narendra and L.S. Valavani, "Stable adaptive controller design-direct control," *IEEE Trans. Automat. Contr.*, vol.AC-23, pp. 570-583, Aug. 1978.

[4] K.S. Narendra, Y.H. Lin, and L.S. Valavani, "Stable adaptive controller design, part ii: proof of stability," *IEEE Trans. Automat. Contr.*, vol. AC-25, pp. 440-448, June 1980.

[5] A.S. Morse, "Global stability of parameter-adaptive control systems," *IEEE Trans. Automat. Contr.*, vol. AC-25, pp. 433-439, June 1980.

[6] B. Egardt, "Unification of some continuous-time adaptive control schemes," *IEEE Trans. Automat. Contr.*, vol. AC-24, pp. 588-592, Aug. 1979.

[7] G.C. Goodwin, P.J. Ramadge, and P.E. Caines, "Discrete time stochastic adaptive control," SIAM *J. Contr. Optimiz.*, vol. 19, pp. 829-853, Nov. 1981.

[8] I.D. Landau and H.M. silveira, "A stability theorem with applications to adaptive control," *IEEE Trans. Automat. Contr.*, vol. AC-24, pp. 305-312, April 1979.

[9] I.D. Landau, "An extension of a stability theorem applicable to adaptive control," *IEEE Trans. Automat. Contr.*, vol. AC-25, pp. 814-817, Aug. 1980.

[10] C.E. Rohrs, L.S. Valavani, and M. Athans, "Convergence studies of adaptive control algorithms-parti: analysis," in *Proc. IEEE Conf. Dec. and Contr.*, (Albuquerque, NM), pp. 1138-1141, 1980.

[11] P.C. Parks, "Lyapunov redesign of model reference adaptive control systems," *IEEE Trans. Automat. Contr.*, vol. AC-11, pp. 362-367, July 1966.

[12] I.D. Landau, *Adaptive Control: The Model Reference Approach*. New York: Marcel Dekker, Inc, 1979.

[13] P.A. Ioannou and P.V. Kokotovic, "Instability analysis and improvement of robustness of adaptive control," *Automatica*, vol. 20, no. 5, pp. 583-594, 1984.

[14] C.E. Rohrs, L.S. Valavani, M. Athans, and G. Stein, "Analytic verification of undesirable properties of direct model reference adaptive control algorithms," in *Proc. IEEE Conf. Dec. and Contr.*, (San Diego, CA), pp. 1272-1284, Dec. 1981.

[15] C.E. Rohrs, L.S. Valavani, M. Athans, and G. Stein, "Robustness of adaptive control algorithms in the presence of unmodeled dynamics," in *Proc. IEEE Conf. Dec. and Contr.*, (Orlando, FL), pp. 3-11, Dec. 1982.

[16] A.P. Morgan, "On the construction of non-autonomous stable systems with applications to adaptive identification and control," *SIAM J. Contr. Optimiz.*, vol. 17, pp. 400-431, May 1979.

[17] A.P. Morgan, "A method of estimating the rate of convergence of an adaptive observer," in *Proc. IEEE Conf. Dec. and Contr.*, (Albuquerque, NM), pp. 1146-1151, 1980.

[18] G.C. Goodwin and K.S. Sin, *Adaptive Filtering, Prediction, and Control.* New Jersey: Prentice-Hall, Inc., 1984.

[19] B.D.O. Anderson and J.C.R. Johnson, "Exponential convergence of adaptive identification and control algorithms," *Automatica*, vol. 18, pp. 1-13, 1982.

[20] K.S. Narendra, ed., *Adaptive and Learning Systems: Theory and Applications.* New York: Plenum Press, 1986.

[21] K.J. Åström, "Analysis of rohrs counter-examples to adaptive control," in *Proc. IEEE Conf. Dec. and Contr.*, (San Antonio, TX), pp. 982-987, 1983.

[22] S. Boyd and S.S. Sastry, "Necessary and sufficient conditions for parameter convergence in adaptive control," *Automatica*, vol. 22, no. 6, pp. 629-639, 1986.

[23] B.B. Peterson and K.S. Narendra, "Bounded error adaptive control," *IEEE Trans. Automat. Contr.*, vol. AC-27, pp. 1161-1168, Dec. 1982.

[24] G.C. Goodwin, D.J. Hill, and M. Palaniswami, "A perspective on convergence of adaptive control algorithms," *Automatica*, vol. 20, pp. 519-531, 1984. *

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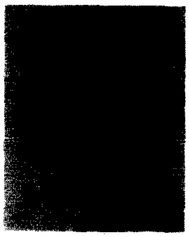

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