Measurement of Air Velocity Using a Slanted Hot-wire

경사진 Hot-wire probe를 사용한 공기속도의 측정에 관한 연구

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— 요 약 —

본 연구에서는 경사진 hot-wire probe의 특성값을 구하고 열선풍속계의 응답식으로부터 수평원통관내에서 3차원 흐름의 속도성분과 난류 강도 그리고 전단 응력을 pitot tube의 결과와 비교 연구하였다.

또한, 속도 및 난류 강도에 관한 유도된 식을 swirling flow에도 적용하여 만족한 결과를 얻었다.

1. INTRODUCTION

The hot-wire technique of measuring velocity data is well established and widely used although much of the published work is not concerned with detailed measurement in truly three dimensional flow. Some investigations on the use of multi-sensor measurements for flows appear in the literature.

However, in ducts of relatively small equivalent diameters those techniques have several disadvantages which will be discussed later. In this investigation, it was decided to employ same of the methods developed for one dimensional flow, which have been described previously by Morsi[6], to determine all mean and fluctuating components using single wire probes.

These have several advantages such as economy, accuracy and low interference compared with multiwire probes. The following section presents hot-wire response equations and the method for relating the anemometer signals to the velocity phenomena. The chosen technique, the experimental equipment and associated instrumentation' are then described in all detail.

2. HOT-WIRE RESPONSE EQUATION

The constant temperature anemometer makes use of the following relationship between the instantaneous voltage signal and the effective cooling velocity (the so called King's

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law) viz.

$$E^2(t) = A + BU_{eff}^m \quad \cdots\cdots (1)$$

where, A,B and m are constants all are normally determined by calibration tests. When the anemometer output voltage E(t) is employed as input value to a linearizer the output signal E from the linearizer unit is concerned to U_{eff} by,

$$E = sU_{eff} \quad \cdots\cdots (2)$$

where s is a slop factor to be obtained from the hot wire calibration.

In the 1971, Jorgensen[4] mentioned that the velocity vector resolution at sensor along the probe axis(x) along the sensor y, and along an axis z perpendicular to sensor prong plane. The effective cooling velocity acting on the sensor can be expressed as:

$$U_{eff}^2 = U_n^2 + H^2 U_p^2 + K^2 U_b^2 \quad \cdots\cdots (3)$$

where, U_n is the velocity component perpendicular to hot-wire along the probe axis, U_p is the velocity component parallel to the sensor, U_b is the velocity component perpendicular to the prongs - sensor plan, H is the yaw factor and K is the pitch factor.

If equation (2) is substituted into equation (3), the following principal equation is obtained

$$(E/s)^2 = U_n^2 + H^2 U_p^2 + K^2 U_b^2 \quad \cdots\cdots (4)$$

From this equation, unknown velocities and fluctuating components can be calculated.

3. THE PREFERRED METHOD OF CALCULATION

The basic problem in hot-wire analysis is to relate analysing the measured hot-wire signal. Usually, two fundamental approaches are used to analyse the electrical output signals from the hot-wire anemometer. The first method is to bring to equation (4) by a series of developments in what is often called the conventional method. When the terms above equation the higher order are neglected, this leads to an approximate equation for E/s.

Therefore, the time mean components are obtained directly by using this method. The second method is the use of the squared signal E^2, rather than E. This method is, therefore, considered superior to the first method in highly turbulent flow.

4. THE FIRST METHOD OF MEASUREMENT

If a normal straight probe is employed and the probe is rotated about its axis so that the hot-wire is parallel to the y or z direction and the probe axis is normal to the main stream (see Fig. 1(a)) then

$$U_{eff}^2 = V^2 + H_1^2 W^2 + K_1^2 U^2 \quad \cdots\cdots (5)$$

Again if the probe is turned through 90° about its axis so that the hot-wire sensor is parallel to the flow as shown in Fig. 1(b) then

$$U_{eff}^2 = V^2 + H_1^2 U^2 + K_1^2 W^2 \quad \cdots\cdots (6)$$

If a right angle wire is employed so that the wire is perpendicular to the main flow Fig. 1(c)

$$U_{eff}^2 = U^2 + H_2^2 V^2 + K_2^2 W^2 \quad \cdots\cdots (7)$$

Hence, the three unknown mean velocities can be obtained from these three equations. This method is useful for deliverating the mean velocity components only, hence a slanted wire may be employed to calculate three velocity, turbulent intensity and shear stress. If a slanted wire is oriented in the x-z plane or the y-z plane, as shown in Fig. 2, the velocities acting on the sensor may be calculated from expression delivered equation (3) as determinationed by Acrivlellis[2] or Morsi[6].

