□ 論 文 □

A Simple Dynamic Model of Maintenance Expenditures: Theory and Empirical Evidence

耐久財 管理費用의 動的인 行態 研究
─ 理論 및 經驗的 分析 ─

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CONTENTS

초 록

交通資産과 같은 耐久財는 일정 期間 동안의 수명을 가지게 된다. 이 期間 동안에 耐久財는 어떠한 형태로 소비되어 지고 이에따라 耐久財는 效率性을 잃어 간다. 耐久財를 效率的으로 使用하기 위해서는 적절한 管理를 필요로 한다.

본 研究는 耐久財를 效率的으로 使用하기 위해 필요한 管理費用의 動的形態를 理論的으로 고찰한 다음, 미국에서의 業務用 비행기의 管理費用資料를 使用하여 經驗的인 結果를 제시한다. 본 研究의 結果는 다음과 같다. (1) 管理費用은 시간이 감에 따라 줄어든다. 그러나 管理費用의 潛在價格(Shadow Price)에 대한 比率은 여러가지 外的變數들에 의해 決定된다. (2) 미국에서의 業務用 비행기의 市場價格에 대한 管理費用의 比率은 시간이 감에 따라 늘어난다.

Ⅰ. Introduction

The expenditures for maintenance of industrial machinery and equipment have increased over the time.[1] This cost of maintenance expenditures shows the high proportion for total investment cost, for example, main-

1) Marinello(1981) surveyed the industrial maintenance costs in U.S.. He reports that the total expenditures for maintenance and replacement of industrial facilities and equipment have increased each year. Furthermore, for the past 10 years, the annual rate of increase has averaged closed to 15 percent.

tenance expenditures for investment in machinery and equipment amounted to 49% in Sweden in 1968(Rapp, 1974). In the study of housing, they have popularly analyzed the behavior of optimal maintenance expenditures, because the level of maintenance expenditures has the important impact on the quality of housing(Arnott and Davidson, 1983 ; Margolis, 1981). Thus the behavior of maintenace expenditure has been concerned as one of decision variables that a firm chooses to maximize its profit over the lifetime of capital stock.

The dynamic pattern of maintenance expenditures has been studied by many researchers.[2] Thompson(1968) theoretically shows that the level of maintenance expenditures is decreased over the time. Following the Thomson's work, there have been many studies about the dynamics of maintenance expenditures with the extension of Thompson's assumptions(Arora and Lele, 1970), or with the inclusion of the relationship between the level of maintenance expenditures and repair decision(Kamien and Schwartz, 1971) or replacement dicision(Sethi, 1973). Hartl(1983) generalizes these model using the nonlinear function, and discusses the optimal properties of maintenance expenditures. All these studies are the theoretical works using the dynamic optimization techniques, especially a control theory approach. The use of a dynamic model gives us the mathematical burden to solve for an optimal function : hence they derive qualitative properties of the optimal level of maintenance expenditures. Otherwise, they need much simplifed assumptions among variables.[3] Moreover, no empirical study has been done to show this dynamic pattern of maintenance expenditures so far.

The purpose of this research is to examine the dynamic pattern of maintenance expenditures in a theoretical model using the explicit functional form for the relationship among variables. Furthermore, we present the empirical evidence for this dynamic pattern of maintenance expenditures using maintenance data of business aircraft.

This paper is organized as follows. First, we develop a theoretical model that explains the dynamic pattern of maintenance over the time. Second, we present our empirical findings for the dynamics of maintenance expenditures. Third, we conclude our findings.

Ⅱ. A Simple Model for the Pattern of Maintenance Expenditures

1. Assumptions

We need to have several assumptions to develop our dynamic model of maintenance expenditures for the tractability.

Assumption 1 : *The firm will not purchase new capital goods and will not resale its holding capital, until the firm use up the holding capital.*

Our interest is in the dynamic patterns of maintenance expenditures on capital stock, which is given at the beginning time period.

2) See McCall(1965), Pierskalla and Voelker(1976), and Sherif and Smith(1981) for the survey of existing models of maintenance expenditures.

3) For example, Thomson(1968) assumes a linear relationship between the level of maintenance expenditures and capital depreciation, and shows that the optimal level of maintenance expenditures is bang—bang, which is discontinuous funtion on time.

