

Design of Lossy Matching Network for Microwave Broadband Amplifier

Using the Relationships Between Gain and Reflection Coefficients

(이득-반사계수 관계를 이용한 마이크로파 광대역

증폭기용 유손실 정합회로의 설계)

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要 約

마이크로파 광대역 증폭기용 유손실 정합회로를 설계하기 위한 새로운 방법을 제시한다. 이 방법은 트랜지스터를 모델링 없이 측정된 산란계수를 이용하여 표시하며, 유손실 정합회로를 무손실 정합회로 사이에 유손실 직렬 임피던스 또는 병렬 어드미턴스가 삽입된 구조로 생각한다. 증폭기의 이득과 반사계수 사이의 선형 관계식을 유도하고 이를 이용하여 적절한 이득과 반사계수를 선택하는 법을 제시한다. 증폭기의 이득 및 반사계수와 유손실 정합 소자의 임피던스 사이에는 쌍 일차 변환의 관계가 있으며 일정 이득 원이나 일정 반사계수 원을 임피던스 평면 또는 스미스 차트에 그려 적절한 정합 소자값을 선택할 수 있다. 본 논문에서 제안된 방법이 유용함을 보이기 위하여 증폭기설계 예를 제시하였다.

Abstract

A new design method of lossy matching network for the microwave broadband amplifier is presented by using scattering parameters instead of modeling of transistor. A lossy matching network is represented as the combination of 2 lossless networks between which lossy serial or parallel immittance is inserted without using specific topology, and so many useful matching circuits can be realized. Also it is shown that linear transforming relation exists between gain and reflection coefficient of the amplifier, and the transforming equation is derived using scattering parameters. With this equation some constant gain circles can be drawn on reflection coefficient plane to get adequate reflection coefficient and gain. And since the relations between amplifier gain/reflection coefficient and the immittance of passive element are bilinear transformations, constant gain or reflection coefficients circles can be drawn on immittance plane or Smith chart to select proper matching immittances. Illustrative examples are presented to show the usefulness of proposed method.

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I. Introduction

Microwave broadband amplifiers are designed to have flat gain characteristics by compensating for the gain roll off of transistor with using matching networks. Designs of amplifiers are possible

with using lossless matching networks, but with lossless matching networks reflection is large at low frequency band, and so amplifiers may be unstable when amplifiers are cascaded. Lossy matching amplifiers dissipate reflection power at matching networks, so give low reflection coefficients and may improve noise figure and gain characteristics.^[1]

Many studies have been done on lossy matching networks but most of them, assuming specific circuit topology, can not be applied to all circuit configurations in general.^[2,3,4] Matching network of arbitrary topology was studied using graphical design method by Perez, but in that study it is uncertain whether realized gain and reflection coefficients are optimum or not.^[5]

In this study, realizable range of amplifier gain and reflection coefficient are derived using transistor scattering parameters, and relations between gain and reflection coefficients are found. Also using the relations between gain and reflection coefficients, design methods to get the specified gain and reflection coefficients are proposed.

II. Gain and Reflection Coefficients of Amplifier

Block diagram of general single stage amplifier is in Fig. 1. At microwave frequency range, characteristics of amplifier are well presented with using scattering parameters, so in this paper transistors are represented as 2 port scattering parameters.

Lossless matching networks can be synthesized by unitary property, if reflection coefficients or transmission coefficients are known, but for lossy matching networks accurate synthesizing processes are very difficult problem if configurations are not specified.

To use the well-known synthesis method of lossless network, a lossy network can be represented as combination of 2 lossless networks

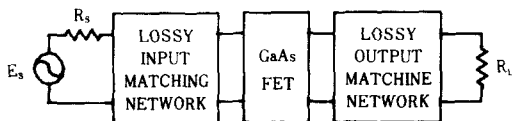


Fig.1. Block diagram of a single stage GaAs FET amplifier.

between which serial impedance or parallel admittance is inserted. As an example, input matching network design is illustrated in Fig.2. In the design of input matching network, lossless matching networks can be designed by real frequency technique which uses measured scattering parameters directly without using transistor modeling or assumption of analytic transfer function.^[6]

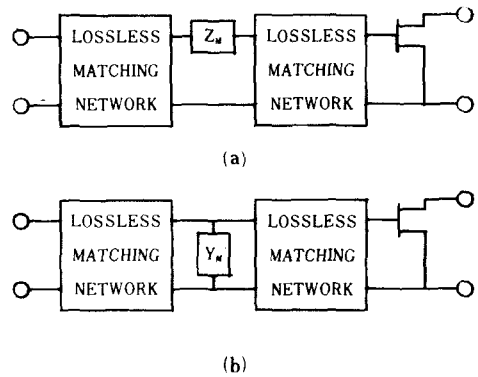


Fig.2. Equivalent circuit of lossy matching network represented as lossless matching network and lossy element.