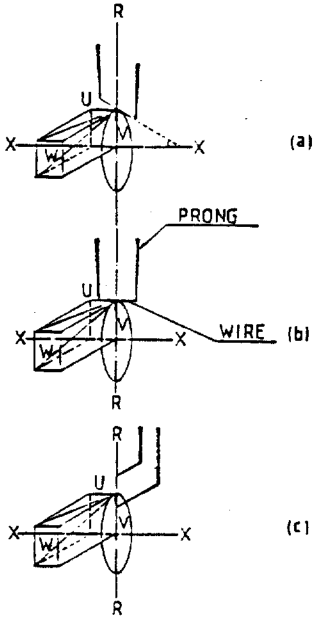

Fig. 1 Positioning of hot-wire probes in the test tube.(a),(b) DISA55P11,(c)DISA 55P13.

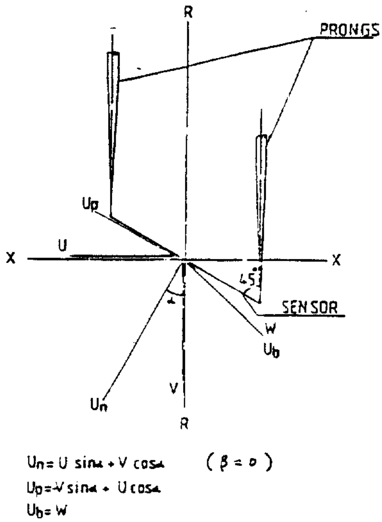

Fig. 2 Schematic of a slanted wire(DISA 55P12) oriented velocity components.

The instantaneous flow velocity $U_{eff}(t)$ can be described as a component U_{eff} so that,

$$U_{eff}(t)=U_{eff}+u_{eff} \quad \cdots\cdots (8)$$

and the instantaneous voltage is

$$E(t)=E+e \quad \cdots\cdots (9)$$

Substituting the above relation into equation (2), one can write,

$$(E^2+e^2)/s^2=U_{eff}^2+u_{eff}^2 \quad \cdots\cdots (10)$$

If a slanted wire is oriented in the y-z plane as shown in Fig.2, the hot-wire response equation is described by the following expression[(2)],

$$E^2(\beta,\alpha)/s^2=\{\overbrace{(V+v)\cos\alpha+[(U+u)\cos\beta-(W+w)\sin\beta]\sin\alpha}^{U_n}\}^2 +H^2\{\overbrace{[(U+u)\cos\beta-(W+w)\sin\beta]\cos\alpha-(V+v)\sin\alpha}^{U_p}\}^2 +K^2\{\overbrace{[(U+u)\sin\beta+(W+w)\cos\beta]}^{U_b}\}^2 \quad \cdots\cdots (11)$$

Using equation (11), the following expression may be delivered

$$\begin{aligned}(E^2+e^2)/s^2=&(V^2+v^2)(\cos^2+H^2\sin^2\alpha)\\&+(U^2+u^2)(\cos^2\beta\sin^2\alpha\\&+H^2\cos^2\beta\cos^2\alpha+K^2\sin^2\beta)\\&+(W^2+w^2)(K^2\cos^2\beta+\sin^2\beta\\&\sin^2\alpha+H^2\cos^2\alpha\sin^2\beta)\\&+2(UV+uv)(1-H^2)\cos\beta\cos\alpha\\&\sin\alpha+2(VW+vw)(H^2-1)\\&\sin\beta\times\cos\alpha\sin\alpha+2(UW+uw)\\&(K^2\cos\beta\sin\beta-H^2\cos^2\alpha\cos\beta\\&\sin\beta-\sin\beta\sin^2\alpha\cos\beta)\cdots (12)\end{aligned}$$

1) Using a single slanted wire (i.e. α=45°)

At $\beta=0$

$$\begin{aligned}E^2(0,45)=(E^2+e^2)/s^2=&0.5(V^2+v^2)\\&(1+H_3^2)+0.5(U^2+u^2)(1+H_3^2)\\&+K_3^2(W^2+w^2)+(1-H_3^2)\\&(UV+uv) \quad \cdots\cdots (13)\end{aligned}$$

At $\beta=45°$

$$E^2(+45, 45)/s^2 = (E^2+e^2)/s^2$$
$$= 0.5(1+H_3^2)(V^2+v^2)$$
$$+(0.25+0.25H_3^2+0.5K_3^2)\times(U^2+u^2)$$
$$+(0.5K_3^2+0.25+0.25H_3^2)(W^2+w^2)$$
$$+0.7071(UV+uv)(1-K_3^2)$$
$$+0.7071(VW+vw)(H_3^2-1)$$
$$+2(0.5K_3^2-0.25H_3^2-0.25)(UW+uw) \quad \cdots\cdots (14)$$

At $\beta = +90°$

$$E^2(+90, 45)/s^2 = (E^2+e^2)/s^2$$
$$= (0.5+0.5H_3^2)(V^2+v^2)+K_3^2(U^2+u^2)$$
$$+(0.5+0.5H_3^2)(W^2+w^2)$$
$$+(H_3^2-1)(VW+vw) \quad \cdots\cdots (15)$$