The decision on optimal capital stock is beyond our interest.[4] We consider the decision of maintaining the optimal capital stock over the time. Thus there is neither resale nor purchase of new capital stock until the lifetime of existing capital stock.

Assumption 2 : *All maintenance expenditures are operating expenses, and are immediately deductible for tax purpose.*

The tax treatment of maintenance expenditure is ambiguous in general. A part of maintenance expenditures can be considered as an operating cost which is an immediately deductible expense for income tax purposes. However, a portion of maintenance expenditures include the replacement of major component parts, which is normally thought of as a capital expenditure. This portion of maintenance should be capitalized and depreciated over time. Thus some maintenance expenses might not be immediately deductible. However, for purposes of this model we assume that all maintenance expenditures are immediately tax deductible operating expenses.

Assumption 3 : *The lifetime of capital good is a predetermined parameter.*

The lifetime of capital goods can be increased or decreased by any amount of maintenance expenditures. However, we assume the lifetime of capital goods is independent of the level of maintenance expenditures. The lifetime of capital goods is exogenous variable, which is predetermined when capital goods are manufactured.

Assumption 4 : *The salvage value at the terminal time of capital goods is zero.*

Assumption 5 : *The prodution function consists with only input variable, capital, and is a linearly homogeneous with respect to capital input.*

We examine the decision on the change of capital stock over the time, and do not consider the labor in our model. We assume that capital is the only input for the production, and that labor input is constant all the time. Also we assume that the production function (F) is a linearly homogeneous with respect to capital input such that :

$$F(K) = \alpha K \qquad (1)$$

where α is parameter and $\alpha > 0$.

Assumption 6 : *The change rate of capital goods is assumed to depend on the level of capital stock, an amount of maintenance expenditures, and time.*

We express the change rate of capital goods (dK/dt) as a function(G) with capital stock(K), maintenance expenditures(M), and time(t) as follows :

$$dK/dt = G[K(t), M(t), t] \qquad (2)$$

where the funtion, G, is assumed to have the following properties

1. $G < 0$ (3)

   Capital goods depreciate over the time. Firm spends the maintenance expenditures to prevent holding capital from a high level of depreciation. However, maintenance expenditures cannot stop the depreciation of capital good, and cannot produce more capital stock than existing capital stock.

2. $\partial G/\partial K < 0$ (4)

4) The optimal capital stock has been popularly examined in the neoclassical investment theory(Hall and Jorgenson, 1967, 1969 ; Jorgenson, 1963).

This relation means that the pattern of capital depreciation follows the accelerated depreciation rather than straight or decelerated depreciation.

3. $\partial G/\partial M > 0$ and $\partial^2 G/\partial M^2 < 0$ (5)

Capital goods will depreciate at a high rate if there exists no maintenance expenditures. Thus, it is obvious that maintenance expenditures will reduce the rate of capital depreciation. However, the effect of maintenance expenditures on a change rate of capital stock will be decreased, as the level of maintenance expenditures is increased.

Thus, we assume that the dynamic pattern of capital stock has the following relationship with the level of maintenance expenditures.

$$dK/dt = aM_t^{\beta} - \delta K_t \quad (6)$$

where δ is the built-in durabilty,[5] and the parameters have the following restrictions : $a > 0$, $0 < \beta < 1$, and $0 < \delta < 1$.

2. Algebraic Statement of the Model

Firm purchases the capital goods, K_0, at the beginnig time, and hold this capital goods until their lifetime, T, which is exogenous. Thus, the objective of firm is to maximize the after-tax profit, W, for the time periods T, with one choice variable, which is the level of maintenance expenditures(M), as follows :

$$W = (1-u)\int_0^T e^{-rt}[p\alpha K_t - M_t]dt - (1-k-uz)qK_0$$

subject to

$$dK/dt = aM_t^{\beta} - \delta_t K_t \quad (7)$$

where

u : the corporate tax rate

r : interest rate

p : output price

k : investment tax rate

z : the present value of the depreciation deduction on one dollar of capital expenditure.

