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Approximate Controllability for the Semilinear Control System with Delay

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1. Introduction

We consider a delay control system with uniformly bounded nonlinear term F ;

$$(N)\qquad \frac{dx(t)}{dt}+Ax(t)=F(t,x_t)+Bu(t)\qquad (t>0)$$

Here $Bu(t)$ corresponds to a finite dimensional control and F is a uniformly bounded nonlinear term, that is, there exists an $M>0$ such that $\|F(t,x_t)\|\leq M$ for each $t\geq 0$, x belonging to some Banach space contained in X.

For (N), we introduce a linear control system (L) :

$$(L)\qquad \frac{dx(t)}{dt}+Ax(t)=Bu(t)\qquad (t>0)$$

In this paper, we discuss whether by choosing an u, we can steer initial state φ to any neighborhood of a given state $x(T)=x_1$ at a given time T. This is called an approximate controllability problem, for nonlinear evolution system, many authors study this problem ([2], [5],[6],[8]).

The purpose of this paper is to prove the equivalence of approximate controllability for the above nonlinear system (N) and one for the linear system (L).

2. Approximate Controllability

We consider the following delay semilinear control system ;

(1)
$$\begin{cases} \dfrac{dx(t)}{dt} + Ax(t) = F(t,x_t) + Bu(t) & t>0 \\ x(\theta)=\varphi(\theta) \quad -h\le\theta<0, \quad x(0)=\varphi_0 \end{cases}$$

Let X is a Banach space over R with norm $\|\cdot\|$, and $-A$ generates an analytic semigroup $\{U(t)\}_{t\ge0}$. Moreover F is a nonlinear term satisfying (4) and (5) stated below u_t has pointwise definition

$$u_t(\theta)\equiv u(t+\theta), \text{ for } -h\le\theta<0.$$

Throughout this paper, we consider the case where $\varphi(\theta)\equiv 0$, $\theta\in[-h,0)$. The function φ is Hölder continuous from $[-h,0]$ to X_α. For sufficiently large $\gamma>0$ and each $\alpha\ge0$, we can define fractional power of $A_1\equiv A+\gamma$, and the Banach space

(2)
$$X_\alpha=\mathscr{D}(A_1^\alpha)$$

with the norm

(3) $$\| x \|_{\alpha} = \| A_1^{\alpha} x \|$$

(e.g. Tanabe [7]). Henceforth we fix a sufficiently large $\gamma>0$.

For some $0 \leq \alpha \leq 1$, let us assume that F is defined on $[0,\infty)\times X_{\alpha}$ and satisfies (4) and (5);

(4) F is uniformly bounded on $[0,\infty)\times X_{\alpha}$, that is, there exists a positive constant M such that $\| F(t,x_t) \| \leq M$ for each $t \geq 0$, $x \in X_{\alpha}$.

(5) F is locally Hölder continuous in t and locally Lipschitz continuous in x. That is for each $r>0$, there exists a constants $L \equiv L(r)>0$ and $\theta \equiv \theta(r) \leq 0$ such that

$$\| F(t,x_t) - F(s,x_s) \| \leq L(| t\text{-}s |^{\theta} + \| x_t - x_s \|_{\alpha})$$

for each $0 \leq s,\ t \leq r$ and $\| x_t \|_{\alpha} \leq r$

The function u in $L^1_{loc}([0,\infty)\,;R^N)$ and B is a linear operator expressed by

(6) $$\begin{array}{cccc} B: & R^N & \rightarrow & X \\ & \Psi & & \Psi \\ & (\eta_1,\cdots,\eta_N) & \longrightarrow & \sum_{k=1}^{N} \eta_k a_k \end{array}$$

for $a_k \subset X$ $(1 \leq k \leq N)$. Here we regard B as a control term, and from the practical point view, we assume that the range of B is finite dimensional.

NOTATION. For $T>0$, we set

$$H([0,T]\,;R^N)=\{B:[0,T]\to R^N\,;\ |B(t)-B(s)|\le c_0|t\text{-}s|^{\theta}$$
$$(t,\ s\in[0,T])\ \text{for some}\ c_0>0\ \text{and}\ \theta>0\}.$$

That is, $H([0,T]\,;R^N)$ is the totality of R^N-valued Hölder continuous functions on $[O,T]$. Here for $\eta=(\eta_1,\cdots,\eta_N)\in R^N$, we set $|\eta|=\sum_{k=1}^{N}|\eta_k|$

Under the assumption (4) and (5), if $\varphi_0\in X_a$ and $u\in H([0,T]\,;R^N)$ for each $T>0$, then there exists a unique strong solution $x(\cdot\,;\varphi_0,u)$ to (1) (e.g. [3] Theorem 3.3.3 and Corollary 3.3.5).

