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Existence theorems for slices and their applications*

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1. Introduction

One of the most interesting things in transformation group (X, G, Φ) is the existence of slices which is a nice local property of the space X.

Gleason ([5]) showed that there exists a local cross section (which is closely related notion to slice) for the orbit map $X \longrightarrow X/G$ if X is completely regular G-space with only one orbit type and G is a compact Lie group.

Koszul ([7]) proved the existence of slices under a differentiable action without the restriction on the orbit type.

Montgomery and Yang ([10]) extended it to a non-differentiable action. Some useful generalizations were given by Mostow ([12]) who proved it with the condition completely regular G-space X where G is compact Lie group. And Palais ([13]) generalized Mostow's theorem to a non-compact Lie group space which is called a Cartan space.

The purpose of this paper is to prove Theorem 2.7 which is an application of the Mostow's theorem and to generalize it to the case of non-compact Lie group space by using the Palais' theorem (Corollarly 3.9). Also we are looking for some general conditions for which an orbit map is a fibration (Proposition 2.4, Theorem 3.8, Theorem 4.3).

In more details, in §1, we write down the general notations which will be used in the following sections. In §2, we sketch the proof of the Mostow's theorem for the existence of slice and prove Theorem 2.7. In §3, we sketch the proof of the Palais' theorem and use it to generalize Theorem 2.7 and Proposition 2.4. In §4, we review the examples which were given by King to show that there is a slice for non

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Cartan G-space and we get a generalized conditions for the fibration of the orbit map $\pi: X \longrightarrow X/G$ of a G-space X (Theorem 4.3, Corollary 4.4).

Throughout this paper G will denote a topological group with identity e and H will denote a closed subgroup of G. By a G-space X we mean a left transformation group (X, G, Φ) where X is a Hausdorff space and Φ is an action on X. If X is a G-space, for each $(g, x) \in G \times X$, $\Phi(g, x)$ will be denoted by gx. For each point x of a G-space X $Gx = \{g \in G \mid gx = x\}$ is the isotropy group of x and $G(x) = \{gx \mid g \in G\}$ is the G-orbit of $x \in X$. If for each $x \in X$ $Gx = e$, then the action is called free. If X is a G-space and K is a subgroup of G

$$X^K = \{x \in X \mid kx = x \text{ for all } k \in K\}.$$

is the set of points fixed by K. For a G-space X and for a subset $A \subset X$, $G(A) = \{ga \mid g \in G,\ a \in A)$. The orbit space of a G-space X will be denoted by X/G which has the quotient topology under the orbit map $\pi: X \longrightarrow X/G$. An equivariant map between two G-spaces X and Y is a map $f: X \longrightarrow Y$ satisfying $f(gx) = gf(x)$ for each $g \in G$ and $x \in X$. An equivariant map $f: X \longrightarrow Y$ which is also a homeomorphism is called an equivalent map.

2. Applications of slice theorem under compact Lie group action

Definition 2.1. A subset S of a G-space X will be called an H-kernel (over $\pi(S)$) if

(1) S is closed in GS

(2) $HS = S$

(3) $(gS) \cap S \neq \phi \Longleftrightarrow g \in H$

An H-kernel S in X will be called an H-slice in X if GS is open. By a slice at $x \in X$ we mean a Gx-slice in X which contains x. If there is a slice for each point of a G-space X we say that there exists a slice in X.

Let A be an H-space. Then the twisted product $G \times_H A$ is the fiber bundle with fiber A over G/H associated to the principal H-bundle $G \longrightarrow G/H$ ([3]).

Proposition 2.2. Let $y \in S \subset X$. Then the following are equivalent.

(a) (1) $G_y S = S$.

(2) The map $\varphi: G\times_{G_y} S \longrightarrow X$

given by $\varphi([g,s])=gs$ is a G-equivalent map, onto an open neighborhood of $G(y)$ in X.

(b) S is a slice at y.

(c) $G(S)$ is an open nbd of $G(y)$ and there is an equivariant retraction

$$f: G(S) \longrightarrow G(y)$$

such that $f^{-1}(y)=S$.

Proof. (a)$\Longrightarrow$(b) It's enough to show that (3) of Definition 2.1 is satisfied. Let $gS\cap S\neq\phi$, then $gs=s_1\in S$ for some $s, s_1\in S$.

