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A NOTE ON S-SETS IN A FIXED GROUP

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1. Introduction

In this paper we introduce $S(X, x_0)$ which is a generalization of Ellis group $G(X, x_0)$, and S-sets in $S(X, x_0)$. In particular we find the sufficient condition for the group $A(I)$ of all automorphisms of I and $K=Iu$ to be isomorphic, where I is a minimal right ideal and u is an idempotent of I.

2. Preliminaries

A transformation group or flow (X, T) will be consist of a jointly continuous action of the discrete topological group T on the compact Hausdorff space X. A minimal set (X, T) is said to be regular if for any almost periodic point (x, y) of $(X\times X, T)$, there exists an automorphism ϕ of (X, T) such that $\phi(x)=y$.

Let βT denote the Stone-Cěch compactification of T. Then $(\beta T, T)$ is a universal point-transitive flow, and βT is an enveloping semigroup for X, whenever X is a flow with acting group T.

Let us fix from now on a minimal right ideal I in βT. We denote by J its set of idempotents and choose a distinguished idempotent $u\in J$. We denote by K the group Iu. Given a minimal flow X, we choose a point $x_0\in Xu=\{xu\,|\,x\in X\}=\{x\,|\,xu=x\}$. Under the canonical map $\pi : (\beta T, e)\to(X, x_0)$, I is mapped onto X and u onto x_0.

Throughout this paper, the set of automorphisms of (X, T) is denoted by $A(X)$ and the set of proximal pairs in (X, T) is denoted by $P(X, T)$.

DEFINITION 2.1 [4]. For this pointed minimal flow (X, x_0), we define $G(X, x_0)=\{\alpha\in K\,|\,x_0\alpha=x_0\}$ or, simply, G, which is called the

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Ellis group of (X, x_0).

LEMMA 2.2 [1]. *Let* $x \in X$, $p \in E(X)$, *the enveloping semigroup of* X, *and* $\phi \in H(X)$, *where* $H(X)$ *denotes the set of all endomorphisms of* X. *Then* $\phi(x)p = \phi(xp)$.

LEMMA 2.3 [1]. *Let* $\phi \in H(I)$, *where* (I, T) *is minimal right ideal. Then there exists* $p \in I$ *such that* $\phi = L_p$, *where* $L_p(q) = pq$ *for all* $q \in I$.

LEMMA 2.4 [1]. *Let* (X, T) *be a minimal set, and* I *a minimal right ideal in* $E(X)$. *Then every* $\phi \in H(X)$ *is induced by some* $L_p \in A(I)$. *If* X *is written as* I/R, *where* R *is closed* T*-invariant equivalence relation on* I, *then* $L_p \in A(I)$ *induces* $\phi \in H(X)$ *if and only if* $pR \subset R$ *and* $L_p \in A(I)$ *induces* $\phi \in A(X)$ *if and only if* $pR = R$.

LEMMA 2.5 [6]. $P(X, T)$ *is on equivalence relation if and only if* $(E(X), T)$ *is regular.*

REMARK 2.6 [3]. Let (X, T) be a flow and $\phi : \beta T \to X$. Then ϕ induces $\theta : (\beta T, e) \to (E(X), e)$ which is independent of ϕ. This permits one to consider an element p of βT as a map of X into X viz. $xp = x\theta(p) (x \in X)$ i.e., identifying p with $\theta(p)$. With this convention $\phi(qp) = \phi(q)p$ $(p, q \in \beta T)$.

3. The role of S-sets in a fixed group

DEFINITION 3.1. For this pointed minimal flow (X, x_0), we define $S(X, x_0) = \{\alpha \in K \mid x_0\phi(\alpha) = x_0$ for some $\phi \in A(I)\}$. For a fixed $\phi \in A(I)$, the set $\{\alpha \in K \mid x_0\phi(\alpha) = x_0\}$ is called the S-set in $S(X, x_0)$ and is denoted by $S_\phi(X, x_0)$ or, simply, S_ϕ.

REMARK 3.2 (1) If $A(I)$ is the trivial group, then $S(X, x_0) = G(X, x_0)$. (2) $G(X, x_0)$ is a subgroup of K.

