변형률 속도를 고려한 철근콘크리트부재 거동 예측을 위한 개선된 해석모델

An Improved Analytical Model for Considering Strain Rate Effects on Reinforced Concrete Element Behavior

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요 지

변형률 속도를 고려할 수 있는 철근과 콘크리트의 재료특성모델을 이용하여 여러가지 하중 재하속도를 받는 철근콘크리트부재의 압축 및 휨 거동 예측이 가능한 해석모델을 개발하였다. 개발된 철근콘크리트부재 해석모델을 적용한 해석 결과는 정적하중에서부터 동적하중을 받는 철근콘크리트부재의 실험결과와 비교적 양호하게 일치하였다.

Abstract

The strain rate-sensitive constitutive models of steel and concrete were incorporated into a refined analytical procedure for loading rate-dependent axial/flexural analysis of reinforced concrete beam-columns. The predictions of the analytical technique compared well with both quasi-static and dynamic test results on reinforced concrete elements.

1. Introduction

Current techniques for analysis and design of reinforced concrete structures subjected to impulsive loads are generally based on the results of quasi-static tests on reinforced concrete(R/C) materials and elements. The rate of straining in such tests is of the order of 10^{-5}/sec[1]. Typical strain rates in the critical regions of R/C structures under impulsive loads, however, can be as high as 10^{-1}/sec, or more[2-10]. Dynamic tests on materials[3-5] and elements[6-10] have revealed considerable dependence of the results on the rate of straining.

In general, the compressive and tensile strengths and stiffnesses of concrete and the yield strength of steel are observed to increase with increasing strain rate(Fig. 1)

As a result of the increases in material strength and stiffness, the axial and frexural strengths and stiffnesses of R/C elements

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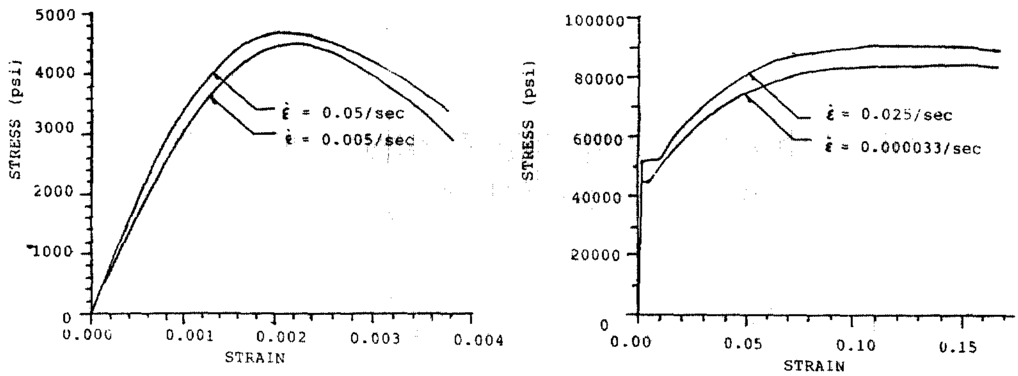
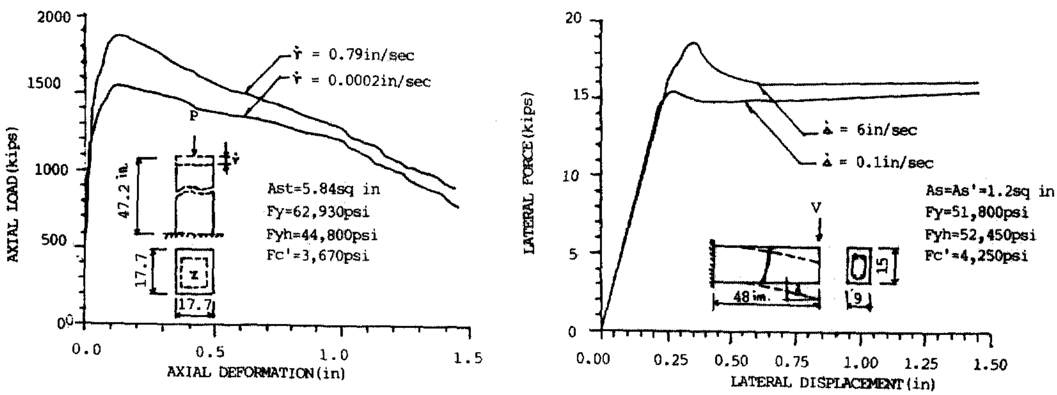