Since $s=\varphi([e,s])$ and $s_1=\varphi([e,s_1])$, $gs=g\,\varphi([e,s])=\varphi([g,s])$

$$\begin{array}{c} gs \\ \parallel \\ s_1=\varphi([e,s_1]) \end{array}$$

Thus $[g,s]=[e,s_1]$ and so there exists $h\in G_y$ such that $gh^{-1}=e$, $hs=s_1$. Therefore $g=h\in G_y$.

(b)$\Longrightarrow$(c)
(c)$\Longrightarrow$(a) } See ([3]).

Now let G be a compact Lie group. If X is a differentiable manifold and the G-action on X is differentiable, then there exists a slice in X ([7] [10]).

We can generalize this to a non-differentiable actions as the following.

Proposition 2.3. Let G be a compact Lie group and let X be a completely regular G-space. Then there exists a slice in X.

Proof. Let $x\in X$. Since G is a comact Lie group there is an orthogonal representation $\rho: G\longrightarrow O(n)$ and a point $v_0\in R^n$(=Euclidean n-space) with $G_{v_0}=G_{x_0}=H$, where G-action on R^n is the $\rho(G)$ action ([3]).

So the composition $\varphi: G(x_o)\longrightarrow G/Gx_o=G/Gv_o\longrightarrow G(v_o)$ is an equivalent map. (Note that the natural map $\alpha: G/Gx_o\longrightarrow G(x_o)$ $(gGx_o\longrightarrow gx_o)$ is equivalent because G/Gx_o is compact).

Since $G(x_o)$ is compact and X is completely regular, by the Tietze-Gleason theorem ([2], [14], [5]) there is an equivariant extension

$$\psi: X\longrightarrow R^n$$

of φ. Since the $O(n)$ action on R^n is differentiable there exists a slice at v_o. By Proposition 2.2 there is an equivariant retraction

$$f: G(U) \longrightarrow G(v_o)$$

such that $f^{-1}(v_o) = U$.

Put $\psi^{-1}(U) = S$. Then $G\psi^{-1}(U) = \psi^{-1}(GU)$ is open in X and the composition

$$GS \xrightarrow{\psi} G(U) \xrightarrow{f} G(v_o) \xrightarrow{\varphi^{-1}} G(x_o)$$
$$gs \longrightarrow \psi(gs) = g\,\psi(s) \longrightarrow gv_o \longrightarrow gx_o$$

$\eta = \varphi^{-1} \circ f \circ \psi$ is an equivariant retraction, where ψ and f are the restrictions on the above domains of them respectively.

Also

$$\begin{aligned} \eta^{-1}(x_o) &= (\varphi^{-1} \circ f \circ \psi)^{-1}\ (x_o) \\ &= (\psi^{-1} \circ f^{-1} \circ \varphi)\ (x_o) \\ &= (\psi^{-1} \circ f^{-1})\ (v_o) \\ &= \psi^{-1}(U) = S. \end{aligned}$$

So S is a slice at x_o by (c) of Proposition 2.2.

If Y is a G-orbit, type (Y) will denote its equivalence class under equivalent map. So if H is conjugate to the isotropy group Gx at a point x of a G-space X, then it is clearly

$$\text{type } (G(x)) = \text{type } (G/G_x) = \text{type } (G/H).$$

For a G-space X we put type $(G, X) = \{\text{type } (G(x)) \mid x \in X\}$. And we say that X has $|\text{type } (G, X)|$ orbit type if it is finite.

Proposition 2.4. Suppose X is a completely regular G-space, G a compact Lie group. Let X have only one orbit type G/H. Then the orbit map $\pi: X \longrightarrow X/G$ is the projection in a fiber bundle with fiber G/H and structure group $N(H)/H$, where $N(H)$ is the normalizer of H in G.

Proof. Let $x_o^* = G(x_o) \in X/G (x_o \in X)$. By Proposition 2.3 there exists a slice S at x_o. By Proposition 2.2 there is an equivalent embedding

$$\varphi: G \times_{G_{x_o}} S \longrightarrow X$$

such that $Im\varphi = GS$ is an open neighborhood of $G(x_o)$ in X.

Since Gx_o is conjugate to H there is a G-equivalent map

$$\psi: G\times_H A \longrightarrow G\times_{G_{x_o}} S,$$

where A is homeomorphic to S and the H action on A is the pull-back action from Gx_o action on S by the conjugation.