LEMMA 3.3 *If* $\phi \in A(I)$, *then* $\phi | K : K \to K$ *is bijective.*

Proof. Let $\alpha \in K$. Then there exists $p \in I$ such that $\alpha = pu$. Since $\phi(\alpha) = \phi(pu) = \phi(p)u \in K$, it suffices to show that $\phi | K : K \to K$ is onto. Suppose there exists $q \in K - \phi(K)$. Then there exist $v \in I - K$ and $w \in I$ such that $\phi(v) = q = wu$. Since $\phi^{-1} \in A(I)$, there exists $p \in I$ such that $L_p=\phi^{-1}$. Hence $v=\phi^{-1}(q)=L_p(wu)=(pw)u\in K$, which is a contradiction for $v\notin K$. Hence $\phi|K : K\to K$ is onto.

COROLLARY 3. 4. *$\alpha\in S_\phi$ if and only if $\phi(\alpha)\in G(X, x_0)$.*

LEMMA 3. 5. $S(X, x_0)=K$.

Proof. Let $\alpha\in K$. Since $(\alpha, u)u=(\alpha, u)$, (α, u) is an almost periodic point of $(I\times I, T)$. Hence there exists $\phi\in A(I)$ such that $\phi(\alpha)=u$ and so $x_0\phi(\alpha)=x_0u=x_0$, which implies $\alpha\in S(X, x_0)$.

THEOREM 3. 6. *Let $\phi\in A(I)$. Then $\alpha\in S_\phi$ if and only if there exists $h\in A(I)$ such that $h|G : G\to G$ is bijective and $\alpha=\phi^{-1}h(u)$.*

Proof. If: Suppose we have $\phi\in A(I)$ and $h\in A(I)$ such that $h|G : G\to G$ is bijective and $\alpha=\phi^{-1}h(u)$. Now let $h(u)=\beta$. Then $\phi(\alpha)=\beta\in G$, because $u\in G$. Hence $\alpha\in S_\phi$.

Only if: Let $\phi\in A(I)$ and let $\alpha\in S_\phi$. Then $\phi(\alpha)=\beta$ for some $\beta\in G$. Since (u, β) is an almost periodic point of $I\times I$ and I is a regular minimal set, there exists $h\in A(I)$ such that $h(u)=\beta$. Hence $\alpha=\phi^{-1}(\beta)=\phi^{-1}h(u)$ and $h(G)\subset G$, because $h(r)=h(ur)=h(u)r=\beta r\in G$ for all $r\in G$. To show that $h|G : G\to G$ is bijective, it is sufficient to show that $h|G : G\to G$ is onto. Suppose there exists $\sigma\in G$ such that $h(w)=\sigma$ for some $w\in K-G$. Then $u=h^{-1}(\beta)=h^{-1}(u\beta)=h^{-1}(u)\beta$. Since G is a subgroup of group K, $h^{-1}(u)=\beta^{-1}$ be in G. Further, $h^{-1}(r)=h^{-1}(ur)=h^{-1}(u)r=\beta^{-1}r\in G$ for all $r\in G$. Hence $h^{-1}(\sigma)\in G$, a contradiction. This proves that $h|G : G\to G$ is onto. Thus $h|G : G\to G$ is bijective.

THEOREM 3. 7. *Let $\delta, \tau\in A(I)$. If $S_\delta\cap S_\tau\neq\phi$. Then there exists $h\in A(I)$ such that $h|G : \to G$ is bijective and $\delta^{-1}\tau h=1$, where 1 is an identity map on I.*

Proof. Let $\delta, \tau\in A(I)$ and let $\alpha\in S_\delta\cap S_\tau$. Then there exist $h_1\in A(I)$ and $h_2\in A(I)$ such that $h_1|G : G$ and $h_2|G : G\to G$ are bijective, and $\delta^{-1}h_1(u)=\alpha=\tau^{-1}h_2(u)$. Hence $\delta^{-1}h_1=\tau^{-1}h_2$ on I and $\tau^{-1}=\delta^{-1}h_3$ for some $h_3\in A(I)$, where $h_3|G : G\to G$ is bijective. Thus $\delta\tau^{-1}h=1$ for some $h\in A(I)$, where $h|G : G\to G$ is bijective and 1 is an identity map on I.

LEMMA 3.8. *Let* $\phi, \tau \in A(I)$. *Then* $S_\phi S_\tau = \cup \{S_{\tau h \phi} | h \in H\}$, *where* $H = \{h \in A(I) \mid h|G : G \to G$ *is bijective*$\}$.

