Lateral Vibration and Elastic Stability of Rectangular Plates with Cutouts

개구부를 가진 직사각형 평판구조의 진동 및 안정성해석

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요 약

環境條件 및 內部의 有孔部 크기와 位置가 變化함에 따라 平板의 두개의 固有値(固有 振動數와 彈性 臨界荷重)는 어떻게 變化하며, 이들 固有値 相互間의 關係는 어떻게 되는지 檢討 하기로 한다.

以上과 같은 檢討 目的에는 數値 解析法이 적당한데 本 論文에서는 그중의 하나인 有限 要素法을 採擇하기로 한다. 有限 要素法에서 利用할 要素는 各 節點에서 自由度가 3, 따라서 要素 全體의 自由度가 12인 四角形 要素이다.

Abstract

Two perforated plates(a square plate and a rectangular plate having an aspect ratio 1.57(L_x=11, L_y=7)) are taken as analysis examples. Each of these plates is given some changes in the boundary conditions. The size of cutouts as well as their locations are also changed in order to examine the variation of two eigenvalues corresponding to the fundamental mode. The relationship between two eigenvalues is established by changing the magnitude of edge thrust.

INTRODUCTION

Two eigenvalues, the elastic critical load and the natural frequency of thin homogeneous rectangular plates, can be determined separately by classical methods, that is, by solving the governing differential equation or by substituting appropriate deflection functions into the total potential energy function. The relationship between cirtical load and natural frequency can also be established by classical methods. The classical methods are not always successful, however, even with the classical boundary conditions (i.e., ideally hinged or completely fixed). That is true, of course, with the

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plate having an inner cutout as shown in Fig. 3.

The difficulties in such cases can be solved by one of the numerical approaches; especially finite element analysis provides a powerful tool to attack the problems. L.G. Tham et al.[1] achieved some successful results of eigenvalue analysis of square plates with cutouts by using the negative stiffness method. The aim of this paper is to examine the variation of the eigenvalues and their relationship of the plates with cutouts and various edge conditons using the finite element method.

BRIEF REVIEW OF CLASSICAL METHODS

When the rectangular plate shown in Fig. 1 is in the form of uniform thin thickness and without cutouts, the following expression[2, 3] can be used to determine two eigenvalues

$$D\left(\frac{\partial^4 \omega}{\partial x^4}+2\frac{\partial^4 \omega}{\partial x^2 \partial y^2}+\frac{\partial^4 \omega}{\partial y^4}\right) = -\left(N_x\frac{\partial^2 \omega}{\partial x^2}+m\frac{\partial^2 \omega}{\partial t^2}\right) \qquad (1)$$

Where

D=flexural rigidity of the plate=$Eh^3/12(1-\mu^2)$

μ=Poisson's ratio(in this paper μ=0.3)

ω=displacement in the z-direction

m=mass per unit area of the plate.

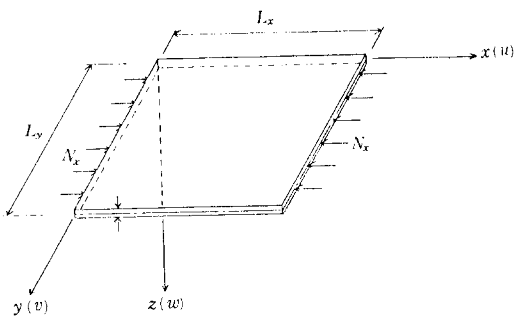


Fig. 1. Rectangular Plate under Edge Thrust without Cutouts

If one wants to apply the energy principle for determining the eigenvalues of the above plate, the following expression[4,5] is usually adopted;

$$\Pi=\int_o^b\int_o^a\left\{\frac{D}{2}\left[\left(\frac{\partial^2 \omega}{\partial x^2}\right)^2+2\mu\frac{\partial^2 \omega}{\partial x^2}\cdot\frac{\partial^2 \omega}{\partial y^2}+\left(\frac{\partial^2 \omega}{\partial y^2}\right)^2+2(1-\mu)\left(\frac{\partial^2 \omega}{\partial x \partial y}\right)^2\right]-\frac{N_x}{2}\left(\frac{\partial \omega}{\partial x}\right)^2-\frac{m}{2}\left(\frac{\partial \omega}{\partial t}\right)^2\right\}dxdy \qquad (2)$$

The static critical load N_{cr} or the natural frequency (without edge thrust) ω_n can be obtained from Eq.(1) or Eq.(2) by omitting the $\partial\omega/\partial t$ term or the $\partial\omega/\partial t$ term respectively. Some of the results can be found in Refs.[6-9].

