부마찰력의 계산적 예측방법

Computational Predictions of Pile Downdrag

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Abstract

A computer program evaluating the pile downdrag is developed using the conventional elastic solid method. Modification of the conventional method has been performed by introducing the concept of critical relative displacement. A simple transfer function method which employes the critical relative displacement as a pile-soil slip criterion and calculates downdrag by Mohr-Coulomb equation, has also been developed.

The results of three methods are all found to be in good agreement with field observations. When they are applied to a centrifuge modeling problem of pile downdrag to predict its result, however, diverse answers are obtained.

Overall, the simple transfer function method developed in this study seems to be the most effective in the evaluation of pile downdrag, considering the quality of its result and its efficiency in computation.

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요 지

종래의 탄성 고체법을 이용하여 말뚝의 부마찰력을 산정하는 전산프로그램을 개발하였다. 그리고 한계 상대 변위 개념을 도입하여 이 전산프로그램의 수정을 시도하였다. 끝으로 한계 상대변위로서 말뚝과 흙사이의 미끄러짐 발생 여부를 결정하고 모아-쿨롱의 파괴 방정식을 이용하여 부마찰력을 산정하는 단순 전이함수법을 개발하였다.

이 세가지 방법에 의한 결과는 모두 현장 측정치와 잘 일치하였다. 그러나, 이들이 원심력을 이용한 모형실험 결과를 예측할 때에는 각기 다른 결과를 나타내었다.

종합적으로 보면, 이 논문에서 제안한 단순전이 함수법이 말뚝 부마찰력 산정시 그 결과의 정확성과 계산상의 효율성들을 고려할 때 가장 능률적이라고 판단된다.

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1. INTRODUCTION

Downdrag forces are exerted on pile shafts under the situations where surrounding soil exhibits a downward movement with respect to a pile shaft. The soil movement can be caused by the self weight consolidation of a under consolidated compressible layer or

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by pumping of water from one of the acquifer strata in the profile or by other surface loads. It is a phenomenon of load transfer in which the load is transferred from soil to pile instead of pile to soil as is more often the case. The load transfer has been approached in several ways, among which the transfer function method, the elastic solid method, and the numerical method (mainly, the finite element method) are included.

In the elastic solid method, the pile is divided into a number of elements, and a solution is obtained by imposing compatibility between the displacements of the pile and the adjacent soil for each element of the pile. the displacements of the pile are obtained by considering the compressibility of the pile under uniaxial loading. The soil displacements are obtained by using Mindlin's equations for the displacements within a soil mass caused by loading within the mass.

The compatibility condition imposed in the elastic method admits no relative motion between pile and soil except when the interaction stress exceeds the shear strength between pile and soil. However, it is widely observed and accepted by the colleagues that the relative movement is always preceded before the interaction stress reached its maximum. The relative displacement which produces the maximum interaction stress is often called a critical relative displacement (CRD). The CRD producing the maximum shaft resistance for in-situ piles is known to have the values between 0.25 in. −0.40 in. [1], regardless of the size of pile diameter and pile length. The very recent research [2] on the same subject established an empirical relationship between the CRD and the maximum pile-soil adhesion.

It is intended, in this paper, to modify the elastic solid method by introducing the concept of the CRD. For th [illegible] computer program (DGSLIP [illegible] conventional elastic method [illegible] loped by the author followi [illegible] developed by D'Appolonia and Ro [illegible], [3]) and Poulos and Mattes(1969, [4]). Next, a modified version (DDRAG) of the program is developed which uses the empirical relationship between the CRD and the maximum pile-soil adhesion to set a slip criterion between pile and soil in terms of the soil movement instead of the shear strength. Finally, a simple method (SMEN) which employes the CRD as a slip criterion and calculates downdrag using the Mohr-Coulomb's shear strength equation has been developed and compared with the previous methods.