(a) concrete[3] (b) steel[4]

Fig. 1. Strain rate effects on the constitutive behavior of concrete and steel(1psi=0.0069 MPa).



(a) axial behavior[6] (b) flexural behavior[10]

Fig. 2. Loading rate effects on the axial and flexural behavior of R/C elements. (1in=25.4mm ; 1Kips=4.45KN)

also increase at higher rates of loading(Fig. 2). Hence, reliable imformation on strength and stiffness of R/C elements at higher strain rates is needed to improve the currently used techniques in dynamic analysis and design of R/C structures.

2. Strain rate-dependent material constitutive models

A refined analytical technique was developed for loading rate-sensitive analysis of R/C elements. This technique uses strain rate-sensitive material constitutive laws to construct the tangent stiffness matrix of R/C sections, which is used for iterative nonlinear analysis of R/C elements.

The adopted empirical strain rate-dependent constitutive law of concrete[3] in this study is :

$$f_c(\varepsilon, \dot{\varepsilon}) = \begin{cases} k_1k_2\, f_c'\left[\dfrac{2\varepsilon}{0.002\ k_1k_3} - \left[\dfrac{\varepsilon}{0.002\ k_1k_3}\right]^2\right] \\ \text{for } \varepsilon < 0.002\ k_1k_3 \\ k_1k_2\, f_c'\,[1-Z(\varepsilon-0.002\,k_1k_3)] > 0.2k_1k_2\, f_c' \\ \text{for } \varepsilon > 0.002\ k_1k_3 \end{cases} \quad (1)$$

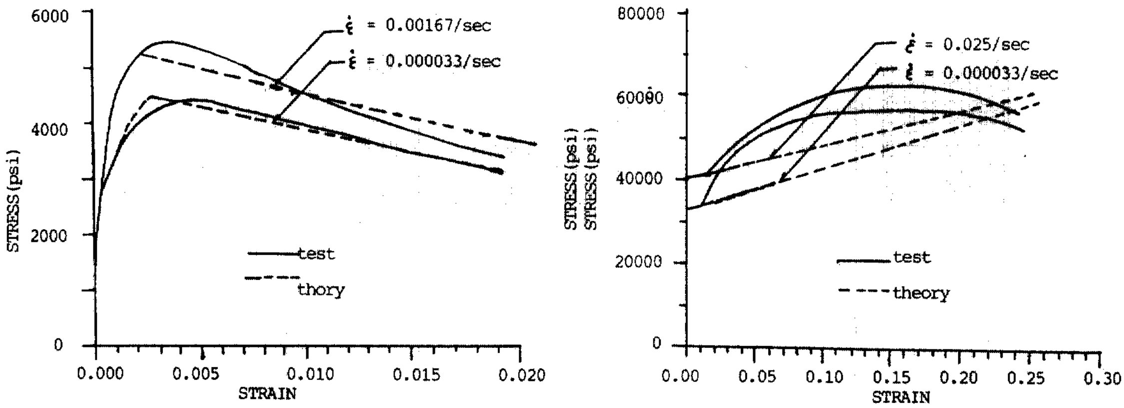
Where, f_c = concrete stress(psi)

ε = strain :

$\dot{\varepsilon}$ = strain rate(1/sec) $\geq 10^{-5}$

$k_1 = 1 + \dfrac{\rho_s f_{yh}}{f_c'}$;