$$z = \int_0^L e^{-rs} D(s) ds$$

L : capital lifetime for tax purpose

D(s) : depreciation schedule for tax purposes

q : the price of an unit of capital stock at the beginning time

As our model is a control problem, we will solve it using the maximum principle technique. The Hamiltonian is as follows :[6]

$$H = (1-u)e^{-rt}[p\alpha K - M] + \lambda[a M^{\beta} - \delta K] \quad (8)$$

Where λ is the shadow price of capital for the given time. Expression(8) has the following form.

$$H(M, K, \lambda, t) = F(M, K, t) + \lambda G(M, K, t) \quad (9)$$

To interpretate this relation(9), we differentiate both sides of (9) with respect to t, as follows :

$$H(M, K, \lambda, t)dt = F(M, K, t)dt + \lambda dK \quad (10)$$

The relation(10) implies that the after-

5) There has been much argument about the determination of built-in durability, which depends on market structure (for example, Parks, 1974 ; Schmalensee, 1979 ; Swan, 1970). However, this topic is beyond our interest, so that we assume that the built-in durability is exogenous.

6) We drop the subscript, t, for the convenience.

tax profit for the given time interval dt(H dt) is the summation of two components : (1) the revenue after the maintenace cost is deducted (F dt), (2) the capital cost due to a capital stock (λ dK), where λ is the shadow price of capital, and dK is the decreased amount of capital stock during the given time dt[7] and is the negative value. Hence λ dK represents the capital cost.

According to the maximum principle, the optimal control of maintenance expenditures maximize the Hamiltonian at each point in time.[8] Thus, the first-order conditions are as follows :

$$H_M = -(1-u)e^{-rt} + \lambda a \beta M^{\beta-1} = 0 \tag{11}$$

$$d\lambda/dt = -H_K = -(1-u)p\alpha e^{-rt} + \lambda\delta \tag{12}$$

$$\lambda[T] = 0 \tag{13}$$

$$dK/dt = aM^{\beta} - \delta K \tag{13a}$$

The relation (11) shows that Hamiltonian function should have the extreme points with respect to the level of maintenance expenditures, M. For the interpretation of the relation (12), we multiply both sides of (12) with dt, and reexpress it as follows :

$$(1-u)p\alpha e^{-rt}dt = -d\lambda + \lambda\delta dt \tag{14}$$

The left-hand side of (14) is the expression from F_K dt, which represents the marginal revenue due to a change in one unit of capital stock. The right-hand side of (14) is the marginal cost due to a change in one unit of capital stock, which has two components, where the shadow price of capital, λ, is decreased over the time because capital stock is decreased over the time from Assumption 6. The first component ($-d\lambda$) is the change in the price of capital due to using up one unit of capital stock, where $d\lambda$ is negative. The second component ($\lambda\delta$ dt) is the cost of capital depreciation for an unit of capital stock during the given time dt. The summation of these two components represents marginal cost due to a change in one unit of capital stock. Thus the relation (14) implies that marginal revenue for one unit of capital stock should be equal to its marginal cost. The relation, (13), shows the transversality condition, which means that the shadow price of capital should be zero at the terminal time, T. This condition is quite simple to interpretate. The lifetime of capital stock is T from Assumption 3, and its salvage value is zero at the terminal T from Assumption 4. Thus the shadow price of capital stock should be zero at the terminal time T. The relation (13a) shows the optimal level of capital stock at each time, which is determined after the optimal decision of maintenance expenditures.

The second order sufficient condition for the maximization problem is as follows :

$$H_{MM} = \lambda a \beta(\beta-1)M^{\beta-2} < 0 \tag{15}$$

where λ is the shadow price of capital, and should be positive for $0 \leq t < T$, because capital stock cannot be negative. At the terminal time T, the shadow price of capital is zero from (13). Then there is no need to maintain the capital stock, which is no maintenance expenditures at t=T. We do not consider the terminal time T in our analysis, and develop our model for $0 \leq t < T$. Thus Hamiltonian, H,

7) Note that capital stock is decreased over the time, irrespective of how much maintenance expenditures are spent, from Assumption 6.

8) See Dorfman(1969) for the economic interpretation of optimal control theory in detail

is strictly concave function with respect to the level of maintenance expenditures (M) for $0 \leq t < T$ from our first and second order conditions, (11) and (15).