2. Descriptions of Computational Methods

As mentioned earlier, three different computer programmes are developed to solve downdrag problems as illustrated in Figs. 1 and 2. Each program is briefly described below.

DGSLIP

Governing Equation :

$$([I]) + dk[{}_sI - {}_sI'][D]^{-1}\{\rho\} = \{S\} + \frac{Pa}{Es\pi d}[D]^{-1}[{}_sI - {}_sI']\{h\}$$

, where

$[I]$: identity matrix,

$[{}_sI - {}_sI']$: matrix of soil displacement influence factor including the effect of 'mirror image' for a rigid base,

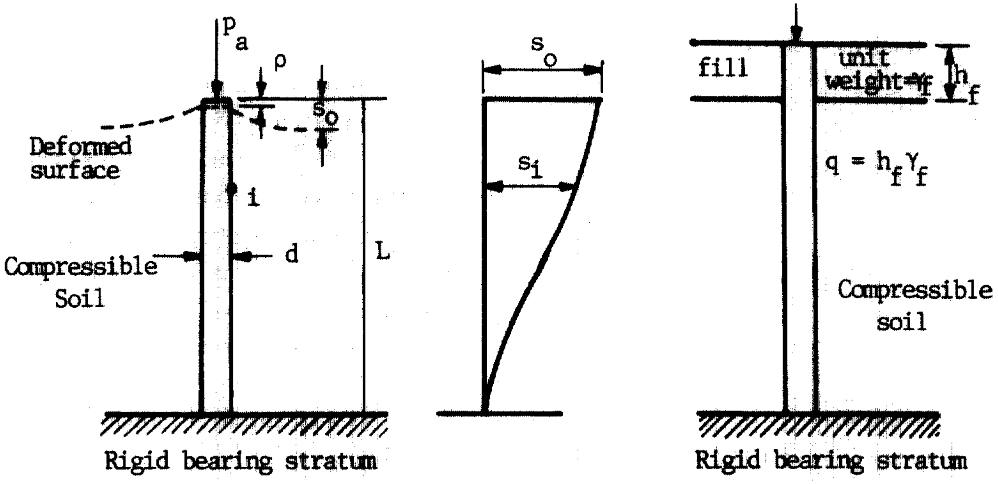
$[D]$: $n \times n$ matrix of soil displacement factors,

$\{\rho\}$: pile-soil displacement vector,

$\{S\}$: soil movement,

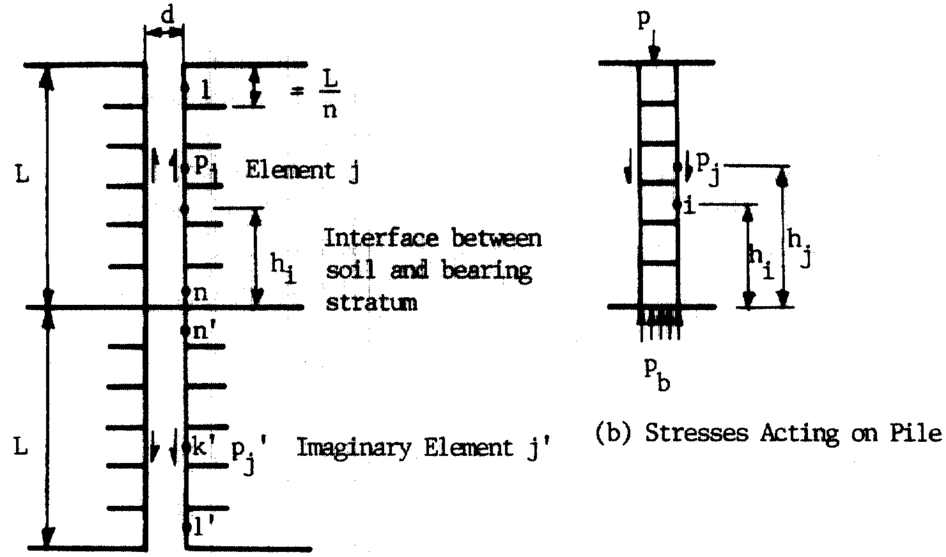
$\{h\}$: distance vector from bottom to the ith element,

k : (Ep/Es) Ra, stiffness factor,



(a) Geometry of pile (b) Distribution of Soil movement with depth (c) Pile soil loaded with fill