At $\beta = 180°$

$$E^2(180, 45)/s^2 = (E^2+e^2)/s^2$$
$$= 0.5(V^2+v^2)(1+H_3^2)$$
$$+0.5(U^2+u^2)(1+H_3^2)$$
$$+K_3^2(W^2+w^2)-(1-H_3^2)(UV+uv) \quad \cdots\cdots (16)$$

Using the R.M.S. reading from equation (13)~(16)

$$e^2(0, 45)/s^2 = 0.5(1+H_3^2)v^2$$
$$+0.5(1+H_3^2)u^2+K_3^2w^2+(1-H_3^2)uv \quad \cdots\cdots (17)$$

$$e^2(+45, 45)/s^2 = 0.5(1+H_3^2)v^2$$
$$+(0.25+0.25H_3^2+0.5K_3^2)u^2$$
$$+(0.5K_3^2+0.25+0.25H_3^2)w^2$$
$$+0.7071(1-K_3^2)uv+0.7071(H_3^2-1)vw$$
$$+2(0.5K_3^2-0.25H_3^2-0.25)uw \quad \cdots\cdots (18)$$

$$e^2(+90, 45)/s^2 = (0.5+0.5H_3^2)v^2$$
$$+K_3^2u^2+(0.5+0.5H_3^2)w^2+(H_3^2-1)w \quad \cdots\cdots (19)$$

$$e^2(180, 45)/s^2 = 0.5(1+H_3^2)v^2$$
$$+0.5(1+H_3^2)u^2+K_3^2w^2$$
$$-(1-H_3^2)uv \quad \cdots\cdots (20)$$

From the equations (13)~(20), one can calculate three unknown mean velocities, turbulent intensities and shear stress by a slanted wire as Appendix A.

5. SECOND METHOD OF MEASUREMENT

The first method has several benefits to calculate unknown velocity and turbulence intensities by using simple equations and suitable traversing mechanism. But, when one employ this methods to solve the components with high turbulent, one may be found the prong interference on the hot-wire signals.

However, if one uses second method in confined flow, a special mechanism to rotate the probes are needed for measuring the components, because the probe axis is aligned with the main stream of the flow. This method employed a similar response equation (12) with the hot-wire probe axis is aligned with the flow in the test tube.

2) Using a single slanted wire (i.e. $\alpha = 45°$)

$$E^2(0, +45)/s^2 = (E^2+e^2)/s^2$$
$$+0.5(1+H_3^2)(U^2+u^2)$$
$$+0.5(1+H_3^2)(V^2+v^2)+K_3^2(W^2+w^2)$$
$$+(1-H_3^2)(UV+uv) \quad \cdots\cdots (21)$$

The D.C. reading from equation (21)

$$E^2/s^2 = 0.5(1+H_3^2)U^2+0.5(1+H_3^2)V^2$$
$$+K_3^2W^2+(1-H_3^2)UV \quad \cdots\cdots (22)$$

A.C. reading from equation (21)

$$e^2/s^2 = 0.5(1+H_3^2)u^2+0.5(1+H_3^2)v^2$$
$$+K_3^2w^2+(1-H_3^2)uv \quad \cdots\cdots (23)$$

At $\beta = +90°$

$$E^2(+90, 45)/s^2 = (E^2+e^2)/s^2$$
$$= 0.5(1+H_3^2)(U^2+u^2)+K_3^2(V^2+v^2)$$
$$+0.5(1+H_3^2)(W^2+w^2)+(1-H_3^2)(UW$$
$$+uw) \quad \cdots\cdots (24)$$

The D.C. reading and A.C. reading from equation (24)

$$E^2(+90, 45)/s^2 = 0.5(1+H_3^2)U^2 + K_3^2V^2 + 0.5(1+H_3^2)W^2 + (1-H_3^2)UW \quad \cdots\cdots (25)$$

$$e^2(+90, 45)/s^2 = 0.5(1+H_3^2)u^2 + K_3^2v^2 + 0.5(1+H_3^2)w^2 + (1-H_3^2)uw \quad \cdots\cdots (26)$$

from these equations (22), (23), (25) and (26), we can solve for u, v, w, uv and uw components.

$$uv = [e^2(0, 45) - e^2(0, -45)]/2s^2(1-H_3^2) \quad \cdots\cdots (27)$$

$$uw = [e^2(90, 45) - e^2(-90, 45)]/2s^2(1-H_3^2) \quad \cdots\cdots (28)$$

2) Using a single normal wire (i.e. $\alpha = 0°$)