(a) serial lossy impedance Z_M .

(b) parallel lossy admittance Y_M .

This paper deals mainly with the design of the lossy matching element, so an amplifier which is constructed with a lossy element and a transistor is considered first. Once a lossy element is determined, lossless matching networks can be used to improve the amplifier characteristics. To derive the relationships among the gain, the reflection coefficient, and the lossy matching element, it is assumed that the amplifier is constructed as in Fig.3. In that figure, transistor is represented as 2 port scattering parameters f_{ij} ($ij=1, 2$), and f_{ij} might be the scattering parameters of cascade connection of lossless matching network and transistor if necessary. Fig.3 is the case for input parallel admittance Y_M connection, and for input serial impedance Z_M connection or output connection similar relationships are derived. (For convenience Y_M and Z_M are assumed to be normalized below.)

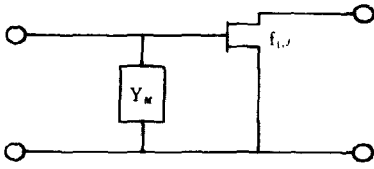


Fig.3. Cascade connection of normalized parallel admittance Y_M and transistor.

In Fig.3 if transistor scattering parameters f_{ij} ($i,j=1, 2$) are known, scattering parameters of total system can be derived. The relationships between scattering parameters of total system S_{ij} ($i,j=1,2$) and normalized parallel admittance Y_M are as follows:

$$S_{21} = \frac{2f_{21}}{Y_M(1+f_{11})+2} \tag{1}$$

$$S_{11} = \frac{-Y_M(1+f_{11})+2f_{11}}{Y_M(1+f_{11})+2} = -1 + \frac{2(1+f_{11})}{Y_M(1+f_{11})+2} \tag{2}$$

It is evident from Eq.(1) and (2) that the relations between S_{21} (or S_{11}) and Y_M are bilinear transformation, so loci of constant gain or constant reflection coefficient on Y_M plane are circles. From Eq.(1) and (2), the relationship between S_{21} and S_{11} is as

$$S_{21} = \frac{f_{21}}{1+f_{11}} S_{11} + \frac{f_{21}}{1+f_{11}} = \frac{f_{21}}{1+f_{11}} (1+S_{11}). \tag{3}$$

Eq.(3) represents that a linear transformation exists between S_{21} and S_{11} , and magnitude of S_{21} is proportional to the length between S_{11} and point $(-1,0)$ on complex plane. From Eq.(3) either of gain and reflection coefficient determines another, and in order to choose the goal of gain or reflection coefficient, it is preferable to know the range of realizable gain and reflection coefficient.

1. Realizable Range of S_{11} and S_{21}

To derive the possible S_{11} range, Y_M can be written from Eq.(2) as

$$Y_M = \frac{2}{1+S_{11}} - \frac{2}{1+f_{11}} \tag{4}$$

In general Y_M is passive, so it holds

$$\text{Re} \left[\frac{1}{1+S_{11}} - \frac{1}{1+f_{11}} \right] \geq 0,$$

and some manipulations lead to

$$\left| S_{11} - \left[-1 + \frac{1}{2n} \right] \right| \leq \left| \frac{1}{2n} \right| \tag{5}$$

where

$$n = \text{Re} \left[\frac{1}{1+f_{11}} \right].$$

In the same way possible region of S_{21} can be derived as follows:

$$Y_M = \frac{2}{1+f_{11}} \left[\frac{f_{21}}{S_{21}} - 1 \right] \tag{6}$$

From $\text{Re } Y_M \geq 0$, it can be shown

$$\left| S_{21} - \frac{f_{21}(1+f_{11}^*)}{2\text{Re}(1+f_{11})} \right| \leq \left| \frac{f_{21}(1+f_{11}^*)}{2\text{Re}(1+f_{11})} \right| \tag{7}$$

Eq.(5) and (7) can be drawn on complex S_{11} and S_{21} plane as Fig.4.