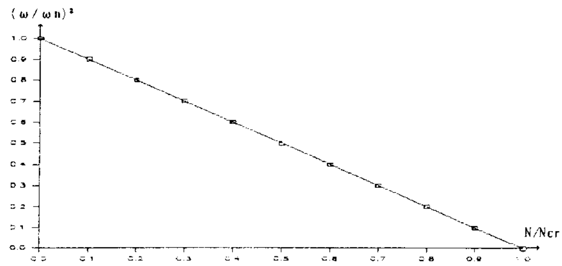


Fig. 2. Variation of Frequencies with Edge Thrust of the Simply Supported Plate

When the natural frequency ω under edge thrust N is required, Eq. (1) or Eq.(2) is effective without omitting the $\partial\omega/\partial t$ or the $\partial\omega/\partial x$ term respectively. When the four edges of plates are simply supported, the relationship between the two eigenvalues is easily obtained. Fig. 2 shows their relationship[10], which indicates that the square of the frequency of lateral vibration is exactly linearly related to the edge thrust intersecting each axis at unit value. It is not difficult for the simply supported beam-column to prove this linearity[11]. The first author[12] showed that approximate linearity also holds for the other beam-column with various end conditions (see Fig. 9 below).

As mentioned above, the authors' primary aim is to establish the relationship of the two fundamental eigenvalues of the plate shown in Fig. 3. In other words, the plates with varying edge conditions will be examined to see whether or not the same linearity shown Fig. 2 holds. The locations and shapes of holes are to be examined for the influence on the relationship between two eigenvalues.

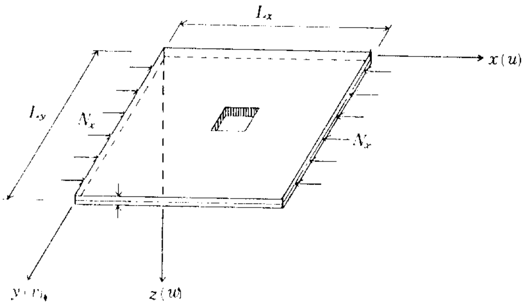
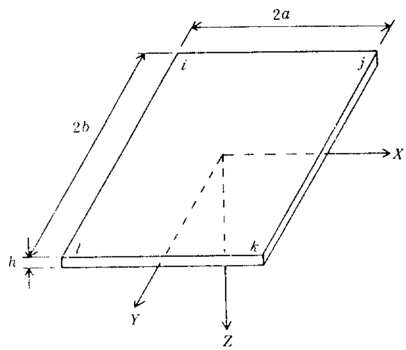
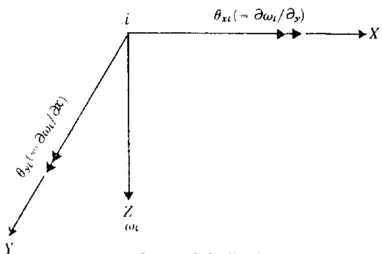


Fig. 3. Rectangular Plate with a Cutout



(a) plate dimension



(b) nodal displacement

Fig. 4. Rectangular Element

NUMERICAL METHOD

The finite element used here is a rectangular element with corner nodes. The typical element chosen in this work is shown in Fig. 4. The degrees of freedom at any node i are deflection w_i and rotations $\theta_{xi}(=\partial \omega_i/\partial y)$ and $\theta_{yi}(=\partial \omega_i/\partial x)$. Therefore, the element has a total of 12 degrees of freedom.

The coefficients of the Eq.(3), A_1, $A_2 \cdots$, A_{12} can be represented by 12 degrees of freedom of the element.

$$W=A_1+A_2x+A_3y+A_4x^2+A_5xy+A_6y^2+A_7x^3 +A_8x^2y+A_9xy^2+A_{10}y^3+A_{11}x^3y+A_{12}xy^2 \quad (3)$$

Representing A_i by the nodal displacements, Eq.(3) takes the following form

$$W=[\overline{N}]\{\delta_i\} \quad (4)$$

Where $[\overline{N}]$ is the shape function and $\{\delta_i\}$ is the diplacement vector of an element. Information about the shape function $[\overline{N}]$ and the procedures how to get the element flexural stiffness matrix $[k_B]$ the geometric stiffness matrix $[k_G]$ and finally the consistent mass matrix $[m_c]$ can be found in several references [13-16]. So these procedures are completely omitted here.

Once the stiffness and mass matrices of an element are known, they can be assembled for the whole plate. After the boundary conditions being considered, the final form of the finite element equation for the eigenvalue analysis takes the following form

$$]\}\{\triangle\}=\omega^2[M_c]\{\triangle\} \quad (5)$$

wher $[K_B]$ and $[K_G]$ are assembled flexural and geometric stiffness matrix respectively, $[M_c]$ is assembled consistent mass matrix and $\{\triangle\}$ is the displacement vector of the whole plate. By letting