Fig. 1. Downdrag Problem



(a) Stresses on Soil Adjacent to Pile

(b) Stresses Acting on Pile

Fig. 2. Analysis of Downdrag on End-Bearing Pile

Ra : $Ap/(\pi d^2/4)$, area ratio,
Pa : pile load acting on top,
Ep : Young's modulus of pile, and
Es : Young's modulus of soil.

For the element in which the computed downdrag stress (p_i) is bigger than the maximum pile-soil adhesion(τ_{ai}), replace the corresponding row of the governing equation by

$$[D]^{-1}[I]\{\rho_i\} = \frac{1}{EpRa}\{\tau_{ai}\} + \frac{Pa}{ApEp}[D]^{-1}$$

, where $\tau_{ai} = C_{ai}' + Ks \cdot \sigma_{vi} \cdot \tan\phi_a'$

The modified system of equations is now resolved and the procedure is repeated until the computed values of shear stress(p_i) do not exceed the limiting values τ_{ai}.

DDRAG

Governing Equation :

$$\left(\frac{[D]}{dk}+[sI-sI']\right)\{p\}$$
$$=\frac{Es}{d}\left(\{S\}-\frac{Pa}{EpAp}\{h\}\right)=\frac{Es}{d}\{\triangle\}$$

, where

{p} : downdrag acting on pile surface,

$\{\triangle\} : \{S\}-\frac{Pa}{EpAp}\{h\}$

For the element in which $\triangle i > CRD+(\delta max)i$, replace $\triangle i$ by $CRD+(\delta max)i$, where $(\delta max)i$ represents the maximum possible pile shortening due to the downdrag at the *i*th element. No iteration needed.

SMEN

1. Calculate the critical relative displacement (CRD) for a given pile-soil system using the empirical relationship, $(\tau_{ai})_{av}=24$ CRD [in terms of lb-in. units].
2. Compare the soil movement with the CRD for each element of the pile.
3. Distinguish slip elements from non-slip elements.
4. Calculate the downdrag for the slip elements by Mohr-Coulmb equation, and for the non-slip elements, calculate the downdrag by $p_i=\tau_{ai}\times\frac{\rho_i}{CRD}$, where, ρ_i = soil movement at the *i*th element.
5. Calculate the magnitude of pile shortening due to the downdrag force.
6. Calculate the net soil movement by subtracting the pile shortening from the original soil movement.
7. Repeat steps from 2 to 6 until identical downdrag forces obtained. Normally, 3 iteratons are enough.

3. Computed Results and Discussions

First of all, to verify the validity of the computer programmes developed in this study by comparing the computed results with in-field observations of pile behavior in the presence of downdrag, three example cases reported in the literature are considered here (Bjerrum et. al., 1969⁽⁵⁾ ; Walker and Darvall, 1973⁽⁶⁾). The data of the three cases are summarized as the followings :

Case 1. Test of Bjerrum et. al. --- Heroya Site, Pile A

L=30m(7.5m of fill included), d=0.3m (7mm wall)

Unit wt. of fill=19.6 KN/m³,

Submerged unit wt. of clay=9.81KN/m³,

$(Ks\cdot\tan\phi_a')$ fill=0.20, $(Ks\cdot\tan\phi_a')$ clay=0.25, $C_a'=0.0$

Es=9.81MN/m², Ep=2.06×10⁵MN/m²,

Poisson's ratio=0.40, top=20cm (hyperbolic dist.)