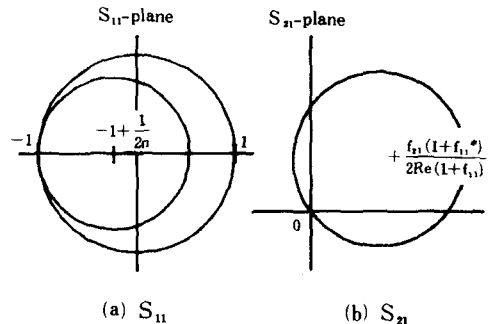


Fig.4. Realizable range of S_{11} and S_{21} .

2. Locus of Y_M That Results Specific Gain and Reflection Coefficients

It is convenient to draw the locus of constant gain or reflection coefficient on Y_M -derived Smith

chart rather than on Y_M -plane.

Let α corresponds to the Y_M derived point on the Smith chart, i.e. $\alpha = (1 - Y_M)/(1 + Y_M)$. Then S_{21} and S_{11} are written as follows:

$$S_{21} = \frac{2f_{21}\alpha + 2f_{11}}{(1-f_{11})\alpha + (3+f_{11})} \quad (8)$$

$$S_{11} = \frac{(1+3f_{11})\alpha + (-1+f_{11})}{(1-f_{11})\alpha + (3+f_{11})} \quad (9)$$

Eq.(8) and (9) show that loci of constant $|S_{21}|$ and $|S_{11}|$ are circles on the Smith chart, because the relationships between S_{21} (or S_{11}) and α are bilinear transformations.^[7] To find the center and radius of circle, let's think a general equation

$$S_{ij} = \frac{A\alpha + B}{C\alpha + D}$$

Locus of $|S_{ij}| = K$ on α plane (Smith chart) can be derived as follows:

$$|S_{ij}|^2 = S_{ij}S_{ij}^* = \frac{A\rho e^{j\theta} + B}{C\rho e^{j\theta} + D} \frac{A^*\rho e^{-j\theta} + B^*}{C^*\rho e^{-j\theta} + D^*} = K^2$$

where $\alpha = \rho e^{j\theta}$.

$$|\alpha - \alpha_0|^2 = r^2 \quad (10)$$

where

$$\text{center } \alpha_0 = \frac{K^2 C^* D - B A^*}{|A|^2 - K^2 |C|^2}$$

$$\text{radius } r = \left[|\alpha_0|^2 - \frac{|B|^2 - K^2 |D|^2}{|A|^2 - K^2 |C|^2} \right]^{1/2}$$

3. Selection of Gain and Reflection Coefficients

The relationship between the gain and the reflection coefficient was derived previously. To design the amplifier, the goal gain and the reflection coefficient are to be selected.

For the amplifier of Fig. 3, when we select the proper magnitude of S_{21} in the possible range of $|S_{21}|$ as in Fig.4(b), the possible range of $|S_{11}|$ may be calculated. And if we select the magnitude of S_{11} , complete S_{11} (i.e. magnitude and argument) can be determined and from which S_{21} is determined.

Let $|S_{21}|$ be

$$|S_{21}| = \left| \frac{f_{21}}{1+f_{11}} \right| |1+S_{11}| = M \quad (11a)$$

where M is in the possible range of $|S_{21}|$.

And this equation may be rewritten as

$$|1+S_{21}| = \frac{M|1+f_{11}|}{|f_{21}|} = M_d \quad (11b)$$

where M_d is the distance between S_{11} and point $(-1,0)$.

Eq. (11) shows that the possible S_{11} is on the circle of radius M_d and center $(-1,0)$, and in Fig.4(a) the range of possible S_{11} was derived. So, realizable S_{11} with $|S_{21}| = M$ is the common part of the circle by Eq.(11b) and the possible range of Fig.4(a), and is represented as the dotted line in Fig.5. In Fig.5 minimum $|S_{11}|$ is $|1-M_d|$ which is the cross point of dotted line and real axis, and maximum value occurs at P and P' point which are the cross points of the possible S_{11} circle and dotted line.