At $\beta = 0°$

$$E^2(0, 0)/s^2 = (E^2+e^2)/s^2 = (U^2+u^2 + H_1^2(V^2+v^2) + K_1(W^2+w^2) \quad \cdots\cdots (29)$$

$$E^2(+45, 0)/s^2 = (E^2+e^2)/s^2 = (U^2+u^2) + 0.5(H_1^2+K_1^2)(V^2+v^2) + 0.5(H_1^2+K_1^2)(W^2+w^2) + (H_1^2+K_1^2)(VW+vw) \quad \cdots\cdots (30)$$

At $\beta = 90°$

$$E^2(90, 0)/s^2 = (E^2+e^2)/s^2 = (U^2+u^2) + H_1^2(W^2+w^2) + K^2(V^2+v^2) \quad \cdots\cdots (31)$$

If one take D.C. voltage from hot-wire anemometer, the equation (29), (30), and (31) can be described as followed

$$E^2(0, 0)/s^2 = U^2 + H_1^2V^2 + K_1^2W^2 \quad \cdots\cdots (32)$$

$$E^2(0.5, 0)/s^2 = U^2 + 0.5(H_1^2+K_1^2)V^2 + 0.5(H_1^2+K_1^2)W^2 + (H_1^2-K_1^2)VW \quad (33)$$

$$E^2(-45, 0)/s^2 = U^2 + 0.5(H_1^2+K_1^2)V^2 + 0.5(H_1^2+K_1^2)W^2 - (H_1^2-K_1^2)VW \quad (34)$$

$$E^2(90, 0)/s^2 = U^2 + H_1^2W^2 + K_1^2V^2 \cdots\cdots (35)$$

From these four equations (32) ~ (35), one can calculate unknown three velocities. However, we can solve u, v, w and vw by a similar procedures from equation (30) with A.C. reading,

$$e^2(+45, 0)/s^2 = u^2 + 0.5(H_1^2+K_1^2)v^2 + 0.5(H_1^2+K_1^2)w^2 + (H_1^2-K_1^2)vw$$

$$vw = [e^2(45, 0) - e^2(-45, 0)]/2s^2(H_1^2-K_1^2) \quad \cdots\cdots (36)$$

But this second method was not used for measuring velocities and the Reynolds stress components. Because the wire orientation was difficult in the 50mm radius test tube.

6. CALIBRATION OF THE HOT-WIRE

DISA type 55 P12 single hot-wire probe was used for this calibration. This probe employs a hot-wire of diameter 5μm, length 1.25mm (1/d = 250).

In order to obtain accurate characteristics, all hot-wires were annealed before use. The hot-wire probe and a pitot-tube were mounted in parallel on the probe supporter at a distance of 30mm. This calibration was made In a low-speed wind tunnel (max.speed is 60m/s) with a open circuit and open test section. The hot-wire probe was mounted on two turn tables with protractors, one turn table rotated the probe in the horizontal plane about a vertical axis through the wire. The other table could change the probe axis in the vertical plan. So, these probes were placed with the wire down stream, and the angle of incidence between the mean flow velocity and the axis of the stem body could be varied in the range $-135° < \alpha < +135°$.

Also, the pitch angle can be varied in the range $-70° < \theta < +70°$. Fig.10 shows a view of the calibration unit with the anemometry used in this calibration experiment. Each hot-wire was calibrated separately by means of

following method. The relation between the cooling effective velocity U_{eff} and the anemometer output voltage.
E being:

$$E^2(t) = A + BU_{eff}^m \quad \cdots\cdots\cdots (1)$$

According to Jorgensen(1971) report, he has performed that using equation (3),

$$U_{eff}^2(\alpha) = U^2(0)(\cos^2\alpha + H^2 \sin^2\alpha)$$
$$\text{for } \theta = 0.0 \quad \cdots\cdots\cdots (37)$$

$$U_{eff}^2(\theta) = U^2(0)(\cos^2\theta + K \sin^2\theta)$$
$$\text{for } \alpha = 0.0 \quad \cdots\cdots\cdots (38)$$

where, α is the yaw angle (or sensor angle) and θ is the pitch angle. Entering the calibration constants (A, B and m) and the value for the exponent in equations (1), (37) and (38) and calculating H and K, respectively, yields:

$$H = 1/\sin\alpha \times [(E(\alpha)^2 - A/E(0)^2 - A) - \cos^2\alpha]^{1/2} \quad \cdots\cdots\cdots (39)$$

$$K = 1/\sin\theta \times [(E(\theta)^2 - A/E(0)^2 - A) - \cos^2\theta]^{1/2} \quad \cdots\cdots\cdots (40)$$

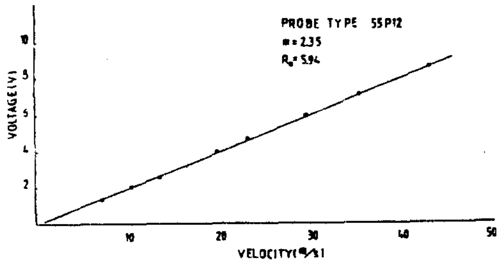

Fig. 3 Hot-wire calibration

These electric symbols E(α), E(θ) and E(0) are the anemometer output voltage measured at yaw angle α, pitch angle θ respectively. The output voltage from the anemometer is fed to DISA type 55D10 linearizer, and the linearizer output is read on a DISA type 55D31 DIGITAL voltmeter and fed to a D.C. compensator. The mean flow velocity in the test section was measured with a pitot-tube. The linearizer was adjusted for each probe so that the calibration curve for the anemometer was a straight line. Fig. 3 shows hot-wire calibration curve, with pitch and yaw factor for type 55P12.