Case 2. Test of Bjerrum et. al. --- Sorenga Site, Pile C

L=55m(13.75m of fill), d=0.5m(8mm wa

Unit wt. of fill=19.6KN/m³,

Submerged unit wt. of clay=9.81KN/m³,

$(Ks\cdot\tan\phi_a')$fill=$(ks\cdot\tan\phi_a')$clay=0.20,

Ca'=0.0,

Es=9.81 MN/m², Ep=2.06×10⁵MN/m²,

Poisson's ratio=0.40, top=27cm(hyperbolic dist.)

Case 3. Test of Walker and Darvall

L=25.5m(8.925m of fill), d=0.76m(11mm wall)

Unit wt. of fill=22.6KN/m³,

Submerged unit wt. of clay=9.81KN/m³,

$(Ks\cdot\tan\phi_a')$fill=0.45, $(Ks\cdot\tan\phi_a')$ clay=0.40,

Ca'=38.3KN/m², Es=16.7MN/m²,

Ep=2.06×10⁵MN/m²,

Poisson's ratio=0.40, top=3.5cm (hyperbolic dist.)

The results obtained for the example cases by the previously described computer pro-

Tabel 1. Comparison of Downdrag Calculation for Case 1

Items / Methods	Accuumlative Ddrag Force(tons)		Pile Shortening at the top (cm)	Computer Time (sec)		Ramarks
	maximum	Element No.***		CPU	Execution	
DGSLIP(IOP* = 0)	107.9	18	1.11	2.32	55.28	NP** = 3
DGSLIP(IOP ≠ 0)	107.9	18	1.11	2.32	55.40	NP** = 4
DDRAG	100.7	18	1.16	1.73	54.63	
SMEN	102.2	19	1.07	0.59	0.14	

*If IOP = 0, Soil movement modification executed.

**NP : number of iteration needed to obtain the final results.

***Total number of element is 20.

Table 2. Comparison of Downdrag Calculation for Case 2

Items / Methods	Accuumlative Ddrag Force(tons)		Pile Shortening at the top (cm)	Computer Time (sec)		Ramarks
	maximum	Element No.***		CPU	Execution	
DGSLIP(IOP* = 0)	389.4	16	4.53	2.32	55.13	NP** = 3
DGSLIP(IOP ≠ 0)	389.4	16	4.53	2.32	55.19	NP** = 4
DDRAG	335.3	16	4.20	1.73	54.48	
SMEN	397.8	15	5.22	0.59	0.14	

*If IOP = 0, Soil movement modification executed.

**NP : number of iteration needed to obtain the final results

***Total number of element is 20.

Table 3. Comparison of Downdrag Calculation for Case 3

Items / Methods	Accuumlative Ddrag Force(tons)		Pile Shortening at the top (cm)	Computer Time (sec)		Ramarks
	maximum	Element No.***		CPU	Execution	
DGSLIP(IOP* = 0)	229.4	14	0.67	2.32	56.40	NP** = 2
DGSLIP(IOP ≠ 0)	229.4	14	0.67	2.32	58.74	NP** = 3
DDRAG	261.0	14	0.83	1.73	56.01	
SMEN	274.7	14	0.82	0.59	0.14	

*If IOP = 0, Soil movement modification executed.

**NP : number of iteration needed to obtain the final results.

***Total number of element is 20.

grammes are tabulated in Tables 1–5, and plotted in Figs. 3–6. In Tables 1–3, identical results of DGSLIP are listed two times for different values of IOP. In the computer program DGSLIP, an optional step is included whose execution is controlled by the value of IOP ; if IOP = 0, the optional step is executed. In the optional step, the magnitude of the maximum efffective soil movement is calculated by adding the maximum possible pile shortening at the pile top due to the downdrag to the CRD, and the soil movements