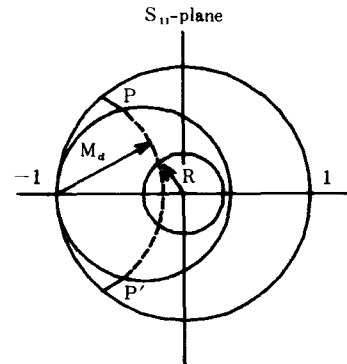


Fig.5. Circles on the S_{11} -plane to calculate S_{11} when $|S_{21}|$ is M .

By calculating the coordinates of these points at which maximum $|S_{11}|$ occurs, the range of possible magnitude of S_{11} can be found as follows:

$$| -1+M_d | \leq |S_{11}| \leq \sqrt{[(1-M_d)^2 + 2M_d(1-n M_d)]} \quad (12)$$

where

$$n = \text{Re} \left[\frac{1}{1 + f_{11}} \right]$$

$$M_d = \frac{M|1 + f_{11}|}{|f_{21}|}$$

If we select $|S_{11}|$ as R which is in the possible range, then S_{11} is the cross point of a circle with radius R from origin and a circle with radius M_d from point $(-1, 0)$. Also angle of S_{11} is given as

$$\theta = \pm \left[\pi - \cos^{-1} \left[\frac{R^2 + 1 - M_d^2}{2R} \right] \right] \quad (13)$$

A complete S_{11} gives S_{21} from Eq.(3) and the matching element value Y_M from Eq.(4), so matching network can be designed to have the specified gain and reflection coefficients.

III. Numerical Examples of Amplifier Design

The theory we have developed can be applied to the amplifier design easily. As an example, possible ranges of S_{11} and S_{21} of an amplifier which consists of normalized parallel admittance Y_M and NE38806 FET at 4GHz are as follows: [8]

- S_{11} - inside of circle with center $(-.204, 0)$ and radius 0.796
- S_{21} - inside of circle with center $(-.919, 1.722)$ and radius 1.952

At this frequency maximum $|S_{21}|$ is 3.903, and with this $|S_{21}|$, magnitude of S_{11} is 0.592, and the value of normalized Y_M that gives this characteristics is $(0, -1.02)$. Using the same transistor, we shall apply this technique to the design of an amplifier operating over 4-8GHz. The scattering parameters of NE38806 FET with the 50Ω transmission line which is 10 degree length at 8GHz and connected to the gate port are as table 1.

The loci of the desired gain and the reflection coefficient on the Smith chart can be drawn as circles at each frequency. For the scattering parameters of Table 1, the maximum realizable values of $|S_{21}|$ at 4, 6 and 8GHz are 4.12, 3.11, 2.22 respectively, so 2.22 can be taken as the goal of flat gain. The centers and radii of the circles which are the loci of $|S_{21}| = 2.2$ on the Smith chart are derived as in the table 2. The matching

Table 1. S parameters of GaAs FET NE38806 with $50\Omega-10^\circ$ at 8GHz transmission line connected to input port.

GHz	f_{11}	f_{21}	f_{12}	f_{22}
4	.78/ -103°	3.03/ 74°	.058/ 23°	.55/ -63°
5	.71/ -130°	2.80/ 50°	.058/ 7.7°	.52/ -82°
6	.68/ -156°	2.51/ 30.5°	.046/ 6.5°	.56/ -100°
7	.68/ -182°	2.14/ 7.9°	.046/ 22°	.62/ -119°
8	.69/ 156°	1.77/ -19°	.054/ 36°	.68/ -137°

element Y_M which exhibits the desired flat gain, can be realized with a circuit of which characteristics crosses the constant gain circles drawn on the Smith chart. Also written are the possible ranges of reflection coefficients in the same table. If proper magnitude of reflection coefficients are selected, the matching admittance can be calculated as shown before. The circuit that has the characteristics of calculated admittances over the desired frequency band, can be found with various methods, for example, with complex curve fitting. [9]

Table 2. Locus of Y_M on Smith chart and realizable $|S_{11}|$ with $|S_{21}| = 2.2$.