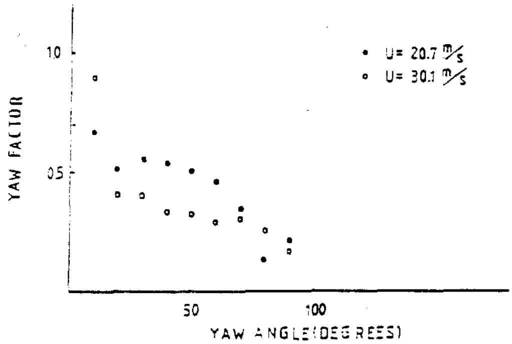

Fig. 4 Type 55P12 wire probe. Variation in yaw factor with yaw angle and velocity.

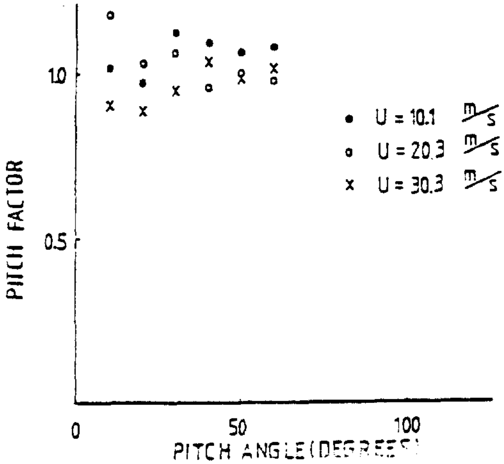

Fig. 5 Type 55P12 Wire probe. Variation in pitch factor with pitch angle and velocity.

7. DISCUSSION OF THE FIRST METHOD

As has been noted in the previous section, a slanted hot-wire is used to measure velocities and turbulence intensities. In this section the results obtained by using a slanted wire, the normal straight wire (DISA type 55P11) with the right angle wire, and a pitot tube are compared. The velocity and turbulence intensity components were measured in the test pipe for a Reynolds number lying in the range 20000 to 63500, with no swirl present.

The axial velocity components (found by using the above wires and pitot-tube) were found to agree very well as is shown in Figs. 6 to 7. Fig. 9 shows the Reynolds stress when calculated using the hot-wire outputs and pitot tube, for Re=20000,30000 and 40000 with no swirl flow

The results show that the shear stress is linear across the test section tube. In addition, the maximum and minimum values are found at the tube wall and along the center line of the flow respectively. Moreover, in Figs. 7 and 8 show the axial velocities and tangential velocities for Re=50000 at L/D=0.8 and 16 or x/D=59 with swirl flow. Fig. 7 shows axial velocities using the three hot-wire and pitot-tube, at a Re=50000 at x/D=59 with swirl.

The axial velocities agree well each other. However, the velocities near the wall show some differences. The measuring points with the three wires are slightly different since it is difficult for a slanted wire to be oriented very close to the wall of the test tube. This is not such a problem for the normal wire or the right angle wire with turbulent flow. The right angle wire can easily be oriented near the wall to measure the velocity. This kind of phenomena can be found at tangential velocity in Fig. 3. From these results, it was deduced that the first method using a straight wire, a right angle wire or a slanted wire is reasonable for measuring velocity and turbulence intensity with swirl and no swirl.

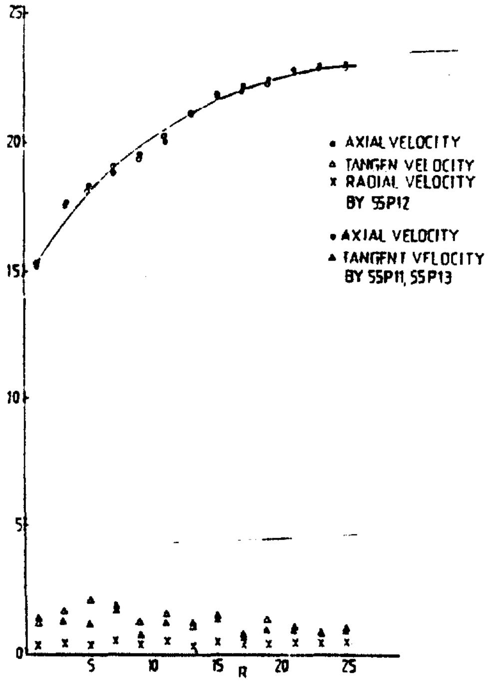

Fig. 6 Comparison of velocity by using 55P12 and 55P11, 55P13 for Re=60000 at X/D=59 with no swirl.