We can state an approximate controllability problem, for example, as follows.

Detemine $a_k\in X$, $1\le k\le N$ such that for given $T>0$, $\varphi_0\in X_a$, $x_1\in X$ and $\varepsilon>0$, there exists an $u\in H([0,T]\,;R^N)$ satisfying

$$\|x(T\,;\varphi_0,\ u)-x_1\|\le\varepsilon$$

For our nonlinear control system (1), we consider a linear control system

$$\frac{dx(t)}{dt}+Ax(t)=Bu(t),\qquad t>0 \tag{7}$$
$$x(0)=x_0$$

For an $u\in H([0,T]\,;R^N)$, we denote a unique (strong) solution to (7) by $x(\cdot\,;x_0,u)$.

We define reachable sets for the semilinear system (1) and linear system (7) ;

Definition 2.1. For $T>0$, $\varphi_0 \in X_\alpha$, $x_0 \in X$, we set

(8) $$R_T(\varphi_0)=\{x(T ; \varphi_0, u) ; x(T ; \varphi_0, u)=U(T)\varphi_0+\int_0^T U(T-s)Bu(s)ds + \int_0^T U(T\text{-}s)F(s,x_s)ds,\ u \in H([0,T] ; R^N)\}$$

and

(9) $$\mathscr{L}_T(x_0)=\{x(T ; x_0, u) ; x(T ; x_0, u)=U(T)x_0 + \int_0^T U(T\text{-}s)Bu(s)ds,\ u \in H([0,T] ; R^N)\}$$

Henceforth we denote the closure of $Y \subset X$ by $\overline{Y}$ and the norm of a bounded linear operator by $\|\cdot\|$.

For the approximate controllability problem, we have only to disscuss whether the reachable set $R_T(\varphi_0)$ is dense in X if and only if a reachable set for (7) with the same B, is dense in X.

Lemma 2.1.([1]) If -A generates an analytic semigroup, then for the linear system (7), the following (a)-(d) are equivalent

(a) $\overline{U_{t>0} \mathscr{L}_t(0)}=X$
(b) $\overline{U_{t>0} \mathscr{L}_t(x_0)}=X$ for each $x_0 \in X$
(c) $\overline{\mathscr{L}_t(x_0)}=X$ for each $x_0 \in X$ and each $t>0$
(d) $\overline{\mathscr{L}_t(0)}=X$ for each $t>0$.

Lemma 2.2.([8]) Let $t,a,b>0$ and $0\leq\alpha\leq 1$. If a continuous function $f:[0,t]\rightarrow[0,\infty)$ satisfies

$$f(s)\leq a+b\int_0^s (s-\eta)^{-\alpha}f(\eta)d\eta,\quad 0\leq s\leq t,$$

then

$$f(s)\leq ca\cdot\exp(cb^{1/(1-\alpha)}s),\quad 0\leq s\leq t,$$

where c is a positive constant depending only on α.

Lemma 2.3. Let $\varphi_0\in x_\alpha=\mathscr{D}((a+\gamma)^\alpha)$ and $u\in H([0,T]);R^N)$ be given and let $r>0$ and $p>1$ be fixed such that $1\leq p/(p-1)<1/\alpha$. If $\upsilon\in H([0,T];R^N)$ satisfies $\|u-v\|_{L^p([0,t];R^N)}\leq r$, then

$$\|x_s(s;\varphi,u)-x_s(s;\varphi,\upsilon)\|_\alpha\leq c_0\|u-\upsilon\|_{L^p([0,t],R^N)},\quad 0\leq s\leq t$$

where a positive constant $c_0=c_0(\|u\|_{L^p([0,t]:R^N)},r,t)$ is bounded as r,t and $\|u\|_{L^p([0,t],R^N)}$ are all bounded.