ρ_s = valumetric ratio of transverse



(a) confined concrete (b) steel

Fig. 3. Comparision of material constitutive models with test results(1Psi=0.0069 MPa).

reinforcement to concrete core ;

f_{yh} = yield strength of transverse reinforcement(psi) ;

f_c' = standard(quasi-static) compressive strength of concrete(psi) ;

$$k_2 = \begin{cases} 1.48+0.160\log_{10}\dot{\varepsilon}+0.0127(\log_{10}\dot{\varepsilon})^2 & \text{for air-dried concrete} \\ 2.54+0.580\log_{10}\dot{\varepsilon}+0.0543(\log_{10}\dot{\varepsilon})^2 & \text{for saturated concrete} \end{cases}$$

$$k_3 = 1.08+0.112\log_{10}\dot{\varepsilon}+0.0543(\log_{10}\dot{\varepsilon})^2$$

$$Z = \frac{0.5}{\dfrac{3+0.002f_c'}{f_c'-100}+\dfrac{3}{4}\rho_5\dfrac{\sqrt{h'}}{s}\quad 0.002\,k_1k_3};$$

h' = width of concrete core measured outside of transverse reinforcemnt ;

s = center to center spacing of transverse reinforcement ;

Fig. 3(a) presents typical comparision between the predictions of the above model and test results performed at two different strain rates on confined concrete specimens.

The empirical strain rate-dependent constitutive law of steel[(4)] adopted in this study is :

$$f_s(\varepsilon,\ \varepsilon) = \begin{cases} E_s\ \varepsilon & \text{for } \varepsilon < \dfrac{f_y'}{E_s} \\ f_y' + E_h'(\varepsilon - \dfrac{f_y'}{E_s}) & \text{for } \dfrac{f_y'}{E_s} < \varepsilon < \varepsilon_u' \\ 0 & \text{for } \varepsilon > \varepsilon_u' \end{cases} \quad (2)$$

where, f_s = steel stress(psi) ;

E_s = modulus of elasticity of steel(psi)

f_y' = dynamic yield strength of steel (psi)

$= f_y[-4.51\times10^{-6}f_y+1.46+(-9.20\times10^{-7}f_y+0.0927)\log_{10}\dot{\varepsilon}]$;

f_y = standard(quasi-static) yield strength of steel(psi) ;

E_h' = dynamic strain hardening modulus of steel(psi)

$\dot{\varepsilon}$ = strain rate(1/sec)$>10^{-5}$

ε_u' = dynamic ultimate strain of steel

$= \varepsilon_u[-8.93\times10^{-6}f_y+0.007+(4\times10^{-6}f_y-0.1850)\log_{10}\varepsilon]$;

ε_u = quasi-static ultimate strain of steel

Typical comparision between the predictions of this model and results of tension tests performed on steel specimens at two different strain rates is shown in Fig. 3(b).

3. Section model

The axial/flexural tangent stiffness matrix of R/C cross sections was derived using the layer modelling technique[11]. In this technique, the cross section is divided into a number of concrete and steel layers(Fig. 4). The strains and strain rates at these layers are calculated from the values of strain and

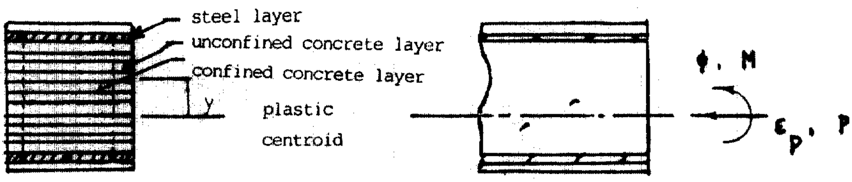


Fig. 4. Layer modeling of R/C sections.