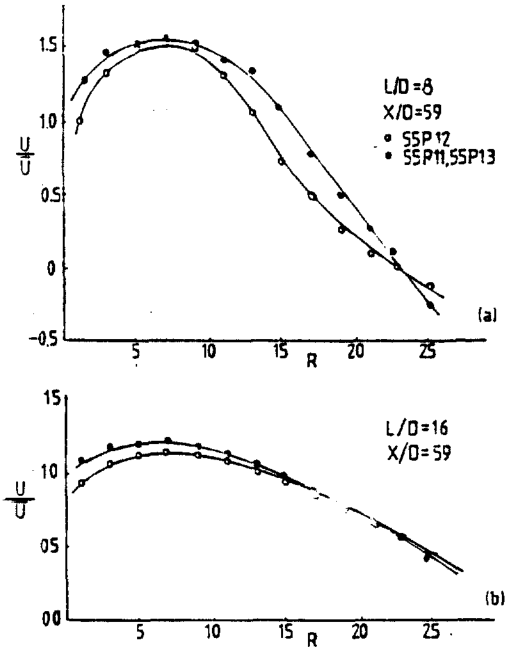

Fig. 7 Comparisons of axial velocity by using type P12 and P11, P13 for Re=50000 with swirl at (a)L/D=8 and (b)L/D=16.

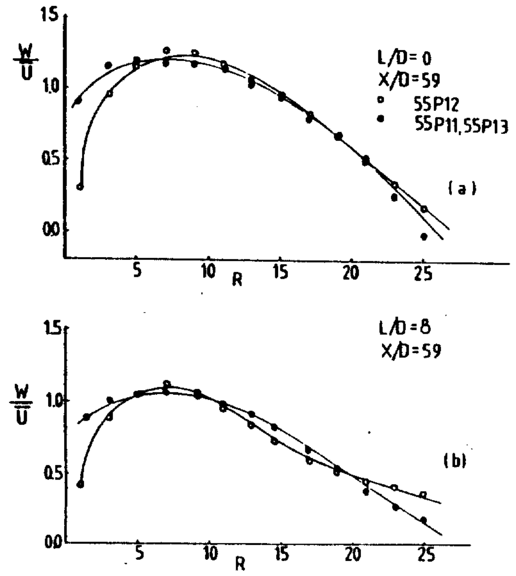

Fig. 8 Comparisons of tangential velocity by using type P12 and P11, P13 for Re= 50000 with swirl at (a)L/D=0 and (b)L/D=8.

8. CONCLUSION OF USING HOT-WIRE

To interpret the electric output signal from hot-wire anemometer, several equations were employed then. Also, to calculate the directional sensitivity of the probes used in this work a number of calibrations were performed to determine the value of H and K. Moreover, the results by using type 55P12 and that of using type 55P11, type 55P13 were compared each other for several Reynolds numbers with swirl and no swirl. Some of the conclusions drawn from using hot-wire may be summarized as follows,

1) The yaw factor H depends on sensor geometry and is found to vary with the angle of inclination, decreasing with increasing values of the yaw angles.
2) The pitch factor K depend on sensor geometry and is nearly independent of

