

Adaptive Spatial Domain FB-Predictors for Bearing Estimation

(입사각 추정을 위한 적응 공간영역 FB-예측기)

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要約

공간영역 예측기의 계수를 계산하기 위한 적응 알고리즘이 제안되었다. 제안된 방법은 LMS 알고리즘을 사용하여 TDL (tapped-delay-line)과 ESC (escalator) 구조를 갖는 공간영역 예측기의 계수를 계산한다. 기존의 일반적인 예측기와 다른점은 순방향과 역방향 예측 오차의 평균 자승값의 합을 최소화하며 예측기의 계수를 계산하므로 향상된 선형예측 공간 스펙트럼을 얻을 수 있다. 제안된 방법을 선형으로 배열된 센서에 의하여 얻어진 현대역신호의 입사각 추정문제에 적용시켜 기존의 적응예측 알고리즘과 컴퓨터 시뮬레이션을 통하여 성능을 비교하였다.

Abstract

We propose adaptive algorithms computing the coefficients of spatial domain predictors. The method uses the LMS approach to compute the coefficients of the predictors realized by using the TDL (tapped-delay-line) and the ESC (escalator) structures.

The predictors to be presented differ from the conventional ones in the sense that the relevant weights are updated such that the sum of the mean squared values of the forward and the backward prediction errors is minimized. Using the coefficients of such spatial domain predictors yields improved linear predictive spatial spectrums.

The algorithms are applied to the problems of estimating incident angles of multiple narrow-band signals received by a linear array of sensors. Simulation results demonstrating the performances of the proposed methods are presented.

I. Introduction

Determining the incident angles of plane waves received by an array of sensors is a classical problem in the areas of sonar, radar, and geophysics

[1-3]. Various modern temporal/spatial spectrum estimation techniques have been widely studied due to the high resolution capabilities [1-3].

The intent of this paper is to present adaptive spatial domain prediction algorithms and to apply the methods to the problems of estimating direction-of-arrivals by computing the modified linear predictive spatial spectrum given by [1].

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$$G(f_c, \theta) = \frac{1}{|1 - H(f_c, \theta)|^2} \quad (1a)$$

where

$$H(f_c, \theta) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} a_{mn} \exp \left\{ -j2\pi f_c \left(Tn + \frac{dm}{v} \cos \theta \right) \right\}, \quad m \neq \tilde{m} \quad (1b)$$

f_c is the center frequency of the narrowband plane waves received by a linear array of sensors, d is the distance between neighbouring sensors, v is the propagation velocity of the plane waves, T denotes the sampling interval, m and n are the sensor and the time indices, respectively, a_{mn} represents the n -th coefficient of the filter taking the m -th sensor output as its input, and \tilde{m} is the index of the sensor whose output is to be estimated as a linear combination of other sensor output signals. The linear prediction method determines the incident angles as $\theta = \theta_i$ at which the spatial spectrum estimate $G(f_c, \theta)$ peaks.

Fig. 1. shows the block diagram of the spatial domain predictor for $\tilde{m}=0$. From Fig. 1, the prediction error $e_{\tilde{m}}(k)$ can be expressed as

$$e_{\tilde{m}}(k) = X_{\tilde{m}}(k) - \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} a_{mn} x_m(k-n) \quad (2)$$

Various approaches computing a_{mn} 's, which minimize the statistically or time averaged value of the squared prediction error, have been presented[2,3].

It has been shown [1] that the choice of \tilde{m} with smaller mean squared prediction error does not imply a better spatial spectrum estimate and the resolution and the bias of the linear predictive spatial spectrum estimates for determining incident angles of sinusoidal plane waves depend

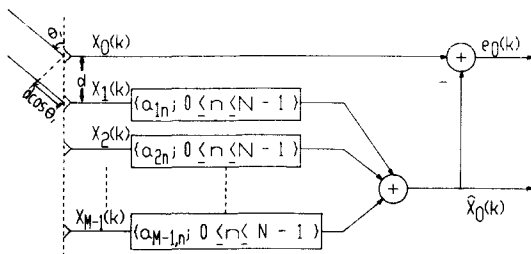


Fig.1. Block diagram of the spatial domain predictor.

on the choice \tilde{m} (i.e., the sensor output to be predicted). In the temporal prediction problems, the selection of \tilde{m} corresponds to the choice of the sample to be predicted and past sample values are usually used to predict the present sample to make the system model causal. However, for a spatially sampled signals, causality is not an important issue. To overcome the problems of selecting \tilde{m} , least square approaches minimizing the time averaged sum of the squared values of the forward($\tilde{m}=0$) and the backward($\tilde{m}=M-1$) prediction errors[4,5] were presented.