strain rate of the section(assuming that plane sections remainplane) :

$$d\varepsilon_i = d\varepsilon_p + d\phi \cdot y_i \qquad (3)$$

$$\dot{\varepsilon}_i = \dot{\varepsilon}_p + \dot{\phi} \cdot y_i \qquad (4)$$

where, $d\varepsilon_i$ = strain increment in the i'th layer ;
$\dot{\varepsilon}_i$ = strain rate in the i'th layer ;
$d\varepsilon_p$ = strain increment at plastic centroid ;
$\dot{\varepsilon}_p$ = strain rate at plastic centroid ;
$d\phi$ = section curvature increment ;
$\dot{\phi}$ = section curvature rate ;
y_i = distance from the centroid of the i'th layer to the plastic centroid

Once the layer strains and strain rates are obtained, tangent stiffness moduli(Ei) of various layers can be calculated using the strain rate-dependent steel and concrete constitutive models presented earlier :

$$E_i = \frac{df(\varepsilon_i, \dot{\varepsilon}_i)}{d\varepsilon_i} \qquad (5)$$

Where, $f(\varepsilon_i, \dot{\varepsilon}_i)$ = constitutive equation expressing the value of stress at the i'th layer in terms of strain and strain rate

The section stiffness matrix(Ks) can then be constructed by proper summation of the layer tangent stiffness moduli :

$$\begin{bmatrix} dM \\ dP \end{bmatrix} = k_s \begin{bmatrix} d\phi \\ d\varepsilon_p \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \cdot \begin{bmatrix} d\phi \\ d\varepsilon_p \end{bmatrix} \qquad (6)$$

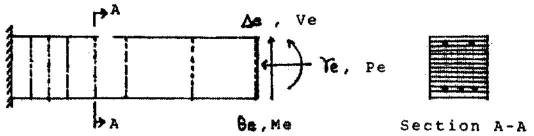
Where, dM = bending moment increment at the cross section ;
dP = axial load increment at the cross section ;
$k_{11} = \sum_i A_i E_i y_i^2$;
$k_{12} = k_{21} = \sum_i A_i E_i y_i$;
$k_{22} = \sum_i A_i E_i$;
A_i = area of the i'th layer

4. Element model

The section stiffness matrix derived above was incorporated into an algorithm for non-linear axial/flexural analysis of R/C cantilever elements. In flexural analysis, the inputs to the element analysis are the increments in lateral displacement, time, bending moment and axial load at the free end of the cantilever element. In axial analysis, the inputs are the increments in axial deformation of the plastic centroid, time, bending moment and lateral load at the free end. The model for axial/flexural analysis of R/C cantilever elements was constructed by : firstly, dividing the element by a number of slices along its length as shown in Fig. 5(a) (the divisions being more refined near the fixed end that is in generally more critically stressed), and then subdividing each slice to a number of steel and concrete layers as discussed in the section model[Fig. 5(b)].

The numerical algorithm for inelastic flexural [or axial] analysis of R/C cantilever elements is presented below. This method accounts for the loading rate effects and the geometric material nonlinearities.

1. Input the incremental values of free end lateral displacement(d△e), time(dt), bending moment(dM_e) and axial load(dP_e) for flexural analysis [or lateral load(dV_e), time(dt), bend-



(a) division of element by slices (b) subdivision of slices by layers

Fig. 5. Modeling of R/C Elements

ing moment(dM_e) and axial displacement($d\gamma_e$) for axial analysis〕; add to the values of previous step to get the total amounts.

2. Estimate the total value of lateral load (V_e) at the free end for flexural analysis 〔or the total value of axial force(P_e) for axial analysis〕 at the completion of the current time step.

3. For each slices :

   (a) calaulate the values of axial force and bending moment at the end of current time step resulting from the application of the input(or estimated) values of end bending moment, axial load and lateral force. The contribution of axial load to the bending moment(P-△effect) should be considered

   (b) using the strain rate-sensitive section analysis formulations, presented earlier obtain the values of curvature and plastic centroid axial strain corresponding to the values of bending moment and axial load applied to section at a specified loading rate.