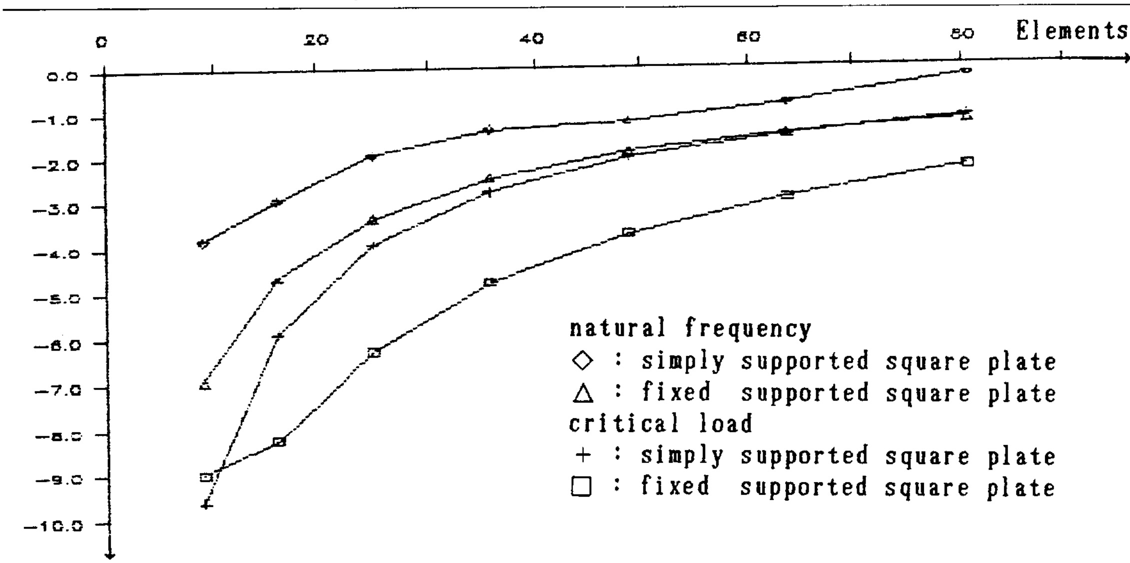


Fig. 5. Number of Elements vs Errors(%)

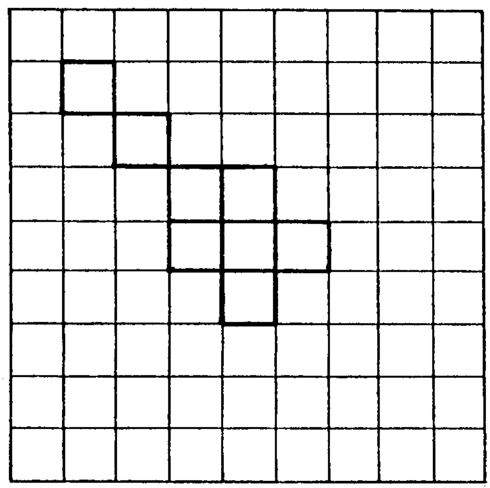
$[M_C]=[0]$ in Eq.(5), one gets the static critical load. With $N_x=0$, similarly one gets the natural frequency of a plate.

The solution procedures (or solution algorithms) are well described in the several texts[17−19] as are the element stiffness or the mass matrix.

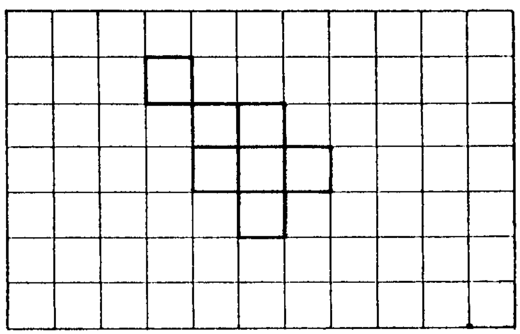
NUMERICAL EXAMPLES

It is generally known that the element division is a very important factor in the numerical analysis. Fig. 5 shows the reducing errors with increasing number of elements. One can see that the natural frequency (without edge thrust) ω_n converges more rapidly to the exact value than the static critical load N_{cr} does. Fig 5 also shows a common trend that the results of the plate with the simply supported edges are more convergent than those of the plate with fixed edges.

Because of computing time and CPU capacity, the square plate is divided by 9×9=81 elements and the rectangular plate by 7×11=77 elements. In both plates the element form is square. Fig. 6 shows the element divisions of each plate.