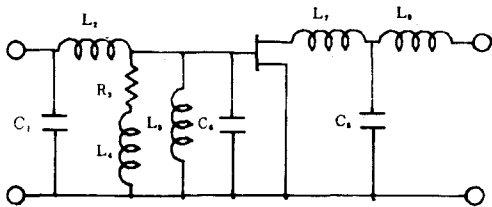
Freq. (GHz)	Locus of Y_M center	radius	possible $ S_{11} $
4	$(-.861, -.540)$	1.098	.186 - .891
6	$(-.570, -.442)$.856	.589 - .766
8	$(-8.10, -7.61)$	10.12	.423 - .437

Here to demonstrate the simplicity of the proposed method, the lossy matching element Y_M is realized by series resistor and inductor (R-L) circuit, which is shunt connected to the FET input. First desirable values of Y_M at each frequency are to be selected by considering the loci of $|S_{21}| = 2.2$ on the Smith chart. For the series R-L circuit which is shunt connected to the FET input, conductance is determined by only 2 variables (R and L), so the real values of Y_M can not be chosen arbitrarily at 4, 6 and 8GHz. So, at two frequencies 4 and 6GHz, the values of Y_M are chosen as $0.9-j2.0$ and $0.68 + j 0.18$. These values are selected by considering such conditions that $|S_{21}|$ is near 2.2, $|S_{11}|$ is as small as possible and Y_M is easily realizable by

simple circuit over the band. The values of R and L which result the conductance of Y_M at 4 and 6GHz are calculated as 40.74Ω and $0.558nH$. By these R and L, only the conductances are satisfied at 4 and 6GHz, so the susceptances need to be compensated. Shunt inductance and capacitance are used to compensate for the susceptances, and the values of inductance and capacitance are calculated to be $0.978nH$ and $1.679pF$ respectively. Then the value of Y_M at 8GHz becomes $0.5 + j 1.834$ from these circuits. The characteristics of the amplifier with these lossy matching elements are shown in Fig.7 with solid line. In this example, series connected R and L, shunt connected C and L are used to realize the real and imaginary part of Y_M at 2 frequencies. If more complicated circuits are used, the values of selected Y_M can be realized at 3 frequencies with those circuits.

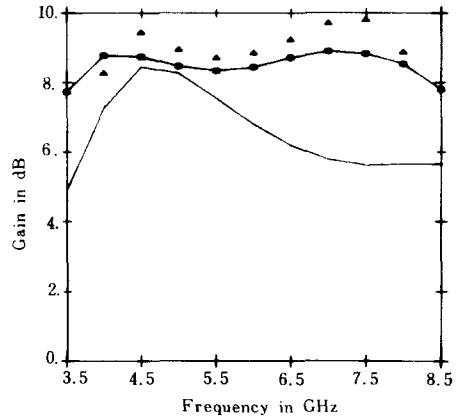
If the improvement of the amplifier characteristics is needed, it can be achieved by the optimization of elements or by adding additional matching circuits. Here the real frequency technique is used for the design of the additional lossless matching network to improve the gain and reflection coefficient.^[10] In the real frequency technique, 3 element lossless matching network is used. The circuit of final amplifier is as Fig.6. In Fig.6, shunt R_3-L_4 , L_5 , and part of C_6 ($1.679pF$) are designed by lossy matching technique, and C_1 , L_2 , part of C_6 ($.246pF$) and output matching elements are designed by real frequency technique. The amplifier characteristics are as Fig.7.

The gain of the amplifier is $8.56 \pm 0.33dB$ over

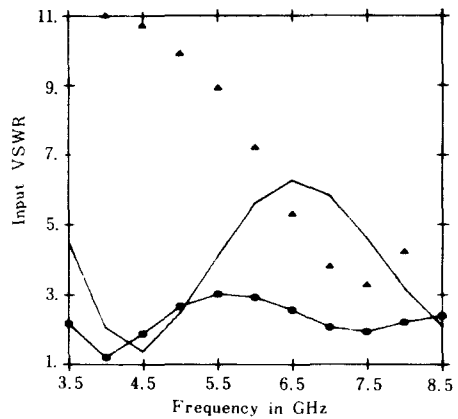


$C_1=0.650pF$, $L_1=0.615nH$, $R_3=40.74ohm$
 $L_2=0.978nH$, $L_3=0.558nH$, $C_4=1.925pF$
 $L_4=1.421nH$, $C_5=0.479pF$, $L_5=1.274nH$
 FET : NE38806 GaAs MESFET

Fig.6. Circuit of NE38806 4.0-8.0GHz amplifier.



(a) Amplifier gain



(b) Input VSWR

Fig.7. Characteristics of NE38806 4.0-16.0GHz amplifier.

the 4-8GHz frequency range. And the input VSWR is less than 3.0 over the same frequency range. These results can be compared with another previously published results of T.T. Ha.^[8] The gain is approximately same and |input VSWR is much improved. For comparison, results of T.T. Ha are plotted also in Fig.7 with small circle.