Proof. First, for $\upsilon\in L^p([0,t];R^N)$ satifying $\|u-\upsilon\|_{L^p([0,t]:R^N)}\leq r$,

we will estimate $\|x_s(s;\varphi_0,\upsilon)\|_\alpha$, $0\leq s\leq t$. To this end, we note

$$\|U(t)\|\leq c_1\ \exp(wt),\quad t\leq 0,$$

for some constants $c_1>0$ and $w>0$, and

(10) $$\| A_1^{\alpha} U(s) \| \leq c_2 s^{-\alpha}, \ 0 \leq s \leq t$$

(e.g. [7]). By

(11) $$x_t(t\ ;\ \varphi_0, u)(\theta) = U(t+\theta)\varphi_0 + \int_0^{t+\theta} U(t+\theta-\eta)Bu(\eta)d\eta + \int_0^{t+\theta} U(t+\theta-\eta)F(\eta, x_\eta)d\eta$$

for each $0 \leq s \leq t$, we have

$$\begin{aligned}
&\| A_1^{\alpha} x_s(s\ ;\ \varphi_0, \upsilon)(\theta) \| \\
&= \| U(s+\theta)A_1^{\alpha}\varphi_0 \\
&+ \int_0^{s+\theta} A_1^{\alpha} U(s+\theta-\eta)B\upsilon(\eta)d\eta + \int_0^{s+\theta} A_1^{\alpha} U(s+\theta-\eta)F(\eta, x_\eta)d\eta \| \\
&\leq c_1 \ \exp(w(s+\theta)) \| \varphi_0 \|_{\alpha} + c_2 M \int_0^{s+\theta} (s+\theta-\eta)^{-\alpha} d\eta \\
&+ c_2 \max_{1 \leq k \leq N} \| a_k \| \int_0^{s+\theta} (s+\theta-\eta)^{-\alpha} | \upsilon(\eta) | d\eta \\
&\leq c_1 \ \exp(w(s+\theta)) \| \varphi_0 \|_{\alpha} + c_2 M_s^{1-\alpha} / (1-\alpha) \\
&+ c_2 \max_{1 \leq k \leq N} \| a_k \| \ \{ \int_0^{s+\theta} (s+\theta-\eta)^{-\frac{p\alpha}{p-1}} d\eta \}^{\frac{p-1}{p}} \ \{ \int_0^{s+\theta} | \upsilon(\eta) |^p d\eta \}^{\frac{1}{p}} \\
&\leq c_1 \ \exp(wt) \| \varphi_0 \|_{\alpha} + c_2 \max_{1 \leq k \leq N} \| a_k \| \ \{ \frac{p-1}{p-1-p\alpha} \}^{(p-1)/p} \\
&\times t^{(p-1-p\alpha)/p} \{ \| u \|_{L^p([0,t], R^N)} + r \} + (c_2 M t(1-\alpha))/(1-\alpha)
\end{aligned}$$

$$\equiv c_3$$

In the last inequality, we use $p/(p\text{-}1)<1/\alpha$ and

$$\| \theta \|_{L^p([0,t]:R^N)}$$

$$\leq \| \upsilon\text{-}u \|_{L^p([0,t]:R^N)} + \| u \|_{L^p([0,t]:R^N)}$$

$$\leq r + \| u \|_{L^p([0,t]:R^N)}$$

Hence we get $\| x_s(x\,;\,\varphi_0,u) \|_\alpha, \| x_s(s\,;\,\varphi_0,\upsilon) \|_\alpha \leq c_3,\ 0\leq s\leq t,$ so that by

(5) there exists a constant $c_4>0$ depend c_3 such that

$$\| F(s,x_s(s\,;\,\varphi_0,u)) - F(s\,;\,x_s(x\,;\,\varphi_0,\upsilon)) \|$$

$$\leq c_4 \| x_s(s\,;\,\varphi_0,u)(\theta) - x_s(s\,;\,\varphi_0,\upsilon)(\theta) \|_\alpha,\ \ 0\leq s\leq t.$$

Now, since $x_s(s\,;\,\varphi_0,\ u)$ and $x_s(s\,;\,\varphi_0,\upsilon)$ are expressed similarly to (11), we obtain

$$\| x_s(s\,;\,\varphi_0,u)(\theta) - x_s(s\,;\,\varphi_0,\upsilon)(\theta) \|_\alpha$$

$$\leq \| \int_0^{s+\theta} A_1^\alpha U(s+\theta\text{-}\eta)B(u(\eta)\text{-}v(\eta))d\eta$$

$$+ \int_0^{s+\theta} A_1^\alpha U(s+\theta\text{-}\eta)(F(\eta\,;\,x_\eta(\eta\,;\,\varphi_0,u) - (F(\varphi\,;\,x_\eta(\eta\,;\,\varphi_0,v)))d\eta \|$$