In this paper, various adaptive spatial domain prediction algorithms performing forward and backward predictions are presented. The predictor structures to be considered are TDL(tapped-delay-line) [7,8] and ESC(escalator)[9,10]. The relevant coefficients are updated using the LMS (least-mean-square) approach such that minimization of the sum of the mean squared forward and backward prediction errors can be performed simultaneously. Computer simulation results comparing the performances of the proposed methods and those of the corresponding spatial domain predictors performing on directional minimization are presented.

In Section II, the algorithms updating the coefficients of the TDL and ESC structures are presented. Computer simulation results for a variety of situations are presented in Section III. Finally, conclusions will be made in Section IV.

II. Adaptive Spatial Domain Forward-Backward Predictor

1. Adaptive TDL Forward-Backward Predictor

A schematic diagram of the spatial domain FB(forward-backward) predictor with the TDL structure is illustrated in Fig.2 where the linear array consists of M sensors; and $e_f(k)$ and $e_b(k)$ are the forward and the backward prediction errors, respectively. From Fig.2, the spatial prediction errors are given by

$$e_f(k) = x_0(k) - A^T X_r(k) \quad (3a)$$

and

$$e_b(k) = x_{M-1}(k-L+1) - B^T X_b(k) \quad (3b)$$

where

$$A = [a_{10}, a_{11}, \dots, a_{1,L-1}, \dots, a_{M-1,0}, a_{M-1,1}, \dots, a_{M-1,L-1}]^T \quad (3c)$$

$$B = [b_{10}, b_{11}, \dots, b_{1,L-1}, \dots, b_{M-1,0}, b_{M-1,1}, \dots, b_{M-1,L-1}]^T \quad (3d)$$

$$X_r(k) = [x_1(k) \ x_1(k-1) \ \dots \ x_1(k-L+1) \ \dots \ x_{M-1}(k) \ \dots \ x_{M-1}(k-L+1)]^T \quad (3e)$$

$$X_b(k) = [x_M(k-L+1) \ \dots \ x_{M-2}(k) \ \dots \ x_0(k-L+1) \ \dots \ x_0(k)]^T \quad (3f)$$

and "T" denotes matrix transpose. In the above equations, L is the number of coefficients of the TDL filter for each sensor output.

Assuming that the sensor output signals are stationary; the additive noises at different sensors are mutually uncorrelated and have the same statistics; and the sensors have identical characteristics and are equally spaced, we can show that the optimum weight vectors A^* and B^* minimizing $E\{[e_f(k)]^2\}$ and $E\{[e_b(k)]^2\}$, respectively, are the same. That is

$$A^* = [R_r]^{-1} P_r = B^* = [R_b]^{-1} P_b \quad (4a)$$

where

$$R_r = E\{X_r(k) X_r^T(k)\} = R_b = E\{X_b(k) X_b^T(k)\} \quad (4b)$$

and

$$P_r = E\{x_1(k) X_r(k)\} = P_b = E\{x_{M-1}(k-L+1) X_b(k)\} \quad (4c)$$

However, if the signal statistics in (4b) and (4c) should be estimated from finite records of sensor output signals by taking time average, then $\hat{R}_f \neq \hat{R}_b$ and $\hat{P}_f \neq \hat{P}_b$. Thus the resulting coefficient vectors \hat{A} and \hat{B} are different. Similarly, computing \hat{A} and \hat{B} using an adaptive algorithm with finite time constant yields different weight vectors.

Now, let us consider the problem of minimizing the cost function C which is the sum of the mean squared values of the forward and the backward prediction errors under the constraint that $A=B$. That is

$$\text{minimizing } C = E\{[e_f(k)]^2 + [e_b(k)]^2\} \quad (5a)$$

$$\text{subject to } A=B \quad (5b)$$

An efficient method of computing the coefficient in a recursive way is the LMS algorithm[7,8] using the estimated gradient given by

$$\hat{\nabla}(k) = \frac{\partial \{[e_f(k)]^2 + [e_b(k)]^2\}}{\partial \hat{A}(k)} \quad (6a)$$

$$= -2[e_f(k) X_r(k) + e_b(k) X_b(k)] \quad (6b)$$

where

$$e_f(k) = x_1(k) - \hat{A}(k) X_r(k) \quad (6c)$$

and

$$e_b(k) = x_{M-1}(k-L+1) - \hat{A}(k) X_b(k) \quad (6d)$$

Using the estimated gradient, the LMS algorithm can be expressed as

$$\hat{A}(k+1) = \hat{A}(k) + \mu[-\hat{\nabla}(k)] \quad (7a)$$

$$= \hat{A}(k) + 2\mu[e_f(k) X_r(k) + e_b(k) X_b(k)] \quad (7b)$$

where $\hat{A}(k)$ is the weight vector estimated at time k.