Since $G_{[e,a]}=Ha\subset H$ $(a\in A)$ and $G_{[e,a]}$ is conjugate to H, $Ha=H$ (Note that $A\longrightarrow G\times_H A(a\mapsto[e,a])$ is an H-embedding and G is compact). Thus H acts trivially on A. So $G\times_H A\cong G/H\times A$ (equivalent).

Also wo know that

$$A\approx A/H\approx(G\times_H A)/G\approx(G\times_{G_{x_o}} S)/G\approx GS/G.$$

Identify A with its homeomerphic image GS/G, then we have a coordinate chart

$$\begin{array}{ccc} \varphi_A: G/H\times A & \xrightarrow{\approx} & \pi^{-1}(A)=GS \\ & \searrow \quad \swarrow{\scriptstyle\pi} & \\ & A & \end{array}$$

over $A\ni x_o^*$.

Let $\varphi_B: G/H\times B\longrightarrow \pi^{-1}(B)$ be a coordinate chart over $B\ni y_o^*\in X/G$. If $A\cap B\neq\phi$,

$$\begin{array}{ccc} \varphi_B^{-1}\circ\varphi_A: G/H\times(A\cap B) & \longrightarrow & G/H\times(A\cap B) \\ & \searrow \quad \swarrow & \\ & A\cap B & \end{array}$$

gives a map $\theta: A\cap B\longrightarrow \mathrm{Homeo}^G(G/H)$. Since G is compact $\mathrm{Homeo}^G(G/H)\approx N(H)/H$ ([3]).

A toral group is a compact, connected, abelian Lie group. Since every connected abelian Lie group is isomorphic to $T^k\times R^{n-k}$ for some n, k (where T is the circle group and R is the real group), toral group is a product of circle groups except for the trivial torus. A maximal torus $T\subset G$ is a torus such that if $T\subset H\subset G$ and H is torus then $T=H$. If T is a maximal torus of H, $N(T)$ will denote the normalizer of T in G. The following result is well known.

Proposition 2.5. Any two maximal tori of a compact Lie group are conjugate ([1], [8]).

Proposition 2.6. Let G be a compact Lie group and let T be a maximal torus of H. Then $(G/H)^T=N(T)/H\cap N(T)$.

Proof. Let $gH \in (G/H)^T$ and so $TgH = gH$. Then $g^{-1}Tg \subset H$ and $g^{-1}\, Tg$ is also a maximal torus of H. By Proposition 2.5 there exists $h \in H$ such that

$$h^{-1}(g^{-1}Tg)h = T.$$

Since $h^{-1}(g^{-1}Tg)h = (gh)^{-1}\, T(gh)$, $gh \in N(T)$. Thus $g \in N(T)H$ and $gH \in N(T)H/H$. Conversely, if $nH \in N(T)H/H(n \in N(T))$, then $TnH = nTH = nH$ and so $nH \in (G/H)^T$. Therefore we proved that

$$(G/H)^T \cong N(T)H/H \cong N(T)/H \cap N(T).$$

Theorem 2.7. Let X be a completely regular G-space, G a compact Lie group and let X have only one orbit type G/H. Let T be a maximal torus of H. Then

(1) $G \times_N X^T \longrightarrow X([g, x] \longrightarrow gx)$ is the projection of a fiber bundle with fiber $H/N \cap H$, where $N = N(T)$.

(2) The cannonical map

$$\mu : X^T/N \longrightarrow X/G$$

is a homeomorphism.

Proof. (1) Let $x \in X$ and let S be a slice at x. Since Gx is conjugate to H, as in the proof of Proposition 2.4, there is an equivarient embedding

$$\eta :\ G/H \times A \longrightarrow X(A \text{ is homeomorphic to } S)$$

such that Im $\eta = GS$. Put

$$p : G \times_N X^T \longrightarrow X.$$

Then

$$\begin{aligned} p^{-1}(GS) &\cong G \times_N ((GS)^T) \\ &\cong G \times_N ((G/H \times A)^T) \\ &\cong G \times_N ((G/H)^T \times A) \ (T \text{ is trivial on } A) \\ &\cong (G \times_N (G/H)^T) \times A \\ &\cong (G \times_N (N/H \cap N)) \times A \ (\text{by Prop. } 2.6) \\ &\cong (G/H \cap N) \times A \end{aligned}$$