Proof. For each $r \in S_\phi S_\tau$, we let $r = \alpha\beta$ for some $\alpha \in S_\phi$ and $\beta \in S_\tau$. Then there exist $h_1 \in H$ and $h_2 \in H$ such that $\alpha = \phi^{-1}h_1(u)$ and $\beta = \tau^{-1}h_2(u)$. Hence $r = \alpha\beta = \phi^{-1}h_1(u)\tau^{-1}h_2(u) = \phi^{-1}h_1(\tau^{-1}h_2(u)) = (\tau h_1^{-1}\phi)^{-1}h_2(u)$. Since $\tau h_1^{-1}\phi \in A(I)$ and $h_1^{-1} \in H$, $r \in S_\tau h_1^{-1}\phi \subset \cup \{S_{\tau h \phi} | h \in H\}$.

For the converse inclusion, let $r \in \cup \{S_{\tau h \phi} | h \in H\}$. Then there exists $h_1 \in H$ such that $r \in S_{\tau h_1 \phi}$ and hence there exists $h_2 \in H$ such that $r = (\tau h_1 \phi)^{-1} h_2(u)$. Then $r = \phi^{-1}h_1^{-1}\tau^{-1}h_2(u) = \phi^{-1}h_1^{-1}(u\tau^{-1}h_2(u)) = \phi^{-1}h_1^{-1}(u)\tau^{-1}h_2(u)$. Now let $\phi^{-1}h_1^{-1}(u) = \alpha$ and $\tau^{-1}h_2(u) = \beta$. Since $h_1^{-1} \in H$, $\alpha \in S_\phi$ and $\beta \in S_\tau$. Thus $r = \alpha\beta \in S_\phi S_\tau$.

THEOREM 3.9. *If* $G(X, x_0) = \{u\}$, *then*
(1) $S_\phi S_\tau = S_{\tau\phi}$ *for all* $\phi, \tau \in A(I)$.
(2) $\Sigma = \{S_\phi | \phi \in A(I)\}$ *is a group.*
(3) Σ *and* K *are isomorphic.*

Proof. (1) Let $H = \{h \in A(I) \mid h|G : G \to G$ is bijective$\}$. Since $G(X, x_0) = \{u\}$ and $H = \{1\}$, $S_\phi S_\tau = S_{\tau\phi}$.

(2) First we show that S_ϕ is a singleton for each $\phi \in A(I)$. Since $G(X, x_0) = \{u\}$, $S_\phi = \{\alpha \in K | x_0\phi(\alpha) = x_0\} = \{\alpha \in K | \phi(\alpha) \in G(X, x_0)\} = \{\alpha \in K | \phi(\alpha) = u\}$ and so S_ϕ is a singleton. By (1), it is easy to show that S_{Lu} is the identity element of Σ, $(S_\phi)^{-1} = S_{\phi^{-1}}$, and $(S_\phi S_\tau)S_\delta = S_\phi(S_\tau S_\delta)$ for all $\phi, \tau, \delta \in A(I)$. Hence $\Sigma = \{S_\phi | \phi \in A(I)\}$ is a group.

(3) To show that Σ and K are isomorphic, we define $f : \Sigma \to K$ by $f(S_\phi) = \alpha$ if $\phi(\alpha) = u$. Then it is clear that f is well defined. Now let $f(S_\phi) = f(S_\tau) = \alpha$. Then $\phi(\alpha) = \tau(\alpha) = u$ and hence $\phi = \tau$ on I, because I is a minimal set. This proves that f is injective. For each $\alpha \in K$, there exists $\phi \in A(I)$ such that $\phi(\alpha) = u$, since that (α, u) is an almost periodic point of $I \times I$ and I is a regular minimal set. This proves that f is onto. Finally we show that f is a group homomorphism. Let $S_\phi, S_\tau \in \Sigma$, and let $f(S_\phi) = \alpha$, and $f(S_\tau) = \beta$. Then $f(S_\phi S_\tau) = f(S_{\tau\phi}) = r$ for some $r \in K$, which implies that $\tau\phi(r) = u$. Hence $r = (\tau\phi)^{-1}(u) = \phi^{-1}\tau^{-1}(u) = \phi^{-1}(u\tau^{-1}(u)) = \phi^{-1}(u)\tau^{-1}(u) = \alpha\beta = f(S_\phi)f(S_\tau)$. Thus f is an isomorphism.