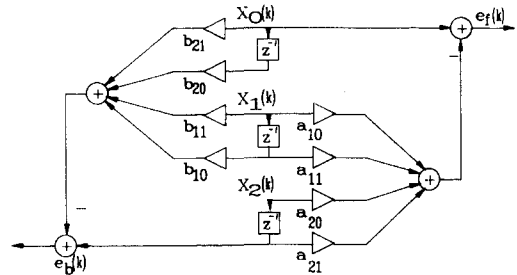


Fig.2. Schematic diagram of the spatial domain forward-backward predictor with the TDL structure for $M=3$ and $L=2$.

2. Adaptive ESC Forward-Backward Predictor

The ESC structure realizes the Gram-Schmidt orthogonalization procedure[9] and has been successfully applied to the problems of adaptive beamforming[10] and temporal prediction[9]. Now, since the coefficients of the ESC structure are computed such that mean squared values of

local errors are minimized, applying the LMS algorithm to compute the relevant weights yields faster convergence speed and more stable steady state with less mean squared error than the TDL counterpart [9,10].

Fig. 3(b) shows the schematic diagram of the spatial domain forward-backward predictor with the ESC structure, the forward part of which is shown in Fig. 3(a) [9,10]. In the remaining discussions, $b_{21}(k)$ and $\beta_{21}(k)$ represent the prediction errors and ESC coefficients, whose forward counterparts are $e_{21}(k)$ and $\alpha_{21}(k)$, respectively. Due to the complexities of expressing the equations relevant to the ESC structure, let us consider $e_{21}(k)$ of the forward predictor and the corresponding intermediate signal $b_{21}(k)$ of the backward predictor. From Fig.3. they can be expressed as

$$e_{21}(k) = e_{12}(k) - \alpha_{21} e_{11}(k) \tag{8a}$$

and

$$b_{21}(k) = b_{12}(k) - \beta_{21} b_{11}(k) \tag{8b}$$

Now, as for the case of the previously discussed adaptive TDL forward-backward predictor, the ESC coefficient minimizing the sum of the mean squared values of the forward and the backward errors can be computed by letting $\alpha_{21} = \beta_{21}$ and using the LMS approach given by

$$\alpha_{21}(k+1) = \alpha_{21}(k) + \mu_{11}(k) [e_{21}(k) e_{11}(k) + b_{21}(k) b_{11}(k)] \tag{9a}$$

where

$$\mu_{11}(k) = (1-\beta) / \sigma_{11}^2(k) \tag{9b}$$

and

$$\sigma_{11}^2(k) = \beta \sigma_{11}^2(k-1) + \frac{(1-\beta)}{2} [\{e_{11}(k)\}^2 + \{b_{11}(k)\}^2] \tag{9c}$$

III. Simulation Results

To demonstrate the performances of the proposed adaptive spatial domain FB-prediction algorithms for the problems of bearing estimation,

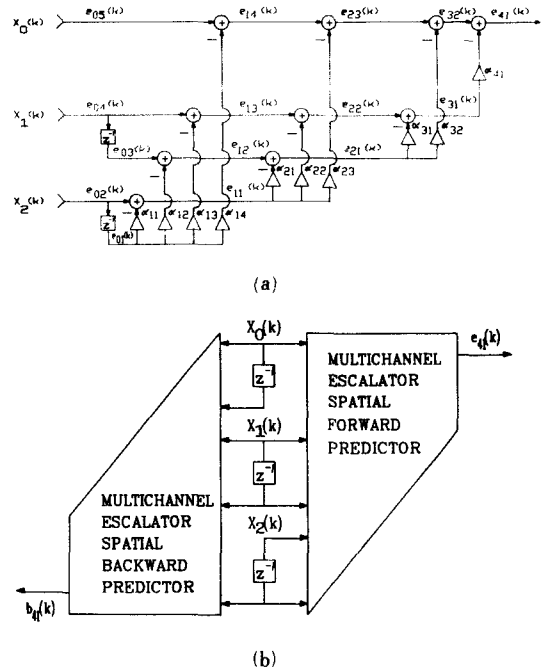


Fig.3. Schematic diagram of the spatial domain (a) forward (b) forward-backward predictor with the ESC structure for M=3 and L=2.