4. Calculate the end lateral displacement in flexural analysis 〔or axial displacement in axial analysis〕 of the element corresponding to the estimated values of end forces by proper integration of the curvatures in flexural analysis 〔or plastic centroid axial strains in axial analysis〕 of slices along the length.

5. Compare the calculated value of end lateral displacement in flexural analysis 〔or end axial displacement in axial analysis〕 with the corresponding input value. If the error is unacceptable, revise the estimated value of lateral force 〔or axial load〕, and repeat the procedure from step 3. Otherwise, the analysis for this time step is complete, and the end values of rotation and axial displacement in flexural analysis 〔or rotation and lateral displacement in axial analysis〕 can be obtained using the curvatures and plastic centroid axial strains of slices along the element length. The analysis can then be continued for the next time step starting from step 1.

5. Comparison with test results

The developed strain rate-sensitive methods for axial/flexural analysis of R/C elements was used to predict the results of some tests performed at different loading rates, on R/C columns[6] and beams[10].

Ref. 6 has reported the results of column tests in which a concentric axial load was applied at a constant displacement rate. The rate of displacement increase varied in different tests. Fig. 6 compares the experimental axial load-displacement relationships reported in Ref. 6 with the predictions of the axial analysis procedure introduced earlier. It can be seen in this figure that the loading rate effects on axial behavior of R/C columns are significant, and the developed theoretical approach is capable of closely predicting the test results.

Experimental lateral load-deflection envelope curves for R/C beams loaded at different displacement rates are reported in Ref. 10. Fig. 7 compares these test results with the predictions of the suggested strain rate-sensitive flexural analysis technique. The discrepancies between the predictions and test results seem to be came from the simple bilinear constitutive model of reinforcing

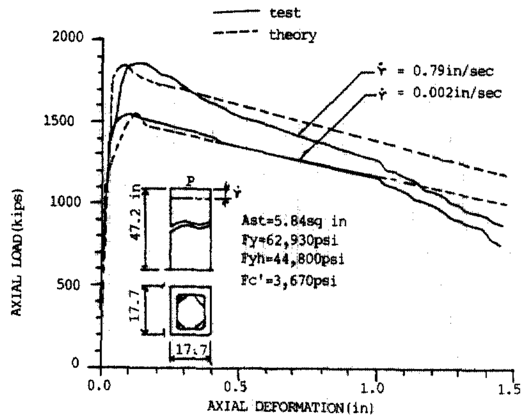


Fig. 6. Experimental[6] and analytical axial performances of R/C columns at different loading rates(1in=25.4m ; 1Kips=0.445KN).

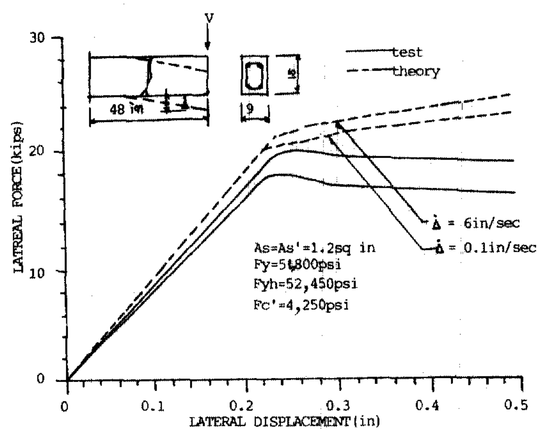


Fig. 7. Experimental[10] and analytical flexural performances of R/C beams at different loading rates(1in=25.4mm ; 1Kips=0.445KN).

steel in Fig. 3(b).

However, the proposed analytical approach is observed to compare satisfactorily with test results before yielding.

6. Conclusions

A strain rate-dependent model was developed for predicting the axial/flexural behavior of R/C elements using two strain rate-sensitive material constitutive laws. The analytical predictions compared well with both quasi-static and dynamic test results.

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