$$\leq c_2 \max_{1\leq k\leq N} \| a_k \| \int_0^{s+\theta} (s+\theta-\eta)^{-\alpha} \mid u(\eta)-\upsilon(\eta) \mid d\eta$$

$$+c_2c_4 \int_0^{s+\theta} (s+\theta-\eta)^{-\alpha} \| x_\eta(\eta ; \varphi_0,u)(\theta)-x_\eta(\eta ; \varphi_0,\upsilon)(\theta) \|_\alpha d\eta$$

$$\leq c_2 \max_{1\leq k\leq N} \| a_k \| \ \{ \int_0^{s+\theta} (s+\theta-\eta)^{p\alpha/(1-p)} d\eta \}^{(p-1)/p} \| u-\upsilon \|_{L^p([0,t] : R^N)}$$

$$+c_2c_4 \int_0^{s+\theta} (s+\theta-\eta)^{-\alpha} \| x_\eta(\eta ; \varphi_0,u)(\theta)-x_\eta(\eta ; \varphi_0,\upsilon)(\theta) \|_\alpha d\eta$$

$$\equiv c_5 \| u-\upsilon \|_{L^p([0,t] : R^N)}$$

$$+c_5 \int_0^{s+\theta} (s+\theta-\eta)^{-\alpha} \| x_\eta(\eta ; \varphi_0,u)(\theta)-x_\eta(\eta ; \varphi_0,\upsilon)(\theta) \|_\alpha d\eta$$

Applying Lemma 2.2, we reach

$$\| x_s(s ; \varphi_0,u)(\theta)-x_s(s ; \varphi_0,\upsilon)(\theta) \|_\alpha$$

$$\leq c_5c_6 \ \exp(c_6c_5^{1/(1-\alpha)}t) \| u-v \|_{L^p([0,t] : R^N)}.$$

Which is the conclusion of Lemma 2.3.

Theorem 2.1. Let $T>0$ be given. Then

$$\overline{\cup_{0<t<T} R_t(\varphi_0)}=X \Rightarrow \overline{\cup_{t>0} \mathscr{L}_t(0)}=X$$

Proof. First we note

$$\| U(t) \| \leq c_7 \ \exp(wt), \ t>0 \tag{12}$$

for some constants $c_7>0$ and $w>0$, because $\{U(t)\}_{t\geq 0}$ is a c_0-semigroup. Assume that

$$(13)\qquad L=\overline{\cup_{t>0}\mathscr{L}_t(0)}\subsetneq X$$

we will derive a contradiction. Since L is a closed linear subspace, there exists an $x_0\in X$ such $\|x_0\|=1$ and

$$\inf\{\|x_0-x\| \ ; x\in L\}>\frac{1}{2}$$

(e.g[4], p131). Hence, for any $r(\neq 0)\in R$, we have

$$(14)\qquad \inf\{\|rx_0-x\| \ ; x\in L\}>\frac{|r|}{2}$$

Let $\varphi_0\in X_a$ and let

$$(15)\qquad |r|>4c_7\exp(wT)+4c_7M\int_0^T \exp(ws)ds$$

Then $rx_0\in\overline{\cup_{0<t<T}R_t(\varphi_0)}$. In fact, by (4) and (12), we have

$$(16)\qquad \left\|\int_0^t U(t-s)F(s,x_s)ds\right\|$$

$$\leq \int_0^t \|U(t-s)\|\ \|F(s,x_s)\|ds$$

$$\leq c_7\int_0^t \exp(ws)ds$$

By

$$x(t\,;\,\varphi_0,u)=U(t)\varphi_0+\int_0^t U(t\text{-}s)Bu(s)ds+\int_0^t U(t\text{-}s)F(s,x_s)ds$$

and

$$\int_0^t U(t\text{-}s)Bu(s)ds\in L(=\overline{\cup_{t>0}\mathrm{L}_t(0)})$$

for each $u\in H([0,T]\,;\,R^N)$, we get for $0\leq t\leq T$,

$$\|x(t\,;\,\varphi_0,u)\text{-}rx_0\|$$

$$\geq\left\|\int_0^t U(t\text{-}s)Bu(s)ds\text{-}rx_0\right\|-\|U(t)\varphi_0\|-\left\|\int_0^t U(t\text{-}s)F(s,x_s)ds\right\|$$

$$\geq\inf\{\|x\text{-}rx_0\|\,;\,x\in L\}\text{-}c_7\exp(w,t)\|\varphi_0\|$$

$$-c_7M\int_0^T \exp(ws)ds$$

$$>\frac{1}{2}\{4c_7\ \exp(wT)\|\varphi_0\|+\int_0^T \exp(ws)ds\}$$

$$-c_7M\int_0^t \exp(ws)ds$$

$$=2c_7\exp(wT)\|\varphi_0\|+c_7M\int_0^t \exp(ws)ds>0$$

This proves $rx_0\notin\overline{\cup_{0<t<T}R_t(\varphi_0)}$, which completes the proof.