the sensor output signals $x_1(k)$, $i=0, 1, \dots, 9$ are generated by adding mutually uncorrelated zero mean white Gaussian random data $n_1(k)$. That is

$$x_i(k) = s_1(k - D_{11}) + s_2(k - D_{21}) + n_1(k) \tag{10a}$$

$$i = 0, 1, \dots, 9$$

where

$$D_{11} = i \frac{d}{v} \cos \theta_1 \tag{10b}$$

$$D_{21} = i \frac{d}{v} \cos \theta_2 \tag{10c}$$

and the signals $S_1(k)$ and $S_2(k)$ were obtained by processing white Gaussian random sequences through a second order Butterworth band-pass filter with the lower and upper cutoff frequencies of 8 Hz and 17 Hz, (i.e., $f_c = 12.5\text{Hz}$) respectively at the sampling rate of 300Hz. The number of time samples of each sensor output to realize the spatial domain predictors with the TDL and ESC structures are 15 and 8, respectively; the con-

vergence parameter μ in (7b) and the smoothing parameters β are 0.9999 and 0.95, respectively. These parameters were chosen after intensive simulations. The incident angles of the narrow-band signals were -3° and $+3^\circ$ for the case-1 and -10° and $+10^\circ$ for the case-2. Different choice of the reference sensor output (which is to be spatially predicted as a weighted sum of other sensor output signals) does not make any difference due to the geometrical symmetry of the two signals $s_1(k)$ and $s_2(k)$, when only one directional (i.e., forward or backward) prediction is performed. Forward or backward prediction algorithms for the TDL and ESC structures compute the relevant coefficients using the LMS approach which can be obtained by omitting $e_b(k)$, $X_b(k)$, $b_{21}(k)$, $b_{11}(k)$, and $(b_{11}(k))^2$ in (7b), (9a), and (9c), respectively.

Simulation results in Fig.4-Fig.7 display the spatial spectrums computed by using (a) FTDL (forward TDL), (b) FBTDL (forward and backward TDL), (c) FESC (forward ESC), and (d) FBESC (forward and backward ESC) spatial domain predictors. These results were obtained by displaying 10 independent normalized spectrums such that consistencies of the algorithms can be examined.

SNR's and incident angles for the simulations are summarized in Table 1.

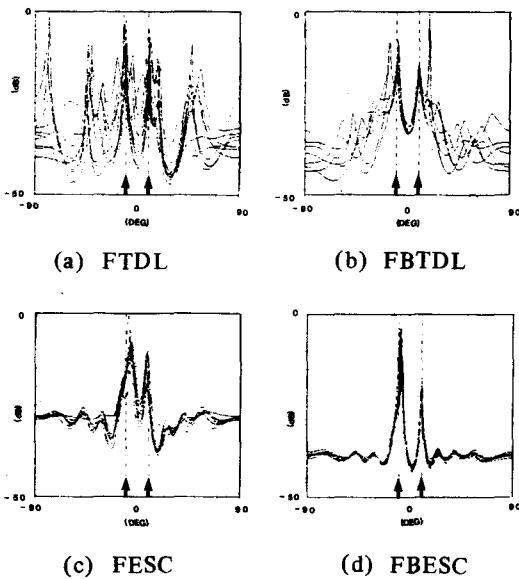


Fig.4. Linear predictive spatial spectrums for SNR=10dB and $\theta_i = \pm 10^\circ$.

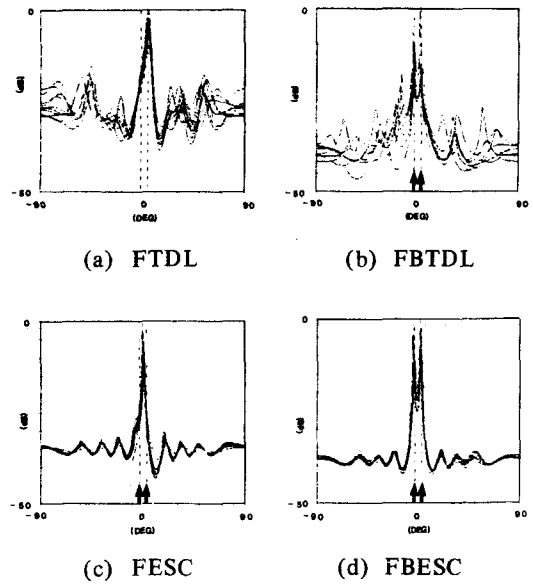


Fig.5. Linear predictive spatial spectrums for SNR=10dB and $\theta_i = \pm 3^\circ$.