COROLLARY 3.10. *If $G(X, x_0)=\{u\}$, then $A(I)$ and K are isomorphic.*

Proof. It suffices to show that $A(I)$ and Σ are group isomorphic. Now we define $g: A(I)\to\Sigma$ by $g(\phi)=S_{\phi-1}$ for each $\phi\in A(I)$. Since I is a minimal set, $S_\phi=S_\tau$ implies $\phi=\tau$. Hence $\phi=\tau$ iff $S_{\phi^{-1}}=S_{\tau^{-1}}$, which shows that g is bijective. But $g(\phi\tau)=S_{(\phi\tau)^{-1}}=S_{\tau^{-1}\phi^{-1}}=S_{\phi^{-1}}S_{\tau^{-1}}$ $=g(\phi)g(\tau)$, which means that g is a group homomorphism. Thus $A(I)$ and Σ are isomorphic.

REMARK 3.11. Let (X, T) be almost periodic minimal set with abelian acting group T. Then $G(X, x_0)=\{e\}$ and $A(E)$, E and X are isomorphic. In particular X is essentially a compact abelian topological group, and T is a dense subgroup of X which acts by right multiplication.

THEOREM 3.12. *Let (X, T) be a distal minimal set and let $x_0\in X$. Then $S(X, x_0)=E(X)$ is a group and $G(X, x_0)=\pi^{-1}(x_0)$ is a subgroup of $E(X)$.*

Proof. Since (X, T) is distal, $E(X)$ is a minimal right ideal and a group. By Lemma 3.5, $S(X, x_0)=E(X)$.

Now let $\Gamma: (X, x_0)\to(Y, y_0)$ be a homomorphism of pointed minimal sets.

THEOREM 3.13. (1) *For each $\phi\in A(I)$, $S_\phi(X, x_0)\subset S_\phi(Y, y_0)$.*
(2) *Γ is proximal iff $S_\phi(X, x_0)=S_\phi(Y, y_0)$ for all $\phi\in A(I)$.*

Proof. (1) Let $\phi\in A(I)$ and $\alpha\in S_\phi(X, x_0)$. Then $y_0\phi(\alpha)=\Gamma(x_0)\phi(\alpha)=\Gamma(x_0\phi(\alpha))=\Gamma(x_0)=y_0$, which implies $\alpha\in S_\phi(Y, y_0)$.

(2) Suppose Γ is proximal and let $\phi\in A(I)$. For each $\alpha\in S_\phi(Y, y_0)$, $\Gamma(x_0\phi(\alpha))=\Gamma(x_0)\phi(\alpha)=y_0\phi(\alpha)=y_0=\Gamma(x_0)$. Hence $(x_0\phi(\alpha), x_0)\in P(X, x_0)$. But since $\phi(\alpha)\in K$, $(x_0\phi(\alpha), x_0)u=(x_0\phi(\alpha)u, x_0u)=(x_0\phi(\alpha), x_0)$. Therefore $x_0\phi(\alpha)=x_0$ and so $\alpha\in S_\phi(X, x_0)$. Thus $S_\phi(Y, y_0)\subset S_\phi(X, x_0)$. This means that $S_\phi(X, x_0)=S_\phi(Y, y_0)$.

Conversely suppose that $S_\phi(X, x_0)=S_\phi(Y, y_0)$ for all $\phi\in A(I)$. Let $y\in Y$ and let $x_1, x_2\in\Gamma^{-1}(y)$. Then there exist $p, q\in I$ such that $x_0p=x_1$ and $x_0q=x_2$. Denote $\alpha=q(pu)^{-1}$, then

$$y_0\alpha=\Gamma(x_0)q(pu)^{-1}=\Gamma(x_0q)(pu)^{-1}=\Gamma(x_2)(pu)^{-1}$$
$$=\Gamma(x_1)(pu)^{-1}=\Gamma(x_0p)(pu)^{-1}=\Gamma(x_0)u=y_0u=y_0.$$

Hence $\alpha\in G(Y, y_0)=S_1(Y, y_0)=S_1(X, x_0)=G(X, x_0)$, where 1 is the identity map on I. Thus

$$x_1u=(x_0p)u=x_0(pu)=x_0\alpha(pu)=x_0q(pu)^{-1}(pu)=x_0qu=x_2u,$$

which implies $(x_1, x_2)\in P(X, T)$. This means that Γ is proximal.