(a) Square Plate



(b) Rectangular Plate

Fig. 6. Element Divisions and Locations of the Cutouts

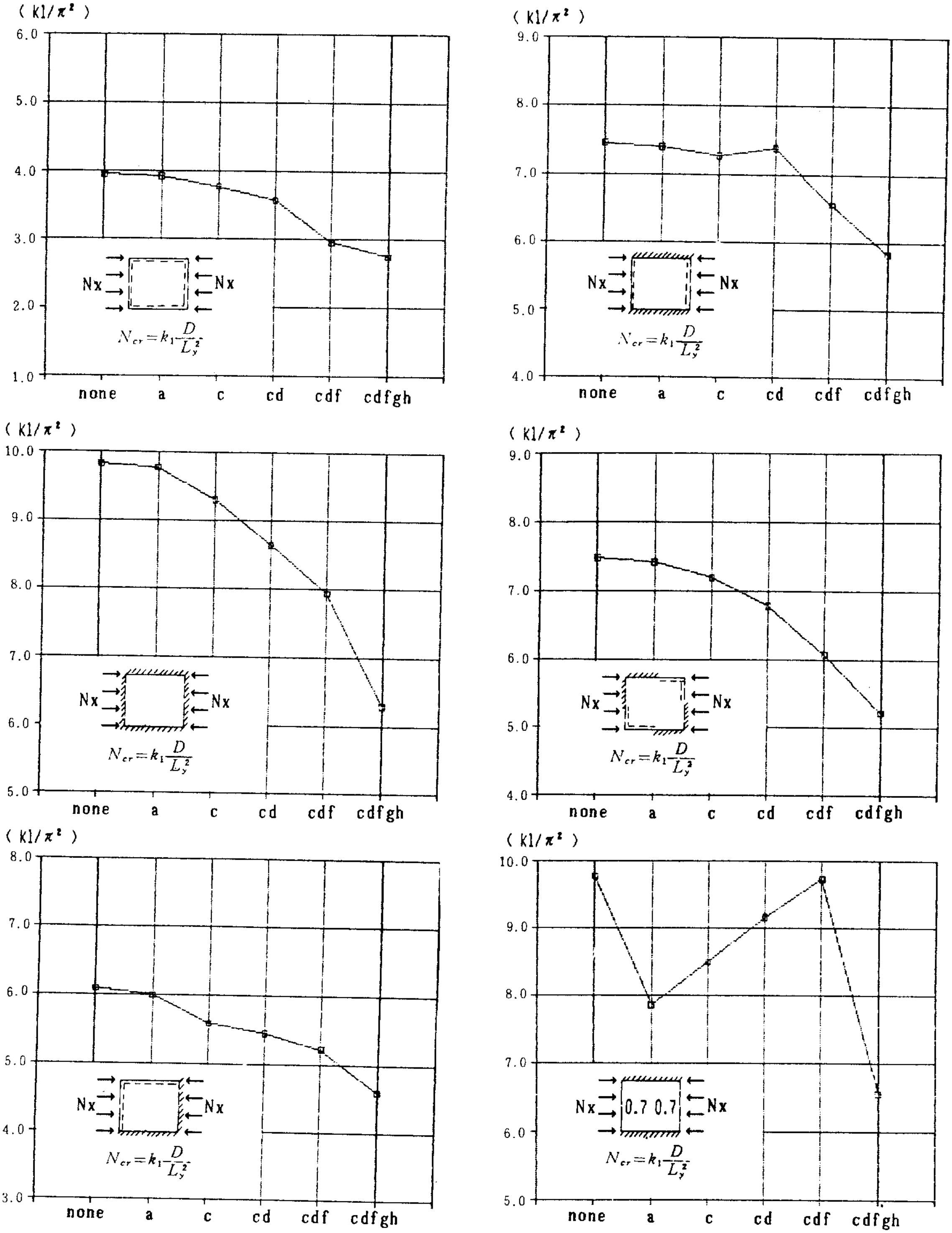


Fig. 7(a). Variation of Static Critical Loads of the Square Plate

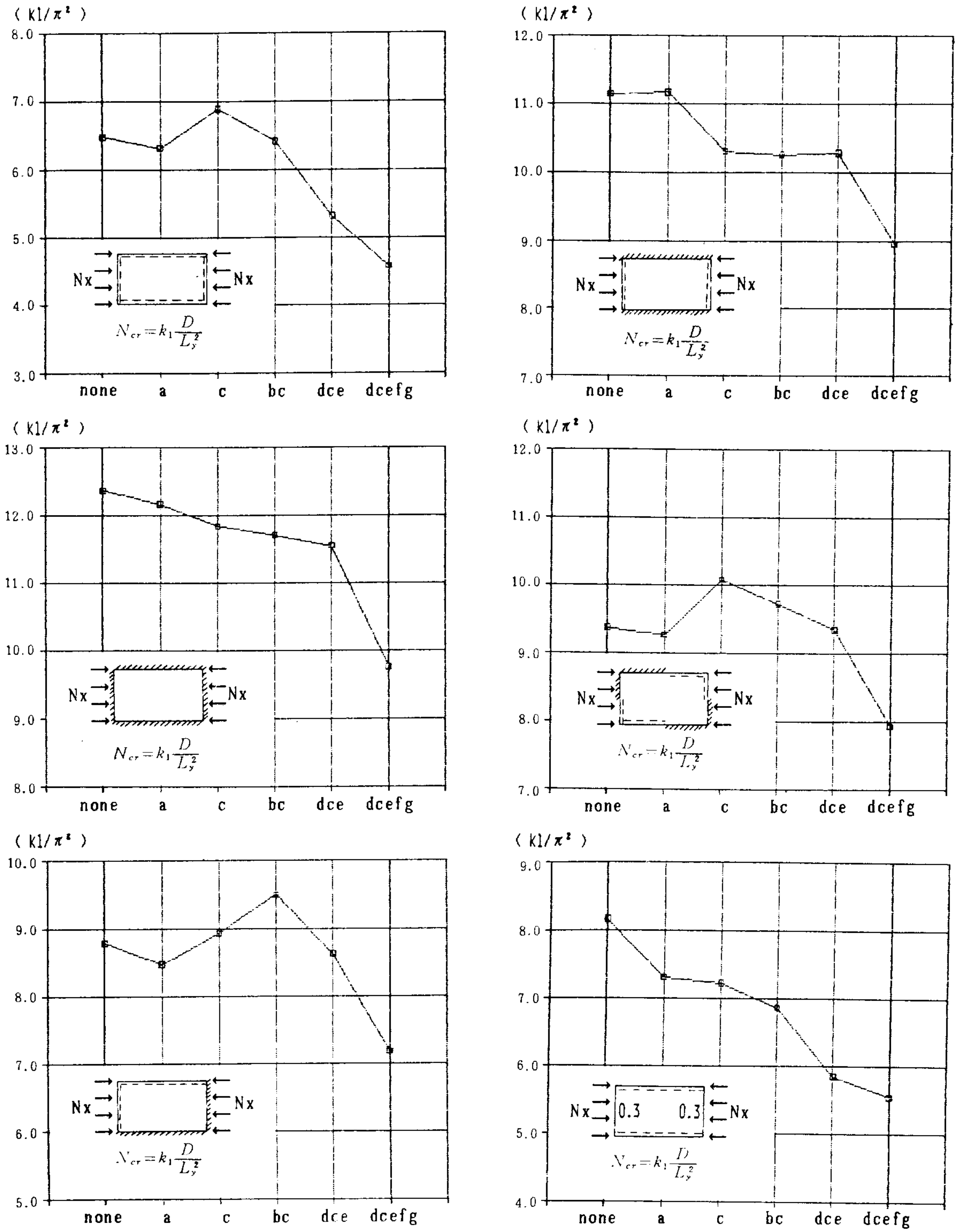


Fig. 7(b). Variation of Static Critical Loads of the Rectangular Plate

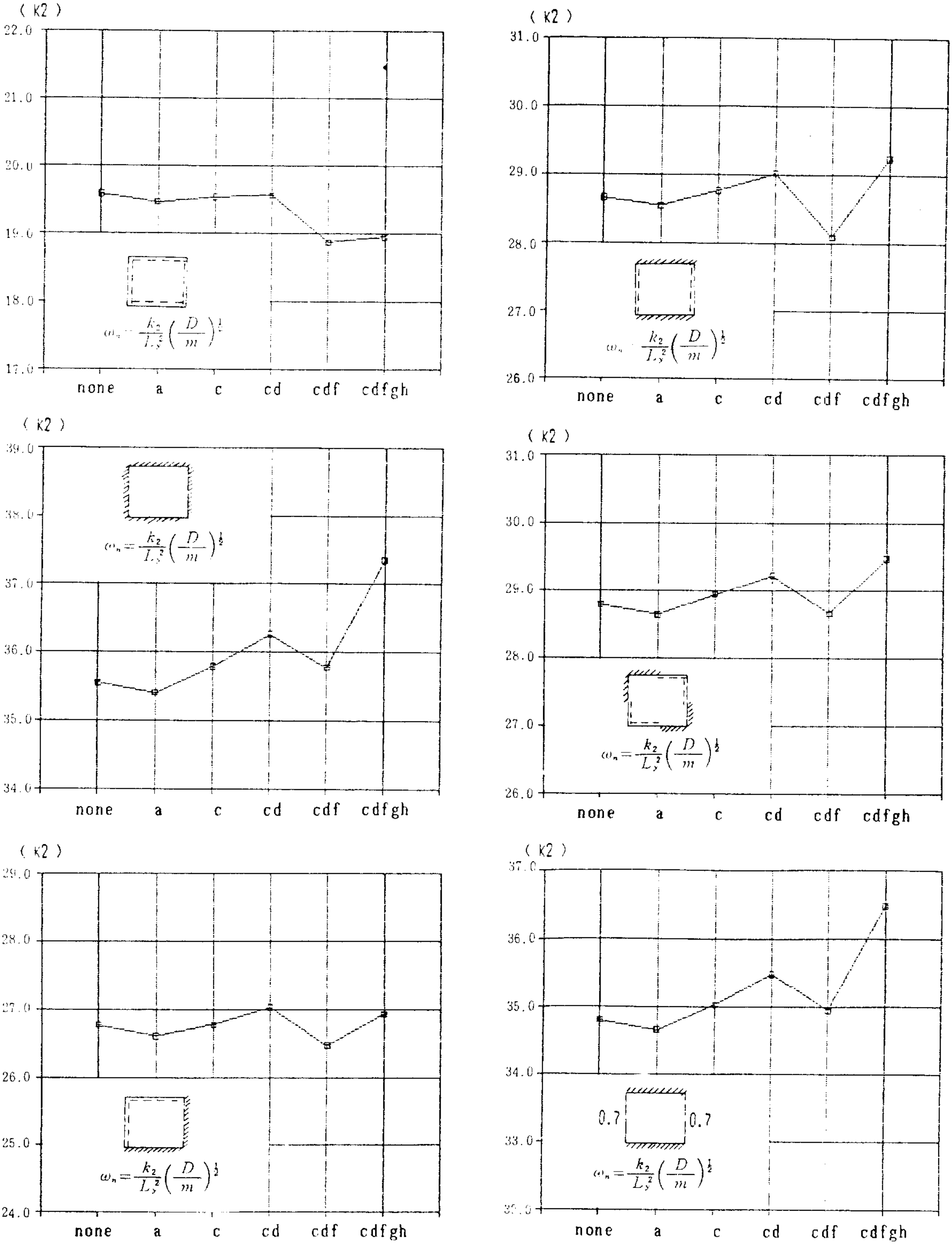


Fig. 8(a). Variation of Natural Frequencies of the Square Plate

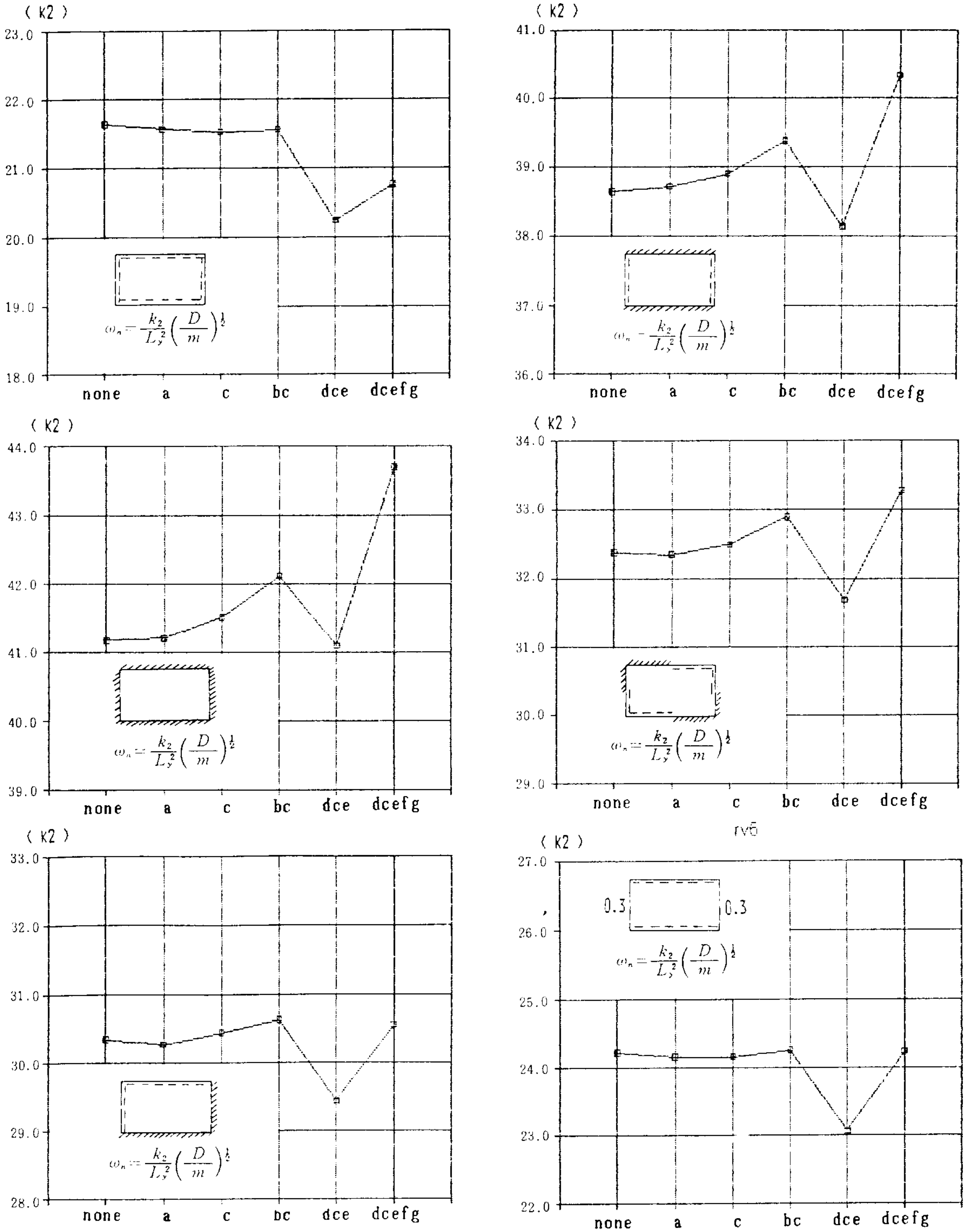


Fig. 8(b). Variation of Natural Frequencies of the Rectangular Plate