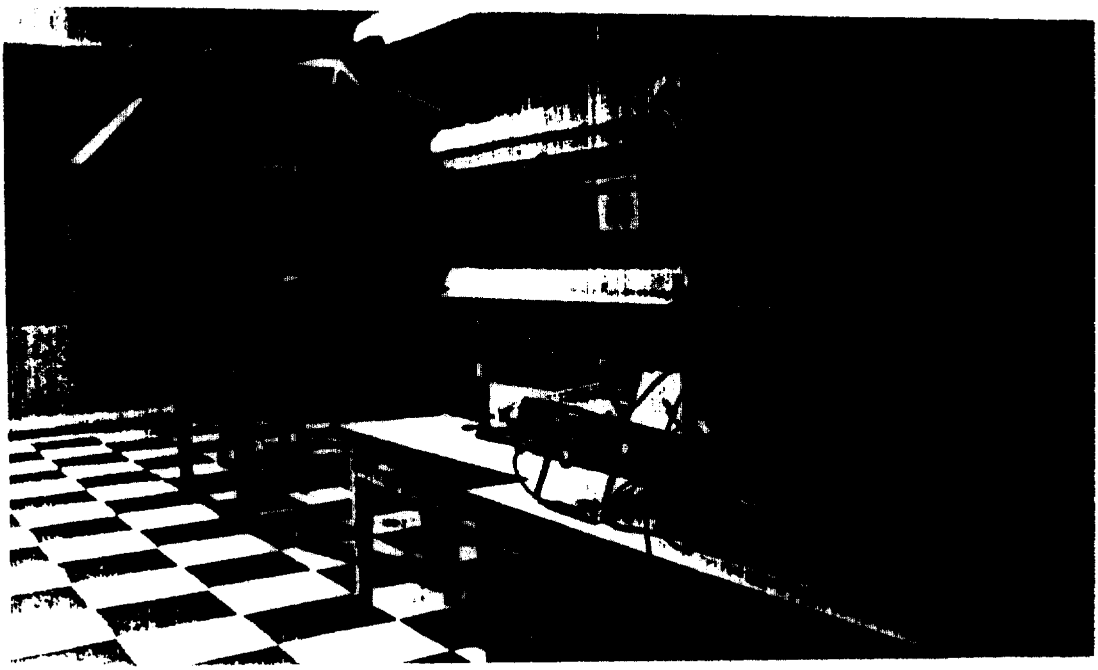

Fig.10 Wind tunnel used in Hot-wire calibration.

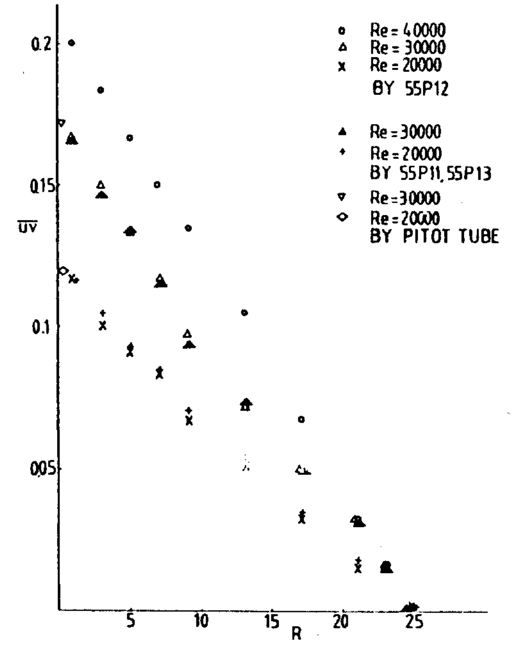

Fig. 9 Distributions of Reynolds stress by using 55P12 and 55P11, 55P13 for Re=20000, 30000 and 40000 at X/D=59 with no swirl.

the pitch angle.

3) The unknown velocity components can be calculated by employing a normal straight probe (DISA type 55P11), a right angle probe (DISA type 55P13)
4) Also, the three velocities and Reynolds stress can be solved by using a slanted wire (DISA type 55P12).
5) The error associated with the signal itself and probe interference.

These two results are compared each other for several Reynolds numbers with swirl and no swirl and the agreements are found to be very good. The yaw and pitch factor are as follows:

For DISA type 55P11 and 55P12.

$H_1 = 0.35$ $H_3 = 0.516$

$K_1 = 0.92$ $K_3 = 1.098$

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NOMENCLATURE

A,D	constant
E	Instantaneous Voltage (D.C)
e	Fluctuating Voltage (A.C)
H, H_1, H_2, H_3	Yaw factor

K, K_1, K_2, K_3	Pitch factor
m	Exponent
R_O	Probe operating resistance
S	The slop of each probe
U	Axial Velocity component
u	Axial fluctuating velocity component
V	Radial velocity component
v	Radial fluctuating velocity component
W	Tangential velocity component
w	Tangential fluctuating velocity component

Greek Symbol

α	The yaw angle or Sensor angle
β	Angle of rotation
θ	The pitch angle

APPENDIX A

Calculate three velocities, turbulent intensity and Reynolds stress from equation

$$E^2(0,45)/s^2=0.5(1+H_3^2)V^2+0.5(1+H_3^2)U^2+K_3^2W^2+(1-H_3^2)UV \quad \cdots (1)$$

$$E^2(180,45)/s^2=0.5(1+H_3^2)V^2+0.5(1+H_3^2)U^2+K_3^2W^2-(1-H_3^2)UV \quad \cdots (2)$$

$$E^2(90,45)/s^2=0.5(1+H_3^2)V^2+K_3^2U^2+0.5(1+H_3^2)W^2+(H_3^2-1)VW \quad \cdots (3)$$

$$E^2(-90,45)/s^2=0.5(1+H_3^2)V^2+K_3^2U^2+0.5(1+H_3^2)W^2-(H_3^2-1)VW \quad \cdots (4)$$

from equation (1) and (2)

$$2UV(1-H_3^2)=[E^2(0,45)-E^2(180,45)]/s^2 \quad \cdots (5)$$

Subtract equation (4) from equation (3)

$$VW=[E^2(90,45)-E^2(-90,45)]/2s^2(H_3^2-1) \quad \cdots (6)$$

from equation (5) and (6)

$$W=U[E^2(90,45)-E^2(-90,45)]/[E^2(0,45)-E^2(180,45)] \quad \cdots (7)$$

from adding equation (1) and (2)

$$[E^2(0,45)+E^2(180,45)]/s^2=(1+H_3^2)V^2+U^2(1+H_3^2)+2K_3^2W^2 \quad \cdots (8)$$

equation (3)2 times- equation (8)

$$[E^2(90,45)+E^2(-90,45)]/s^2=(1+H_3^2)V^2+2K_3^2U^2+(1+H_3^2)W^2 \quad \cdots (9)$$