Since $q : G/H \cap N \longrightarrow G/H$ is a fiber bundle with fiber $H/H \cap N$, there exists a neighborhood $U \ni H$ such that

$$H/H\cap N\times U\cong q^{-1}(U)$$
$$((G/H\cap N)\times A\xrightarrow[(q,i)]{}G/H\times A\xrightarrow{\eta}X).$$

Let $V=\bigcup_{x^*\in U}x^*$. Then

$$\begin{aligned}p^{-1}(VS)&\cong((H/H\cap N)\times U)\times A\\ &\cong(H/H\cap N)\times(U\times A)\\ &\cong(H/H\cap N)\times VS.\end{aligned}$$

(2) μ is the map defined by $\mu(N(x))=G(x)$ for each $x\in X^T$.
for each $[g,x]\in G\times_N X^T$,

$$\pi p([g,x])=\pi(gx)=G(x)$$

and so we have the following commutative diagram.

$$\begin{array}{ccc} G\times_N X^T & \xrightarrow{\ p\ } & X\\ {\scriptstyle /G}\downarrow & & \downarrow{\scriptstyle \pi}\\ X^T/N & \xrightarrow{\ \mu\ } & X/G\end{array}$$

Since π and p are surjective, μ is surjective. Let $x,y\in X^T$ and let $G(x)=G(y)$. Then $y=gx$ for some $g\in G$. Since $x,y\in X^T$, $T\subset G_x$ and $T\subset gG_xg^{-1}$. Since T is a maximal torus of H and Gx is conjugate to H, T and $g^{-1}Tg$ are maximal torus of Gx. As in the proof of Proposition 2.6, there exists $\alpha\in Gx$ such that $g\alpha\in N$. Let $g\alpha=n\in N$. then $g=n\alpha^{-1}$ and $N(y)=N(gx)=N(n\alpha^{-1}x)=N(nx)=N(x)$. Thus μ is injective.

To show that μ is a homeomorphism, it is enough to show that μ is a closed map. Since G is compact, π is a closed map. So it's enough to prove that p is a closed map. By the following commutative diagram

$$\begin{array}{ccc} G\times X^T & \xrightarrow{\ \Phi\ } & X\\ & \searrow & \uparrow{\scriptstyle p}\\ & & G\times_N X^T\end{array}$$

it suffices to show that Φ is a closed map.

Since X^T is clearly closed in X, the closedness of Φ follows from the following Lemma.

Lemma 2.8. If $\Phi: G\times X \longrightarrow X$ is an action of a compact group G on X, then Φ is a closed map ([3]).

3. Palais' generalization and its application.

Now let G be a locally compact group with identity e. Let $((U,V))=\{g\in G \mid gU\cap V\neq\phi\}$ where U and V are subsets of a G-space X.

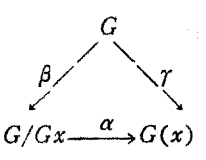
Definition. 3.1 A G-space X is a Cartan G-space if for each $x\in X$ there exists a neighborhood $U\ni x$ such that $Cl\ ((U,U))$ is compact in G, where Cl means closure in G.

Proposition 3.2. If X is a Cartan G-space then each orbit of X is closed in X and each isotropy group of X is compact.

Proof. Let $x\in X$ and let $U\ni x$ be a neighborhood such that $Cl\ ((U,U))$ is compact. Since Gx is closed in G and $Gx\subset((U,U))$, Gx is compact. Let $y\in ClGx$ and choose $V\ni y$ so that $Cl\ ((V,V))$ is compact. Let $\{g_\alpha x\}$ be a net in V converging to y. Fixing β, $(g_\alpha g_\beta^{-1})(g_\beta x)=g_\alpha x$. Thus $g_\alpha g_\beta^{-1}\in((V,V))\subset Cl((V,V))$ and so we can assume that $\{g_\alpha\}$ converges to a point $g\in G$. Therefore $y=\lim g_\alpha x=gx\in Gx$ and so Gx is closed.

Proposition 3.3. If X is a Cartan G-space and $x\in X$ then the map $\alpha: G/G_x\longrightarrow G(x)$ $(gGx\mapsto gx)$ is an equivalent.