Using (12) and (13), we solve the differential equation with respect to λ, and get the optimal shadow price of capital goods at the time t, λ_t^* :

$$\lambda_t^* = (1\text{-}u)p\alpha e^{\delta t}[e^{-(r+\delta)t} - e^{-(r+\delta)T}]/(r+\delta) \quad (16)$$

where u, p, r, δ are constant, and are not changed over the time. The optimal shadow price of capital, (16), is expressed as the function of time t, hence we can get the dynamic pattern of maintenance expenditures (16). We get the relationship between the shadow price of capital(λ) and the level of maintenance expenditures (M) using (11) as follows :

$$M_t^* = [a\beta e^{rt}\lambda^*/(1\text{-}u)]^{1/1-\beta} \quad (17)$$

We differentiate (17) with respect to λ_t^* as follows :

$$\partial M_t^*/\partial\lambda_t^* = [\alpha\beta e^{rt}/(1\text{-}\beta)(1\text{-}u)][\alpha\beta e^{rt}\lambda^*/(1\text{-}u)]^{\beta/(1-\beta)} > 0 \quad (18)$$

The relation (18) means that, for the given time t, when the shadow price of capital is decreased, firm spends less expenditures on maintaining its existing capital stock. Firm will make less efforts to maintain its existing capital, because the capital cost is relatively lower. Less expenditures on maintaining capital goods leads to a higher rate of capital depreciation.

We reexpress (17) using (16), and get the optimal level of maintenance expenditures for each time t as follows :

$$M_t^* = [a\alpha\beta e^{(r+\delta)t}(e^{-(r+\delta)t} - e^{-(r+\delta)T})/(r+\delta)]^{1/1-\beta} \quad (19)$$

where r, δ are constant over the time. The function (19) shows the optimal level of maintenance expenditures for each time $t(0 \leq t \leq T)$.

3. The Dynamic Pattern of Maintenance Expenditures

We use the comparative statics to derive the dynamic pattern of maintenance expenditures over the time. Differentiate (16) and (19) with respect to the time, t, $(0 \leq t < T)$ to get the dynamic patterns of the shadow price of capital and maintenance expenditures as follows :[9]

$$d\lambda^*/dt = -(1\text{-}u)\alpha[re^{-rt} + \delta e^{\delta t-(r+\delta)T}]/(r+\delta) < 0 \quad (20)$$

$$dM_t^*/dt = -a\alpha\beta e^{-(r+\delta)(T-t)} M_t^{\beta}/(1\text{-}\beta) < 0 \quad (21)$$

The relation (20) implies that the shadow price of capital is decreased as time goes. It is because capital goods depreciate at the rate of δ when there is no expenditure on maintaining capital goods. Even though there is certain amount of expenditures on maintaining capital stock, Assumption 6 indicates that the level of maintenance expenditures cannot produce more capital stock than existing capital stock. As a consequence, the shadow price of capital stock, which reflects the amount of capital stock, should be decreased as time goes. The relation (21) implies that the level of maintenance expenditures is decreased as time goes (for $0 \leq t < T$). Firm finds that the shadow price of its holding capital is decreased over the time. The decrease in the shadow price of holding capital causes firm to spend less

9) Note that u, r, δ are not changed over the time, hence λ_t^* and M_t^* are functions of the time t.

expenditures on maintainting its existing capital goods, because maintenance cost might be more expensive than the capital cost.

Our next question is about the dynamic pattern of the ratio of maintenance expenditures to the shadow price of capital, which is $d[M^*_t/\lambda^*_t]/dt$. From (17), we get the following expression :

$$M^*_t/\lambda^*_t = [a\beta/(1-u)]^{1/(1-\beta)} e^{rt/(1-\beta)} \lambda^{*\beta/(1-\beta)} \qquad (22)$$