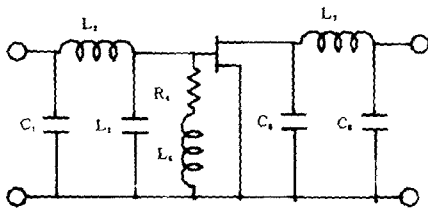
With the similar procedure, another broadband amplifier is designed with $0.3\mu m$ GaAs MESFET NE71000 over the frequency range 6.0-16.0GHz.^[11] Using the given S parameters of NE71000 with $V_{DS}=3V$ and $I_{DS}=30mA$, we can calculate the possible ranges of gain $|S_{21}|$ which is realizable with parallel lossy element and FET

at several frequencies such as 6, 11, and 16GHz respectively. From these calculations, 2.5 may be taken as the goal of flat gain. As in the previous example, series resistor and inductor circuit is used to realize parallel lossy admittance. At 6 and 11GHz, we select the values of Y_M as $0.9-j2.12$ and $0.3-j1.29$, and with these Y_M , $|S_{21}|$ is 2.51 and 2.63 respectively. These values of Y_M can be realized with $R=8.5 \Omega$ and $L=0.53$ nH. With this R-L circuit, $|S_{21}|$ is 2.14 at 16GHz. In this case, this R-L circuit satisfies the conductance and susceptance of selected Y_M simultaneously. As previous example, real frequency technique is used to design additional lossless matching networks in the input and output. In the lossless network design, 11 sample frequencies from 6 to 16GHz and 3rd order circuits are used. The

circuit and characteristics of designed amplifier are as in Fig.8. The gain of the designed amplifier is 8.83 ± 0.52 dB and input VSWR is less than 2.37 over the 6-16GHz frequency range. Adding of lossy elements to the interstage or output matching networks can be done in a similar way.

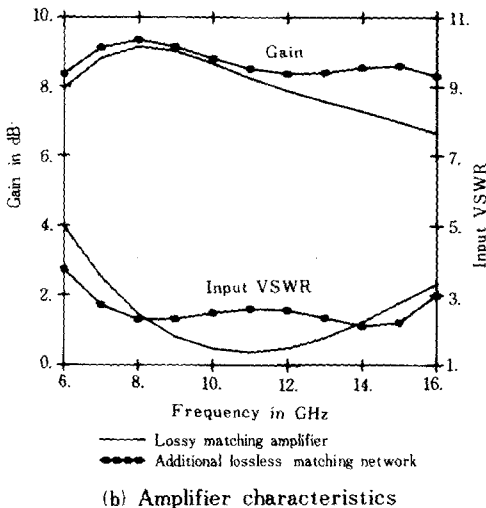
IV. Conclusions

In this study a lossy matching network is represented as a combination of 2 lossless matching networks between which serial impedance or parallel admittance is inserted. And for the amplifier of this configuration, realizable ranges of gain and reflection coefficients are calculated, and the relations between gain and reflection coefficients are derived. Also we propose the design method of lossy matching networks to have the goal gain and reflection coefficients, and if it is necessary to improve the amplifier characteristics, the real frequency technique can be used to design additional lossless matching networks. According to the theory, some numerical examples are presented about the realizable range of gain and reflection coefficients. And an octave bandwidth amplifier operating over 4 to 8GHz frequency is designed with gain of 8.56 ± 0.33 dB and maximum input VSWR of 3.0. Also another broadband amplifier is designed with gain of 8.83 ± 0.52 dB and maximum input VSWR of 2.37 in the frequency range 6.0-16.0GHz.



$C_1=0.227$ pF, $L_1=0.527$ nH, $C_2=0.135$ pF
 $R_1=8.50$ ohm, $L_2=0.530$ nH, $C_3=0.079$ pF
 $L_3=0.706$ nH, $C_4=0.191$ pF
 FET : NE71000 GaAs MESFET

(a) Circuit of designed amplifier



(b) Amplifier characteristics

Fig.8. Circuit and characteristics of NE71000 6.0-16.0GHz amplifier.

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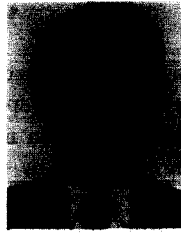
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