LEMMA 3.14. *$G(X, x_0)=G(Y, y_0)$ if and only if $S_\phi(X, x_0)=S_\phi(Y, y_0)$ for all $\phi\in A(I)$.*

Proof. Only if: Suppose $G(X, x_0)=G(Y, y_0)$ and let $\phi\in A(I)$. For each $\alpha\in S_\phi(Y, y_0)$, $\phi(\alpha)\in G(Y, y_0)=G(X, x_0)$. Since $\alpha\in S_\phi(X, x_0)$, it follows that $S_\phi(Y, y_0)\subset S_\phi(X, x_0)$. Hence $S_\phi(X, x_0)=S_\phi(Y, y_0)$.

If: Let $S_\phi(X, x_0)=S_\phi(Y, y_0)$ for all $\phi\in A(I)$. Then $G(X, x_0)=S_{Lu}(X, x_0)=S_{Lu}(Y, y_0)=G(Y, y_0)$, because L_u is the identity automorphism on I.

COROLLARY 3.15[4]. *Γ is proximal if and only if $G(X, x_0)=G(Y, y_0)$.*

Proof. By (2) of Theorem 3.13 and Lemma 3.14.

LEMMA 3.16. *Let (X, T) be a minimal set, and $(E(X), T)$ regular. Then the following are true.*

(1) $X=I/R$ for some closed T-invariant equivalence relation R on I, where I is the only minimal right ideal in $E(X)$.

(2) Suppose, for each $\tau\in A(I)$ there exists $r\in I$ such that $\tau=L_r$ and $rR=R$. Then (X, T) is regular. (By rR we mean the set of pairs (rq, rq'), where $(q, q')\in R$.)

Proof. (1) Since $(E(X), T)$ is regular, $E(X)$ contains exactly one minimal right ideal I. For each $x\in X$, the map $\theta_x: q\to xq$ of I onto X is an epimorphism. We define a relation on I by $(q_1, q_2)\in R$ if $\theta_x(q_1)=\theta_x(q_2)$. Then R is the closed T-invariant equivalence relation on I, and we may write $X=I/R$.

(2) Let $x, y\in X$. Then there exist $p\in I$ and $v\in J(I)$ such that $xp=y=yv$. Since (I, T) is regular minimal, there exists $r\in I$ and a net $\{t_\alpha\}$ in T such that $rR=R$ and $\lim L_r(p)t_\alpha=\lim vt_\alpha=w$ for some $w\in I$. Now we can define $\phi\in A(X)$ by $\phi(\theta_x(q))=\theta_x(L_r(q))$ for all

$q\in I$. Then

$$xw=x\lim L_r(p)t_\alpha=\lim x(L_r(p)t_\alpha)=\lim (xrp)t_\alpha=\lim \phi(xp)t_\alpha$$
$$=\lim \phi(yv)t_\alpha=\lim \phi(yvt_\alpha)=\phi(y\ \lim\ vt_\alpha)=\phi(yw)=\phi(y)w.$$

Hence $(\phi(y), x)\in P(X, T)$. This shows that (X, T) is regular.

THEOREM 3. 17. *Suppose that X is written as I/R, where R is a closed T-invariant equivalence relation on I and for each $\tau\in A(I)$ there exists $r\in I$ such that $\tau=L_r$ and $rR=R$. Let $P(X, x_0)$ be an equivalence relation on X. Then*

(1) There exists $\phi\in A(I)$ *such that* $\phi|G(X, x_0) : G(X, x_0)\to G(X, x_0)$ *and* $\phi|G(Y, y_0) : G(Y, y_0)\to G(Y, y_0)$ *are bijective, and* $S_\phi(X, x_0)=S_\phi(Y, y_0)$.

(2) $G(X, x_0)=G(Y, y_0)$.