Theorem 2.2. Let $T>0$ be give. Then

$$\overline{\cup_{t>0}\mathscr{L}_t(0)}=X\Rightarrow\overline{R_T(\varphi_0)}=X$$

Proof. Let $\varphi_0 \in X_\alpha$. Then, for given $T>0$, $\varepsilon>0$ and $x_1 \in X$, we have to prove that there exists an $u=u_{\varepsilon x_1} \in H([0,T];R^N)$ satisfying

$$\|x(T;\varphi_0, u)-x_1\| \le \varepsilon \tag{17}$$

Since $\overline{\mathscr{D}(A)}=X$, we may assume that $x_1 \in \mathscr{D}(A)$, without loss of generality. Now we set

$$\delta(\varepsilon)=w^{-1}\log\left(1+\frac{\varepsilon w}{2c_7 M}\right) \tag{18}$$

Since we consider only a small $\varepsilon>0$, we may further assume that $\delta(\varepsilon)<T$.

We denote the value of $x(t;\varphi_0,0)$ at the time T-$\delta(\varepsilon)$ by $\tilde{x}_0$;

$$\tilde{x}_0=x(T\text{-}\delta(\varepsilon);\varphi_0)$$

Here we note that $x(t;\varphi_0,0)$ $(0\le t\le T-\delta(\varepsilon))$ is the solution to (1) with $u=0$.

By Lemma 2.1 and $\overline{\cup_{t>0}\mathscr{L}_t(0)}=X$, we have $\overline{\mathscr{L}_T(x_0)}=X$ for each $x_0 \in X$, so that by regarding T-$\delta(\varepsilon)$ as an initial state x_1, there exists an $u \in H([T\text{-}\delta(\varepsilon),T];R^N)$ such that the solution $y(t)$ to

$$\frac{dy(t)}{dt}+Ay(t)=Bu_1(t),\ \ T\text{-}\delta(\varepsilon)< t \le T$$

with $y(T\text{-}\delta(\varepsilon))=\tilde{x}_0$, satisfies

$$\| y(T) - x_1 \| \leq \frac{\varepsilon}{6}.$$

Since

$$y(t) = U(t-T+\delta(\varepsilon))\tilde{x}_0 + \int_{T-\delta(\varepsilon)}^{t} U(t-s)Bu_1(s)ds$$

$t \geq T$-$\delta(\varepsilon)$. We have

$$\text{(19)} \qquad \| U(\delta(\varepsilon))\tilde{x}_0 + \int_{T-\delta(\varepsilon)}^{T} U(T-s)Bu_1(s)ds - x_1 \| \leq \frac{\varepsilon}{6}.$$

Now we set

$$\text{(20)} \qquad \tilde{u}(s) = \begin{cases} 0, & 0 \leq s \leq T\text{-}\delta(\varepsilon) \\ u_1(s), & T\text{-}\delta(\varepsilon) < s \leq T. \end{cases}$$

We have to take a control function u within the class of Hölder continuous functions, so that we approximate $\tilde{u}$ by $u \in H([0,T]; R^N)$. To this end, we fix $p>1$ such that $1 \leq p/(p-1) < 1/\alpha$. Since $C_0^\infty([0,T]; R^N)$ is dense in $L^P([0,T]; R^N)$, also $H([0,T]; R^N)$ is dense in $L^P([0,T]; R^N)$. Hence ther exists an $u \in H([0,T]; R^N)$ such that

$$\text{(21)} \quad \| u-\tilde{u} \|_{L^P([0,T]; R^N)}$$

$$\leq \frac{\varepsilon}{6} \min \{ c_7^{-1} \exp(-\delta(w)w) T^{(1-p)/p} (\max_{1<k<N} \| a_k \|)^{-1}, \ c_7^{-1} c_1^{-1} \exp(-\delta(\varepsilon)w) \}.$$