Proof. Consider the following commutative diagram.



So it is enough to show that γ is open, where β is the projection and $\gamma(g)=gx$ for each $g\in G$. It suffices to show that if K is a neighborhood of e in G, then $K(x)$ is a neighborhood of x in $G(x)$. If $K(x)$ is not a neighborhood of x, then there is a net $\{g_\alpha\}\subset G$ such that $g_\alpha x\notin K(x)$ and $g_\alpha x\longrightarrow x$. Since $g_\alpha x\in K$ if and only if $g_\alpha\in KGx$, it follows that $g_\alpha\notin KGx$.

Since KGx is a neighborhood of Gx, no subset of $\{g_\alpha\}$ can converge to an element of Gx. Let U be a neighborhood of x such that $Cl\ ((U,U))$ is compact. Since $g_\alpha x$ is eventually in U, by passing to a subnet we can suppose that $\{g_\alpha\}\subset((U,U))$ and so again by passing to a subnet we can suppose that $g_\alpha \longrightarrow g$. But then $gx=\lim g_\alpha x=x$ and so $g\in Gx$ a contradiction.

Definition 3.4. A G-space X is proper if each point $x\in X$ has a neighborhood N such that for each $y\in X$ there exists a neighborhood $V\ni y$ so that $Cl\ ((N,V))$ is compact.

A Lie group is of type S if there is a slice in every proper G-space.

Proposition 3.5. Every Lie group is of type S ([13]).

Proposition 3.6. Let G be a Lie group and let X be a G-space. Then the following two conditions are equivalent.

(1) For each $x\in X$, Gx is compact and there is a slice at x.

(2) X is a Cartan G-space.

Proof. (1)$\Longrightarrow$(2)

Let $x\in X$ and let S be a slice at x. Then, clearly $Cl\ ((S,S))=Gx$. Since Gx is compact, X is a Cartan G-space. Before proving (2)$\Longrightarrow$(1) we have to prove the following Lemma.

Lemma 3.7. Let U be a neighborhood of x in X such that $Cl\ ((U,U))$ is compact. Then for each $\gamma,\delta\in G$, $Cl\ ((\gamma U,\ \delta U))$ is compact.

Proof.
$$\begin{aligned} g\in((\gamma U,\ \delta U)) &\Longleftrightarrow g(\gamma U)\cap\delta U\neq\phi \\ &\Longleftrightarrow \delta(\delta^{-1}g\gamma U\cap U)\neq\phi \\ &\Longleftrightarrow \delta^{-1}g\gamma\in((U,U)) \\ &\Longleftrightarrow g\in\delta((U,U))\gamma^{-1} \end{aligned}$$

Thus $Cl\ ((\gamma U,\ \delta U))=Cl\ (\delta((U,\ U))\gamma^{-1})=\delta\ Cl\ ((U,\ U))\gamma^{-1}$. So $Cl\ ((\gamma U,\ \delta U))$ is compact.

Proof of ((2)$\Longrightarrow$(1)).

Let X be a Cartan G-space and let $x\in X$. Then there exists a neighborhood $U\ni x$ such that $Cl\ ((U,U))$ is compact. Show that GU is a proper G-space. Let $gu\in GU$ and take gU as a neighborhood of u. Then for each $\gamma u\in GU$ the neighborhood γu in GU satisfies that $Cl\ ((gU,\ \gamma U))$ is compact by Lemma 3.7. Clearly, proper G-space is a Cartan G-space and so G_x is compact. By Proposition 3.5 there is a slice S at x in GU. Since GU is open in X, S is also a slice at x in X.

Theorem 3.8. Let X be a completely regular G-space such that every isotropy group is conjugate to H and let G be a locally compact Lie group. If every subgroup of H which is conjugate to H is equal to H then the orbit map $\pi: X \longrightarrow X/G$ is a fiber bundle with fiber G/H.

Proof. Let $x_o^* = G(x_o) \in X/G$. By Proposition 3.3 $G/Gx_o \cong G(x_o)$ and by hypothesis $G/Gx_o \cong G/H$. So $\pi^{-1}(x_o^*) \cong G/H$.