Subtract equation (8) from equation (9)

$$[E^2(90,45)+E^2(-90,45)]-(E^2(0,45)+E^2(180,45)]/s^2=U^2(2K_3^2-1-H_3^2)+W^2(1+H_3^2-2K_3^2) \quad \cdots (10)$$

from equation (10) and (7)

$$U^2=[E^2(90,45)+E^2(-90,45)-E^2(0,45)-E^2(180,45)]/s^2\times[E^2(0,45)-E^2(180,45)]^2/(2K_3^2-1-H_3^2)\{[E^2(0,45)-E^2(180,45)]^2-[E^2(90,45)-E^2(-90,45)]^2\} \quad \cdots (11)$$

from equation (11)

$$G=[E^2(0,45)-E(180,45)]^2/(2K_3^2-1-H_3^2)[E^2(0,45)-E^2(180,45)]+[E^2(90,45)-E^2(-90,45)]^2(1+H_3^2-2K_3^2)$$

$$U^2=G[E^2(90,45)+E^2(-90,45)-E^2(0,45)-E^2(180,45)]/s^2$$

from equation (7) and (11)

$$V=-E^2(0,45)-E^2(180,45)/2Us^2(H_3^2-1)=E^2(90,45)-E^2(-90,45)/2Ws^2(H_3^2-1) \quad \cdots (12)$$

If one take A.C. voltage from hot-wire anemometer,

$$e^2(0,\ 45)/s^2 = 0.5v^2(1+H_3^2)+0.5u^2(1+H_3^2)+K_3^2 w^2+(1-H_3^2)uv \quad \cdots\cdots (13)$$

$$e^2(180,\ 45)/s^2 = 0.5v^2(1+H_3^2)+0.5u^2(1+H_3^2)+K_3^2 w^2-(1-H_3^2)uv \quad \cdots\cdots (14)$$

$$e^2(90,\ 45)/s^2 = 0.5(1+H_3^2)v^2+K_3^2 u^2+0.5(1+H_3^2)w^2+(H_3^2-1)vw \quad \cdots\cdots (15)$$

$$e^2(-90,\ 45)/s^2 = 0.5(1+H_3^2)v^2+K_3^2 u^2+0.5(1+H_3^2)w^2-(H_3^2-1)vw \quad \cdots\cdots (16)$$

$$e^2(+45,\ 45) = 0.5(1+H_3^2)v^2+(0.25+0.25H_3^2+0.5K^2)u^2+(0.5K_3^2+0.25+0.25H_3^2)w^2+0.7071(1-K_3^2)uv+0.7071(H_3^2-1)vw+2uw(0.5K_3^2-0.25H_3^2-0.25) \quad \cdots\cdots (17)$$

Using similar procedure

$$u^2 = [e^2(90,\ 45)+e^2(-90,\ 45)-e^2(0,\ 45)-e^2(180,\ 45)]/s^2 \times [e^2(0,\ 45)-e^2(180,\ 45)]^2/(2K_3^2-1-H_3^2)\{[e^2(0,\ 45)-e^2(180,\ 45)]^2-[e^2(90,\ 45)-e^2(90,\ 45)]^2\} \quad \cdots\cdots (18)$$

$$w^2 = u^2[e^2(90,\ 45)-e^2(-90,\ 45)]^2/[e^2(0,\ 45)-e^2(180,\ 45)]^2 \quad \cdots\cdots (19)$$

$$v^2 = [e^2(0,\ 45)-e^2(180,\ 45)]^2/4u^2s^4(H_3^2-1)^2 \quad \cdots\cdots (20)$$

from equation (15) and (16)

$$vw = e^2(90,\ 45)-e^2(-90,\ 45)/2s^2(H_3^2-1) \quad \cdots\cdots (21)$$

subtracting equation (14) from equation (13)

$$uv = 1/2s^2(1-H_3^2)\times[e^2(0,\ 45)-e^2(180,\ 45)] \quad \cdots\cdots (22)$$

from equation (17)

$$uw = [1/(0.5K^2-0.25H_3^2-0.25)]\times[e^2(45,\ 45)-e^2(-45,\ 45)]/s^2-1.414(H_3^2-1)vw \quad \cdots\cdots (23)$$