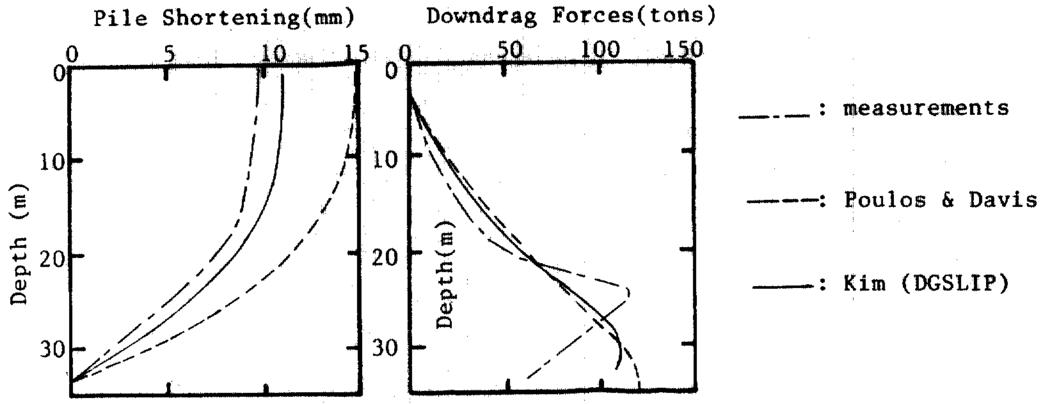


Fig. 3. Comparison with Pile Tests of Bjerrum et. al.(1969) at Heroya

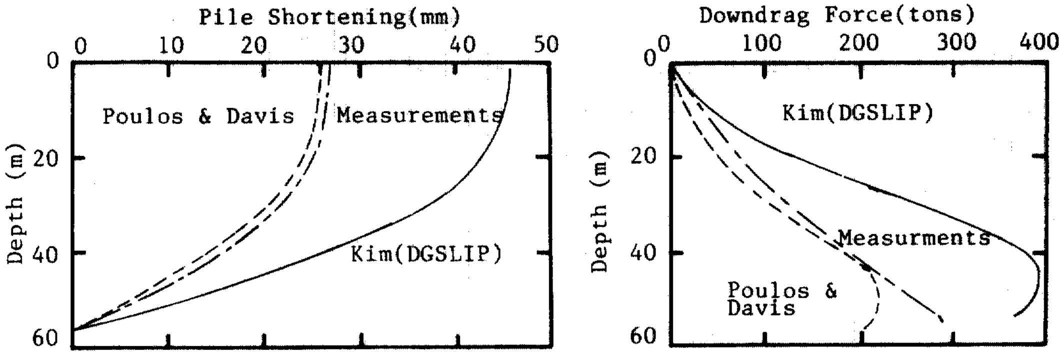


Fig. 4. Comparison with Pile Test of Bjerrum(1969) at Sorenga

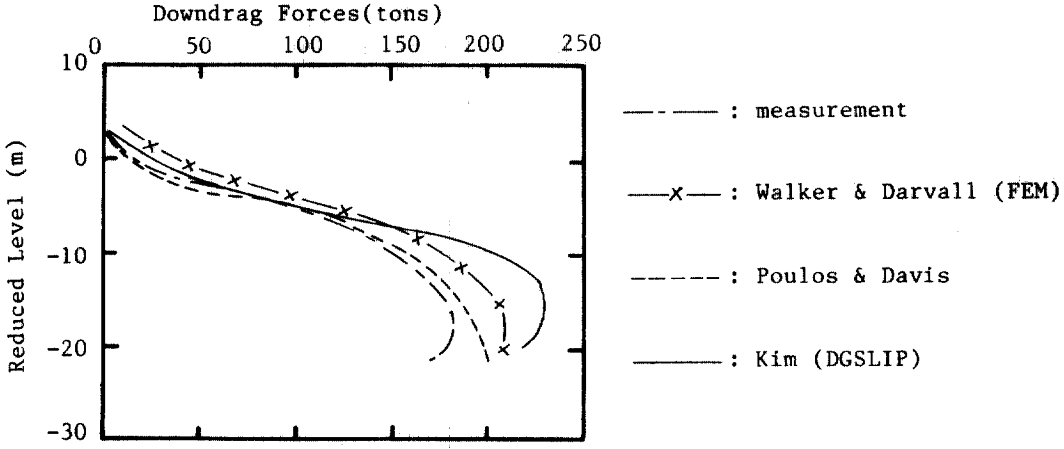
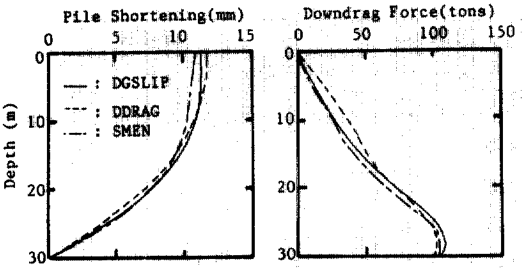