Also by Proposition 3.6 there is a slice S at x_o. As in the proof of Proposition 2.4 there is an equivariant embedding $\psi: G \times_H A \longrightarrow X$ such that $\operatorname{Im} \psi = GS$ is an open neighborhood of $G(x_o)$ in X. Since $G_{[e,a]} = Ha \subset H (a \in A)$ and $G_{[e,a]}$ is conjugate to H, by the hypothesis $Ha = H$. Thus H acts trivially on A and we get a chart

$$\varphi_A : G/H \times A \longrightarrow \pi^{-1}(A)$$

at $x_o^* \in X/G$.

Corollary 3.9. In addition to the conditions of Theorem 3.8, let T be a maximal torus of H and let $N = N(T)$, then

$$G \times_H X^T \longrightarrow X$$

is a fiber bundle with fiber $H/N \cap H$.

Proof. It's clear from the proof of Theorem 2.7 and Theorem 3.8.

The following model was given by B. Halpern and Kulkarni ([9]). We use this model as an example that a G-space which does not satisfy Cartan condition of Theorem 3.8 is not a fiber bundle.

Example 3.10. Let $G = R$ and let $X_1 = \{(x, y) \in R^2 \mid -1 < x < 1, xy \leq 1\}$

Let $X = X_1/\sim$, where $(x, y) \sim (x', y')$ iff either $(x, y) = (x', y')$ or $(x, y) = (-x', -y')$ and $xy = 1$. Then vertical unit vector field on X_1 induces a vector field on X and hence a flow ϕ_t which is defined for all $t \in R$ and so we have an R-action on X.

Take a neighborhood $[(0,0)] \in U \subset X$. Then $((U, U))$ is not bounded and so $Cl\ ((U, U))$ is not compact. Since the action is free, the isotropy groups are compact. So there is not a slice at $[(0,0)] \in X$. Also $\pi: X \longrightarrow X/G$ is not a fiber bundle because there is no chart over every neighborhood of $G[(0,0)] \in X/G$.

4. On the generalization to a non-Cartan G-space

L. M King ([6]) gave the following example which shows the existence of slices for a non-Cartan G-space and generalized the existence theorem of slice to a non-Cartan G-space.

Example 4.1. Let $X=R^2$ be the Euclidean plane and let H and K be the copies of the real group R. Let $G=H\times K$ and let the G action on X be defined by $(h,k)(x,y)=(x,y+h-k)$ where $h\in H$, $k\in K$, $(x,y)\in X$.

Let $(x_o,y_o)\in X$. Then for each neighborhood $U\ni(x_o,y_o)$ $G_{(x_o,y_o)}=\text{diag } R\times R\subset ((U,U))$. So this G-space is not a Cartan. But $\{(x',y_o)\mid x'\in R\}$ is clearly a slice at (x_o,y_o).

A subtransformation group (X,H,Φ_H) of a transformation group (X,G,Φ) is the H-space whose action Φ_H is the restriction of Φ to H.

King also gave the following example to show that there exists a slice in a subtransformation group but there is no slice in the transformation group.

Example 4.2. Under the same X and G as in Example 4.1, define the G-action on X as follows.

For each $(h,k)\in G$ and $(x,y)\in X$ define $(h,k)(x,y)=(x,y-h-kx)$. Then, for $(x_o,y_o)\in X$, $G(x_o,y_o)=\{(kx_o,k)\mid k\in K\}$ and so a slice S should contain a strip containing (x_o,y_o). But it is not a slice. But clearly subtransformation groups (X,H) and (X,K) each has a slice in X. So he found out necessary and sufficient conditions for the existence of a slice in a subtransformation group to imply the existence of a slice in the transformation group.

One thing we knew is that the fibration of an orbit map is closely related to the existence of slices. From the proofs of Proposition 2.4 and Therem 3.8 we have the following generalized throrem for the fibration of an orbit map.

Theorem4. 3 Let X be a G-space such that each isotropy group is conjugate to H and for each $x\in X$ let $G/Gx\cong G(x)$. Suppose that each isotropy group of H which is conjugate to H is H itself. Then if there is a slice in X the orbit map $\pi:X\longrightarrow X/G$ is a fiber bundle with fiber G/H and sturcture group $\text{Homeo}^G(G/H)$.

Corollary 4.4. Let X be a G-space and let the action be free. If there is a slice in X then the orbit map $\pi:X\longrightarrow X/G$ is a fiber bundle with fiber G.

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