(3) *Γ is a proximal homomorphism.*

(4) *If (X, x_0) is distal, then Γ is isomorphism, i. e., (X, x_0) is isomorphic to (Y, y_0).*

Proof. First we show that $G(Y, y_0)\subset S_\phi(X, x_0)$ for some $\phi\in A(I)$. For each $\alpha\in G(Y, y_0)$, $\Gamma(x_0\alpha)=y_0\alpha=y_0=\Gamma(x_0)$. By Lemma 2. 5 and (2) of Lemma 3. 16, (X, x_0) is regular. Hence there exists $f\in A(X)$ such that $(f(x_0\alpha), x_0)\in P(X, x_0)$. Since $E(X)$ contains exactly one minimal right ideal I, $f(x_0\alpha)q=x_0q$ for all $q\in I$. By Lemma 2. 4, f is induced by some $L_p\in A(I)$, where $p\in I$. Then $x_0=x_0u=f(x_0\alpha)u=x_0L_p(\alpha)u=x_0L_p(\alpha u)=x_0L_p(\alpha)$, which implies $\alpha\in S_{L_p}(X, x_0)$. Now let $\phi=L_p$. Then

$$G(X, x_0)\subset G(Y, y_0)\subset S_\phi(X, x_0)\subset S_\phi(Y, y_0).$$

By Corollary 3. 4, $\phi(G(X, x_0))\subset\phi(S_\phi(X, x_0))\subset G(X, x_0)$. As in the proof of Theorem 3. 6 we prove that $\phi|G(X, x_0) : G(X, x_0)\to G(X, x_0)$ is onto, i. e., $\phi(G(X, x_0))=G(X, x_0)$. Hence $\phi|G(X, x_0) : G(X, x_0)\to G(X, x_0)$ is bijective. Similarly, $\phi|G(Y, y_0) : G(Y, y_0)\to G(Y, y_0)$ is also bijective. But $G(Y, y_0)=\phi(G(Y, y_0))\subset\phi(S_\phi(X, x_0))\subset G(X, x_0)$, which implies $G(X, x_0)=G(Y, y_0)$. Thus $S_\phi(X, x_0)=S_\phi(Y, y_0)$ and so Γ is proximal. Finally we show that Γ is an isomorphism if (X, x_0) is distal. Let (X, x_0) be distal. Then $P(X, x_0)=\Delta$. Let $y\in Y$ and let $x_1, x_2\in\Gamma^{-1}(y)$. Since Γ is proximal, $(x_1, x_2)\in P(X, x_0)$, and $x_1=x_2$. This shows that Γ is one to one, and thus we obtain Γ is an isomorphism.

THEOREM 3.18. *For each* $\phi \in A(I)$, $S_\phi(X, x_0)$ *is a closed subset of* I, *and* $S_\phi(X, x_0)$ *is compact* T_2.

Proof. First we show that $G(X, x_0)$ is closed. Let $\{\alpha_n\}$ be a net in $G(X, x_0)$ such that α_n converges to α. Then $x_0\alpha = x_0 (\lim \alpha_n) = \lim x_0\alpha_n = x_0$, and so $\alpha \in G(X, x_0)$, which implies $G(X, x_0)$ is closed. Now let $\alpha \in A(I)$ and $\{\beta_n\}$ be a net in $S_\phi(X, x_0)$ such that β_n converges to β. Then $\lim \phi(\beta_n) = \phi(\beta)$, because ϕ is continuous. Since $G(X, x_0)$ is closed and $\phi(\beta_n) \in G(X, x_0)$, $\phi(\beta) \in G(X, x_0)$. This means that $\beta \in S_\phi(X, x_0)$. Thus $S_\phi(X, x_0)$ is also closed. It is clear that $S_\phi(X, x_0)$ is compact T_2.

References

1. J. Auslander, *Endomorphisms of minimal sets*, Duke Math. J. **30** (1963), 605–614.
2. ______, *Regular minimal sets I*, Trans. Amer. Math. Soc. **123** (1966), 469–479.
3. R. Ellis, *Lectures on topological dynamics*, W.A. Benjamin, Inc, 1969.
4. S. Glasner, *Proximal flows*, Springer Verlag, Berlin New York.
5. P. Shoenfeld, *Regular homomorphisms of minimal sets*, doctoral dissertation, University of Maryland, 1974.
6. M.H. Woo, *Regular transformation groups*, J. Korean Math. Soc. **15** (1979), 129–137.

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