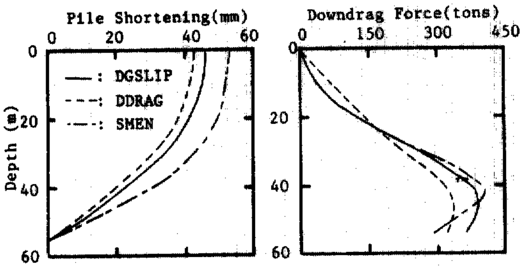
Fig. 5. Comparison of Pile Test of Walker and Darvall(1973)

bigger than the maximum value are adjusted to be equal to that before the system of governing equations is initially solved.

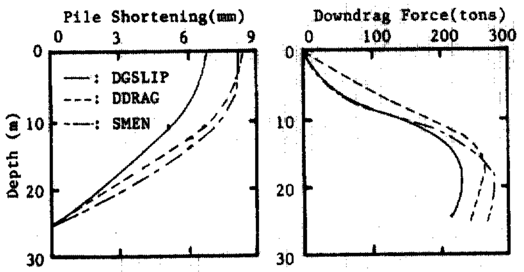
In Figs. 3−5, The computed results by DGSLIP are compared, in terms of distributions of pile shortening and downdrag forces, with measured quantities and with the predictions of Poulos and Davis(1980[7]).



(a) Case 1



(b) Case 2



(c) Case 3

Fig. 6. Comparison of Computed Results for Example Cases

The agreement between measured and computed result is good for Cases 1 and 3, but the computed result is bigger than the measured one for the Case 2. This somewhat large discrepancy in Case 2 may be attributed to the conservative selection of the magnitude of input parameters which are not clearly stated in the literature.

Both the modified version (DDRAG) and the simple method (SMEN) give almost identical results with DGSLIP when they are applied to the practical cases as is confirmed in Figs. 6(a)−(c). The maximum downdrag and the length of pile top shortening are listed in Tables 1−3 along with the CPU and execution times for computer runs. Comparing the computer times, compilation of DGSLIP takes more time than DDRAG by about 35 percents and than SMEN by 300 percents. For the execution of the computer programmes, DGSLIP takes slightly longer time than DDRAG, while SMEN takes only 1/350 of the execution time of either DGSLIP or DDRAG. In the case of DGSLIP, if the optional step is executed (i. e., IOP=0), the total number of iterations (NP) may be reduced by 1, which, in turn, reduces the execution time by a very small amount, as noticed from Tables 1−3. So, the execution of the optional step, in which initially the soil movement is limited to a certain value, does not improve the efficiency of the program run much but, at least, it helps one to deal with numbers which make sense from the beginning (e. g., the maximum probable effective soil movement is of the order of magnitude of 0.5−1.0 in., when the in-field soil movement can be as large as 10 feet).