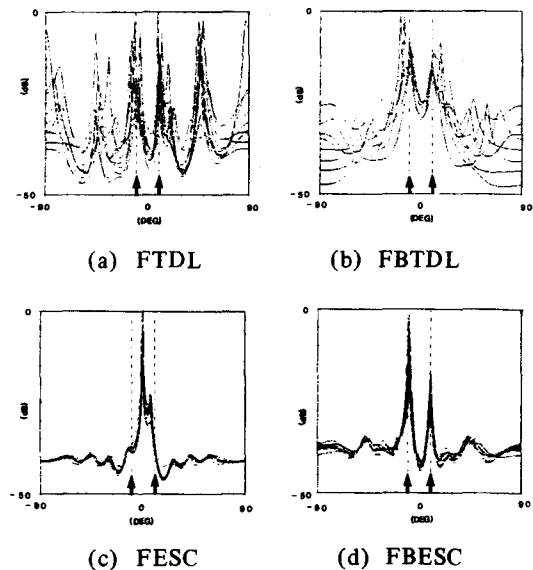


Fig.6. Linear predictive spatial spectrums for SNR=0dB and $\theta_i = \pm 10^\circ$.

From the results in Fig.4-Fig.7, we can observe that higher resolution (separability of two closely spaced signals) can be obtained by minimizing the

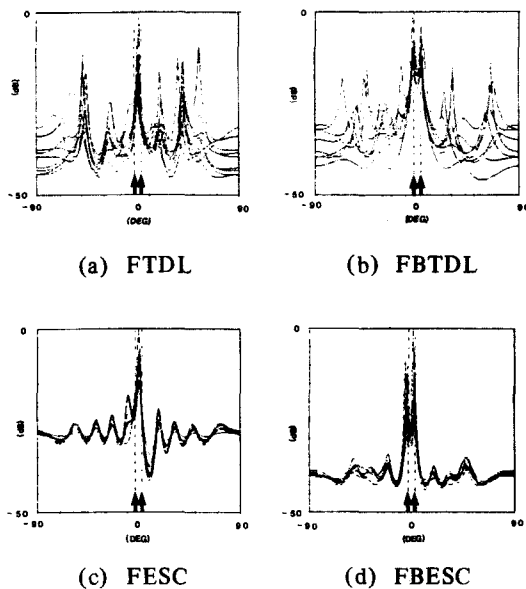


Fig.7. Linear predictive spatial spectra for SNR=0dB and $\theta_1 = \pm 3^\circ$.

Table 1. SNR's and incident angles for the simulations.

	Fig. 4	Fig. 5	Fig. 6	Fig. 7
SNR (dB)	10dB	10dB	0dB	0dB
Incident Angles	$\pm 10^\circ$	$\pm 3^\circ$	$\pm 10^\circ$	$\pm 3^\circ$

forward and backward prediction errors simultaneously; more consistent linear predictive spatial spectra are obtained by using the ESC realization; and the spurious spectral estimates of the FTDL and the FBTDL may result in incorrect bearing estimates when the angles at which dominant peaks occur are selected as the incident angles.

IV. Concluding Remarks

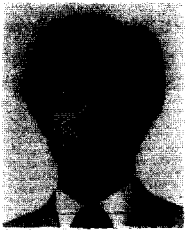
Adaptive spatial domain prediction algorithms minimizing the sum of the mean squared value of the forward and the backward prediction errors are presented. The predictors were realized using the TDL and the ESC structures. The algorithms were applied to the signals received by a linear array of equally spaced sensors and linear predictive spatial spectra were computed using the resulting coefficients.

Computer simulation results indicate that simultaneous minimization of the forward and the backward spatial prediction errors yields spatial spectra with higher resolution and less bias than the case of one directional minimization. Also, it has been observed that more consistent spectrum estimates can be obtained using the ESC structure.

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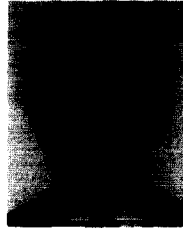
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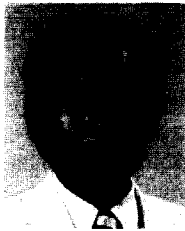
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