For the u, we can obtain (17) in the following manner. Applying Lemma 2.3 for $t = T$-$\delta(\varepsilon)$ and noting that

$$\tilde{x}_0 - x(T-\delta(\varepsilon);\varphi_0,u) = x(T-\delta(\varepsilon);\varphi_0,u) - x(T-\delta(\varepsilon);\varphi_0,u),$$

we get

$$\| \tilde{x}_0 - x(T-\delta(\varepsilon);\varphi_0,u) \| \leq \varepsilon c_7^{-1} \exp(-\delta(\varepsilon)\omega)/6$$

by (21), and so, by (12)

$$(22) \qquad \| U(\delta(\varepsilon))\tilde{x}_0 - U(\delta(\varepsilon))x(T-\delta(\varepsilon);\varphi_0,u) \|$$

$$\leq c_2 \exp(\delta(\varepsilon)\omega) \| \tilde{x}_0 - x(T-\delta(\varepsilon);\varphi_0,u) \|$$

$$\leq \frac{\varepsilon}{6}$$

Moreover, by (10),(21) and Hölder's inequality, we have

$$(23) \qquad \| \int_{T-\delta(\varepsilon)}^{T} U(T-s)Bu_1(s)ds - \int_{T-\delta(\varepsilon)}^{t} U(T-s)Bu(s)ds \|$$

$$\leq \int_{T-\delta(w)}^{T} c_7 \exp(\omega(T-s)) \max_{1<k<N} \| a_k \| \; | \tilde{u}(s)-u(s) | \; ds$$

$$\leq \frac{\varepsilon}{6}$$

From (19),(22) and (23), we get

$$\| U(\delta(\varepsilon))x(T-\delta(\varepsilon);\varphi_0,u) + \int_{T-\delta(\varepsilon)}^{T} U(T-s)Bu(s)ds - x_1 \| \leq \frac{\varepsilon}{2}$$

Furthermore, by (4),(10) and (19), we have

(25) $$\left\| \int_{T-\delta(\varepsilon)}^{T} U(T-s)F(s,x_s)ds \right\|$$
$$\leq M \int_{T-\delta(\varepsilon)}^{T} \| U(T-s) \| ds$$
$$\leq c_7 M \int_0^{\delta(\varepsilon)} \exp(ws)ds = \frac{\varepsilon}{2}$$

By

$$x(T ; \varphi_0,u) = U(\delta(\varepsilon))x(T-\delta(\varepsilon) ; \varphi_0, u) + \int_{T-\delta(\varepsilon)}^{T} U(T-s)Bu(s)ds + \int_{T-\delta(\varepsilon)}^{T} U(T-s)F(s,x_s)ds$$

the estimates (24) and (25) imply (17).

Since u is Hölder continuous on $[0,T]$, the proof is complete.

Theorem 2.3. Let $T>0$ be given, and let us assume

(26) $$\| U(t) \| \leq c_8 \exp(-\tau t), \quad t>0$$

for some constants $c_1>0$ and $\tau>0$. Then

$$\overline{\cup_{t>0} R_t(\varphi_0)} = X \Rightarrow \overline{\cup_{t>0} \mathscr{L}_t(0)} = X$$

Proof. Contrarily assuming (13), we can derive a contradiction in a manner similar to Theorem 2.1. There exists an $rx_0 \in X$ satisfying (12). For any $\varphi_0 \in X_\alpha$, let us put

(27) $$|r| > 4c_8 M/\tau.$$

Then by (26), we have

(28) $$\left\| \int_0^t U(t-s)F(s,x_s)ds \right\|$$

$$\leq M \int_0^\infty \| U(s) \| ds$$

$$\leq c_8 M/\tau$$

$u \in H([0,t]; R^N)$. For any $t>0$ and any $u \in H([0,t]; R^N)$, we get

$$\| x(t; \varphi_0, u) - rx_0 \|$$

$$\geq \left\| \int_0^t U(t-s)Bu(s)ds - rx_0 \right\| - \| U(t)\varphi_0 \|$$

$$+ \left\| \int_0^t U(t-s)F(s,x_s)ds \right\|$$

$$\geq 2c_8 \| \varphi_0 \| + 2c_8 M/\tau - c_8 M/\tau - c_8 \| \varphi_0 \|$$

$$= c_8 \| \varphi_0 \| + c_8 M/\tau > 0$$

This implies $rx_0 \notin \overline{\cup_{t>0} R_t(\varphi_0)}$, which contradicts $\overline{\cup_{t>0} R_t(\varphi_0)} = X$. Thus the proof is complet.

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