In table 4, the results of parametric study to Case 1 are summarized. It is observed from the Table that the characteristics of soil movement distribution along the pile depth affects the results more than the increment of the soil movement by a factor of 10 does, which is not a very surprising result considering the magnitude of the effective soil movement to mobilize full downdrag is of such a small value. Young's modulus of soil (Es) affects the result of DDARG the most, and that of SMEN is not affected at all because it is not included in the input parameters for SMEN. For practical cases where other parameters such as the coefficient of lateral pressure (Ks), the friction angle between pile and soil

Table 4. Calculated Downdrag Forces by Different Methods for Different Cases(case 1)

Method / Item / Parameters		DGSLIP		DDRAG		SMEN	
		Max. Ddrag (KN/m²) (El. No.)	Top Pile (cm) Shortening	Max. Ddrag (KN/m²) (El. No.)	Top Pile (cm) Shortening	Max. Ddrag (KN/m²) (El. No.)	Top Pile (cm) Shortening
Original Case*		749 (18)	1.11	699 (18)	1.16	710 (18)	1.07
Triangular Dist. of Soil Movement		1020 (20)	1.19	913 (20)	1.24	903 (20)	1.13
10 Times Magnif. of Soil Movement		990 (20)	1.19	891 (20)	1.24	885 (20)	1.13
Es	9.81×10^2	426 (17)	0.81	131 (19)	0.24	710 (19)	1.07
	9.81×10^4	885 (19)	1.17	1520 (17)	1.83	710 (19)	1.07
Ep	2.06×10^7	468 (13)	7.61	459 (13)	7.81	99.5 (11)	11.4
	2.06×10^9	814 (19)	0.12	696 (19)	0.11	776 (20)	0.11
Ap	6.40×10^{-4}	468 (13)	7.61	459 (13)	7.81	99.5 (11)	11.4
	6.40×10^{-2}	814 (19)	0.12	696 (19)	0.11	776 (20)	0.11

*In the original case, hyperbolic distribution of soil movement assumed with $Es=9.81\times10^3$, $Ep=2.06\times10^8$(KN/m²), and $Ap=6.40\times10^{-3}$(m²)

(ϕ_a), and soil unit weight (γ) are adjusted accordingly to the change of Es, the results of SMEN may become closer to DGSLIP. The effects of Ep on the results are exactly same with that of pile cross-sectional area (Ap), as expected.

Since the CRD at which the pile-soil adhesive stress is fully developed is not proportionally related with the pile geometry, it is questioned whether the quantitative model tests of reduced scale in a centrifuge will porduce identical results with prototypes. This may very well be verified if such model tests are executed and compared with the in-field measure ments. It is attempted here, numerically, to predict the model pile behavior in terms of the maximum downdrag and pile top shortening using the computer programmes. The results are tabulated in Table 5. From the Table, it is shown that DGSLIP predicts an identical result and DDRAG gives higher values, while SMEN predicts much lower values compared with those of prototype. It is believed by the author that the

Table 5. Scaling Effect on Downdrag Force (Case 1)

Iteam / Methods	Max. Ddrag (KN/m²)		Top Pile Shortening (cm)	
	Prototype	Model ($\lambda=100$)	Prototype	Model ($\lambda=100$)
DGSLIP	749.0	749.0	1.11	0.0111
DDRAG	699.0	4160.0	1.16	0.101
SMEN	710.0	77.0	1.07	0.00156

result of SMEN would be the most realistic one among the three methods considering the similarity rule's violation by the CRD quantity. However, since it cannot be verified experimentally at this moment, it remains to be seen if the belief is true.

4. Conclusions and Recommendations

It is concluded and recommended from the results obtained in this study as summarized below :

1. The computer programmes developed in this study compare well each other and well with in-field observations.

2. The simple method (SMEN) developed in this study seems to be the most effective in the evaluation of downdrag considering the quality of tis result and its efficiency in computations.

3. Model tests on shaft-adhesion-related problems may lead to an erroneous result.

4. Execution of model tests on the shaft-adhesion-related problems is recommended to verify conclusion 3 as well as the computational predictions.

5. Acknowledgment

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