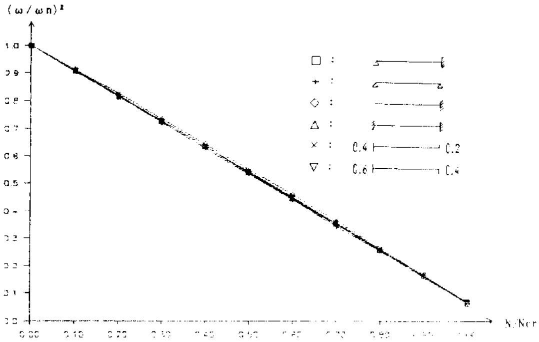


Fig. 9. Variation of Frequencies with End Load of Prismatic Bar

The cutout is made to change its location and size as shown in Fig. 6 to check its influence on the resulting eigenvalues. In this paper, a hole is assumed to be formed when the thickness of the plate at that location is relatively very thin(1/1000) compared to that of the other part of the plate.

Two eigenvalues corresponding to fundamental mode, the static critical load N_{cr}, and the natural frequency (without edge thrust) ω_n are given by Fig. 7 and Fig. 8 respectively. k_1 in Fig. 7 and k_2 in Fig. 8 stand for the coefficients of following expressions.

$$N_{cr}=k_1\frac{D}{L_y^2} \qquad \omega_n=\frac{k_2}{L_{y2}}\left(\frac{D}{m}\right)^{1/2}$$

In Fig. 7 and 8 "none" means that there is no cutout in the plate and "a" means there is only one square hole in the palte at the site indicated by Fig. 6. Similarly, "edf" means a rectangular opening at the central part of the palte.