We differentiate (22) with respect to t ($0 \leq t < T$) to find the dynamic pattern of the ratio of maintenance expenditures to the shadow price of capital over the time

$$d[M^*_t/\lambda^*_t]/dt = [1/(1-\beta)][a\beta/(1-u)]^{1/(1-\beta)} e^{rt/(1-\beta)} \lambda^{*\beta/(1-\beta)} [r + \beta(d\lambda^*_t/dt)\lambda^*_t] \qquad (23)$$

where $0 < \beta < 1$, and $\lambda^*_t > 0$ for $0 \leq t < T$. The sign direction of (23) depends on $[r + \beta(d\lambda^*/dt)/\lambda^*)]$. As we discussed earlier, λ is interpretated as the shadow price of capital. We interpretate $[d\lambda^*_t dt]/\lambda^*_t$ as the rate of economic depreciation at the given time t. As $d\lambda^*_t/dt$ is negative from (20), we difine $d(t) = -[d\lambda^*_t/dt]/\lambda^*_t$ to represent a positive rate of economic depreciation at time t, where $0 \leq d(t) \leq 1$. Then the sign direction of $d[M^*_t/\lambda^*_t]/dt$ will depend on the sign direction of $[r-\beta d(t)]$, where $0 < r < 1$ and $0 < \beta < 1$. The dynamic pattern of the ratio of maintenance expenditures to the shadow price of capital is as follows :

$$\text{If } 0 \leq d(t) \leq r/\beta, \text{ then } d[M^*_t/\lambda^*_t]/dt > 0.$$
$$\text{If } d(t) > r/\beta, \text{ then } d[M^*_t/\lambda^*_t]dt < 0 \quad (24)$$

The relation (24) implies that the dynamic pattern of the ratio of maintenance expendituress to the shadow price of capital depends on the rate of economic depreciation. If the rate of economic depreciation is less than a certain point (r/β), then the ratio of maintenance expenditures to the shadow price of capital is increased as the capital ages. On the contrary, if the rate of economic depreciation is greater than a certain point (r/β), then the ratio of maintenance expenditures to the shadow price of cápital is decreased as the capital ages. This breaking point (r/β) consists with the interest rate and the degree of the effect of maintenance expenditures on capital stock. A firm decides the level of maintenance expenditures for its existing capital to maximize its present value over the capital lifetime. This decision is done through the comparison among the interest rate, the degree of the effect of maintenance expenditures on capital stock, and the rate of economic depreciation of existing capital.

Ⅲ. Empirical Evidence

In this section, we show the empirical evidence for the dynamic pattern of the ratio of maintenance expenditures to the shadow price of capital that our theoretical model discussed. Our theoretical model shows that an amount of maintenance expenditures is decreased as time goes. We need panel data for this empirical estimates to trace the pattern of maintenance expenditures for the given capital good over the time. However, we could not get any useful data for this study. Our theoretical model also discusses the dynamic pattern for the ratio of maintenance expenditures to the shadow price of capital which depends on several parameters. We test this theoretical finding using our data of maintenance expenditures on business aircraft. Our maintenance data from business aircraft was available for this research.[10] This data has the observations of 39.

For the statistical analysis, we need to transform our data to standardize the values of maintenance expenditures. Business aircraft are heterogenous goods in size and in the market price, and hence show the wide variation in maintenance expenditures (M) and in the market price (P) of aircraft. Large aircraft is higher in the market price and needs more expenditures on maintenance than small aircraft. For the aggregation of these different kinds of aircraft, we use the ratio of maintenance expenditures to the market price of each aircraft model and examine this ratio over the time. This ratio is used to represent the ratio of maintenance expenditures to the shadow price of capital, which was discussed in our theoretical model. We assume that the second hand market of business aircraft reflects the shadow price of capital perfectly. For more convenience in interpretation, we use $100^{*}M/P$ as a dependent variable, which represents the percent rate of an amount of maintenance expenditures with respect to the market price of business aircraft. We get the information of the market price of each different aircraft model for each age from aircraft bluebooks.[11] The prices in aircraft bluebooks reflect the real market transaction price.

For an independent variable, we use AGE of each aircraft model to get the dynamic pattern for maintenance expenditures, because our data is from cross section, which is from 1987. Thus model is $[100^{*}M_i/P_i] = \beta_0 + \beta_1 AGE_i$, where $i = 1, \cdots, 39$, and β_1 indicates the dynamic pattern of the ratio of maintenance expenditures to the market price of capital. Table 3−1 shows our empirical results using business aircraft data.

Table 3−1 : Dynamic Pattern of Maintenance Expenditures

$$[100^{*}M/P] = -1.174 + 1.199\,AGE$$
$$(5.3)$$

$R^2 = 0.43$, Sample Size = 39

(The value in parenthesis is t ratio)

The estimate(1.199) that AGE variable has is positive, and is statistically significant at the 5% level. This implies that the percent ratio of maintenance expenditures to the market price of capital is increased as capital good ages. Moreover, business aircraft needs almost 1.2% increase rate in the ratio of maintenance expenditures to capital price for an additional year. Our empirical result shows that the pattern of the ratio of maintenance expenditures to the market price of capital is increased over the time for the case of business aircraft.