The expression f=0.7 or f=0.3 in Fig. 7 or Fig. 8 denotes Kinney's[20] edge fixity f. In other words, the plate is partially fixed at that edge. For example, the plate edge with f=0.3 will have end moment which is 0.3 times the end moment which would exist if the edge were completely fixed. At this time the plate edge will rotate 0.7 (=1−0.3) times the end rotation which would occur if the edge were the ideal hinge. Thus "f=0.0" means an ideal frictionless edge, while "f=1.0" corresponds to the completely fixed edge. Works similar to Fig. 7 and Fig. 8 are found in Refs.[21−25].

On the basis of the results of Fig. 7 and Fig. 8 the relationship between two eigenvalues is established by calculating the corresponding values of ω/ω_n through changing N/N_{cr} values. The calculations are depicted in Fig. 10 and Fig. 11. In Fig. 10 and Fig. 11, "+a" is a square hole at the left corner and "* dce" indicates a rectangular opening at the center part of the rectangular plate as is seen in Fig. 6.

RESULTS AND DISCUSSIONS

Eigenvalues of thin perforated plates are evaluated. The perforated plates are confined to a square plate and a rectangular one with aspect ratio 1.57(L_x=11, L_y=7). To check the influence of perforation on eigenvalues, some changes of the size of the cutouts and their locations are made in each plate.

The coefficient k_1, static critical load N_{cr}, has two general trends. First, k_1 decreases when the size of cutouts increases without relation to boundary conditions. Second, it also decreases when the distance of cutouts from the center of the plate decreases with some exceptions. The coefficient k_2, natural frequency ω_n, is influenced by boundary conditions. That is, in the case of simply supported square plates, k_2 shows similar phenomena to k_1. While in the case of fixed square plates, it shows the opposite phenomena to k_1.

The variation of natural frequencies of each plate is examined by changing the magnitude of edge thrust, which reveals the relationship between two eigenvalues. In the case of square plates,