Ⅳ. Conclusion

Our theoretical model finds that an amount of maintenance expenditures is decreased as time goes. However, the dynamic pattern of the ratio of maintenance expenditures to the shadow price of capital depends upon the interest rate, the degree of effect of maintenance expenditures on capital

10) See Appendix for the models of business aircraft.

11) We used the bluebook of "Aircraft Bluebook−Price Digest" published by Aircraft Bluebook Corporation. Oklahoma, 1987.

stock, and the rate of economic depreciation of capital good. We empirically examine the dynamic pattern of the ratio of maintenance expenditures to the market price using maintenance data of business aircraft. The percent ratio of maintenance expenditures to the market price of business aircraft is increased as business aircraft ages, which is 1.2% increase rate per one additional age.

More empirical evidence should be done using the different kinds of capital goods to support our theoretical findings which depend on several different parameters. Our empirical evidence is based on the cross sectional data. However, the study about the dynamic pattern of maintenance expenditures should be done using available panel data to carefully examine the level of maintenance expenditures for a given capital good over the time.

V. Appendix

No.	Model	Sample Size
1	Aero − Commander 690	2
2	Beech King Air 100	4
3	Beech King Air 200	5
4	Beech Super King7 Air 200C	
5	Beech 99	2
6	Cessna 441	3
7	Falcon 50	4
8	Falcon 20	5
9	Gates Lear Jet 25	4
10	Gates Lear Jet 55	3
	Total	39

Ⅵ. References

Arnott, Richard and Russell Davidson, 1983, Housing Quality, Maintenance, and Rehabilitation, Review of Economic Studies, 467−194.

Dorfman, Robert, 1969, The Economic Interpretation of Optimal Control Theory, American Economic Review, 817−831.

Hall, Robert and Dale Jorgenson, 1967, Tax Policy and Investment Behavior, American Economic Review 57, 391−414.

Hall, Robert and Dale Jorgenson, 1969, Tax Policy and Investment Behavior : Reply and Further Results, American Economic Review 59, 388−401.

Hartl, Richard, 1983, Optimal Maintenance and Production Rates for a Machine, Journal of Economic Dynamics and Control 6, 281−306.

Jorgenson, Dale, 1963, Capital Theory and Investment Behavior, American Economic Review 53, 247−259.

Kamien, Morton and Nancey Schwartz, 1971, Optimal Maintenance and Sale Age for a Machine Subject to Failure, Management Science 17, 495−504.

Margolis, Stephen, 1981, Depreciation and Maintenance of Housing, Land Economics 57, 91−105.

Marinello, R., 1981, Maintenance Costs, Plant Engineering, 58−63.

McCall, John, 1965, Maintenance Policies for Stochastically Failing Equipment : A Survey, Management Science, Vol. 11, No. 5,493−524.

Parks, Richard, 1974, The Demand and Supply of Durable Goods and Durability, American Economic Review 64, 37–55.

Pierskalla, William and John Voelker, 1976, A Survey of Maintenance Models : The Control and Surveillance of Deteriorating System, Naval Research Logistics Quarterly 23, 353–388.

Rapp, Birger, 1974, Models for Optimal Investment and Maintenance Decisions, (Halsted Press).

Schmalensee, Richard, 1979, Market Structure, Durability, and Quality : A Selective Survey, Economic Inquiry, 177–196.

Sethi, Suresh, 1973, Simultaneous Optimization of Preventive Maintenance and Replacement Policies for Machine : A Modern Control Theory Approach, AIIE Transactions 5, 156–163.

Sherif, Y., and M. Smith, 1981, Optimal Maintenance Models for Systems Subject to Failure–A Review, Naval Research Logistics Quarterly 32, 47–74.

Swan, Peter, 1970, Durability of Consumption Goods, American Economic Review 60, 884–894.

Thompson, Gerald, 1968, Optimal Maintenance Policy and Sale Date of a Machine, Management Science 14, 543–550.