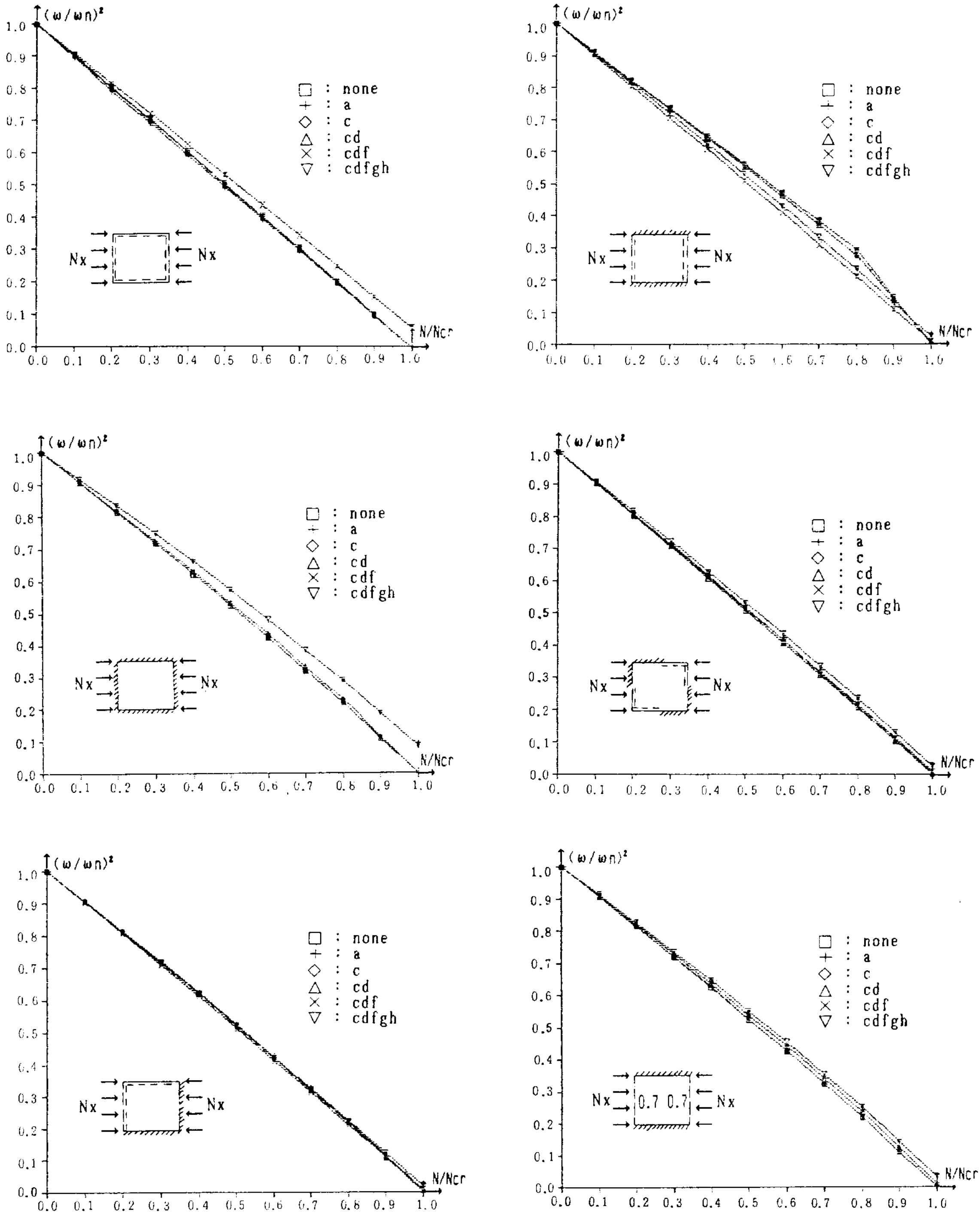


Fig. 10. Variation of Frequencies with End Load of Square Plate

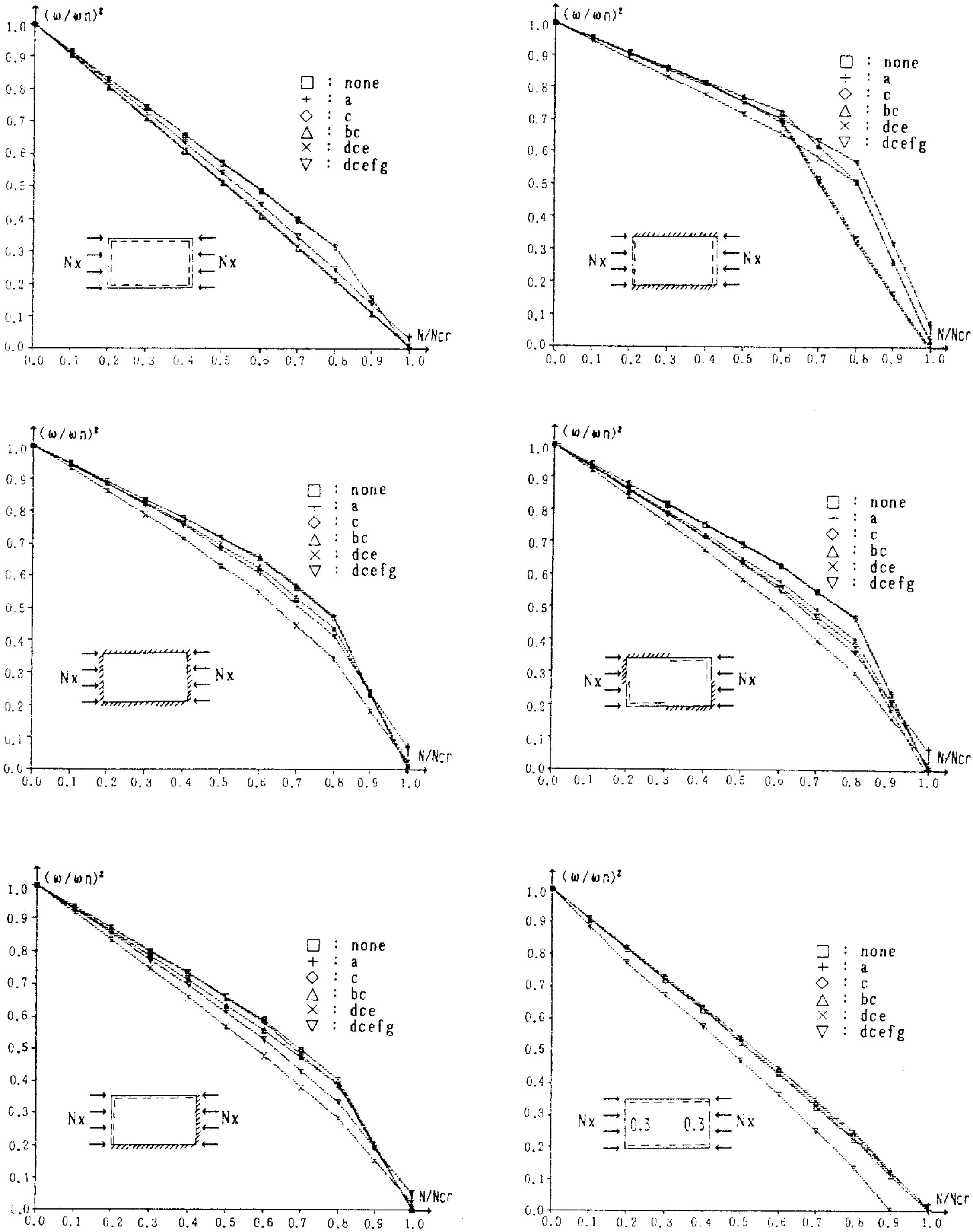


Fig. 11. Variation of Frequencies with End Load of Rectangular Plate

the square of the frequency of lateral vibration is approximately linearly related to the end load without regard to the edge supporting conditions and cutout variations (its sizes and locations). In the case of rectangular plate, however, the relationship between the two eigenvalues shows some deviation from linearity, except the simply supported plates. The relationship for rectangular plates with other boundary conditions tends to be parabolic.

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