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A Note on Bayesian Reliability Estimation for the Lognormal Model

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ABSTRACT

The problem of estimating the reliability using the Bayesian approach and the prior information about the reliability of a lognormal distribution is considered.

Some Bayes estimators are proposed and studied under the squared error loss and the Harris loss. Also Monte Carlo simulations are carried out to examine the performances of the proposed estimators and results are provided in the tables.

1. Introduction

In the problem of estimating the reliability of a certain system, a lognormal distribution is frequently used among many failure time distributions. For example, Howard and Dodson(1961) and Adams(1962) proposed the lognormal as a failure model for semiconductor devices and (computer) transistors. Mann, Schafer, and Singpurwalla(1974) showed that it is an applicable model for failure due to fatigue cracks. In addition, Ratnaparkhi and Park(1986) derived the lognormal model for the fatigue life of a composite material.

For Bayesian estimations, Evans(1969) discussed the benifits of Bayes estimations in the reliability problem. Padgett and Tsokos(1977) proposed and studied Bayes estimators of parameters, the mean time before failure and the reliability function. Also Padgett and Wei(1977) studied Bayes estimators of the reliability function and compared them with the maximum likelihood estimator(MLE) and the minimum variance unbiased estimator. The Bayesian lower bounds on the reliability function were investigated by Padgett and Wei(1978) and Padgett and Johnson(1983).

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However, in the most of these studies for estimating the reliability, the prior informations about the parameters of a lognormal distribution were used rather than the prior knowledges of the reliability itself. Hence in this paper we consider the problem of estimating the reliability using the Bayesian approach and the prior information about the reliability.

In Section 2, we formulate the problem of estimating the reliability and some motivations are given.

In Section 3, some Bayes estimators are proposed and studied under the squared error loss and the Harris loss.

In Section 4, Monte Carlo simulations are carried out to examine the performances of the proposed estimators and concluding remarks are given.

2. Motivation and Formulation

Suppose a random sample $\underline{X} = (X_1, X_2, \cdots, X_n)$ is available from a lognormal distribution with the probability density function(pdf), denoted by $\mathcal{LN}(\mu, \sigma^2)$,

$$f(x|\mu,\sigma^2) = \frac{1}{\sqrt{2\pi}\sigma x}\exp\left\{-\frac{(\ln x-\mu)^2}{2\sigma^2}\right\}, x > 0, -\infty < \mu < \infty, \sigma^2 > 0. \quad (2.1)$$

Our goal is to derive a Bayes estimator of the reliability function, at a given time t,

$$\theta = P(X > t) = 1 - \Phi\left(\frac{\ln t - \mu}{\sigma}\right), \qquad t > 0, \quad (2.2)$$

where $\Phi(\cdot)$ denotes the cumulative distribution function(cdf) of a standard normal distribution.

Assuming σ^2 is known, Padgett and Tsokos(1977) proposed the Bayes estimator of θ with respect to a normal prior $\mathcal{N}^{\mu}(\lambda, \xi^2)$ for μ and the squared-error loss, which is given by

$$\hat{\theta}_B = \Phi\left\{\left[\frac{\sum \ln x_i + \lambda\sigma^2/\xi^2}{n+\sigma^2/\xi^2} - \ln t\right]\sigma^{-1}\left[1+\frac{1}{n+\sigma^2/\xi^2}\right]^{-\frac{1}{2}}\right\}, \quad t > 0. \quad (2.3)$$

Also they obtained another Bayes estimator with respect to a uniform prior $\mathcal{U}^{\mu}(\alpha,\beta)$, $\alpha < \beta$.

Under the assumption of unknown μ and σ^2, Padgett and Wei(1977) suggested a Bayes estimator for θ with respect to a normal-gamma prior

$$\Pi(\mu,\sigma^2) = \Pi_1(\mu|\sigma^2)\Pi_2(\sigma^2) \propto \mathcal{N}^{\mu|\sigma^2}(\lambda,\tau\sigma^2)\mathcal{G}^{\sigma^2}(\alpha,\beta)$$

and the squared-error loss, which is given by

$$\tilde{\theta}_C = 1 - P\left[T_{2\alpha^*} < \sqrt{\frac{\alpha^*}{\beta^*}} \cdot \frac{\ln t - \lambda^*}{\sqrt{1+\tau^*}}\right], \quad t > 0, \tag{2.4}$$

where

$$\lambda^* = \frac{n\bar{y} + \lambda/\tau}{n + 1/\tau}, \; \tau^* = \frac{1}{n+1/\tau}, \; \alpha^* = \alpha + \frac{n}{2},$$

$$\beta^* = \beta + \frac{1}{2}\sum(\ln x_i - \bar{y})^2 + \frac{n(\bar{y}-\lambda)^2}{\tau}\left[2(n+\frac{1}{\tau})\right]^{-1},$$

$$\bar{y} = \frac{1}{n}\sum \ln x_i,$$

and $T_{2\alpha^*}$ is a random variable having a Student's t distribution with $2\alpha^*$ degrees of freedom. They also obtained a Bayes estimator with respect to the vague prior, $g(\mu,\sigma) \propto 1/\sigma$, which is given by

$$\tilde{\theta}_\nu = 1 - P\left[T_{n-1} < \frac{(\ln t - \bar{y})\sqrt{n-1}}{S\sqrt{1+1/n}}\right], \quad t > 0, \tag{2.5}$$

where

$$S^2 = \sum(\ln x_i - \bar{y})^2.$$

Here one can notice that most Bayes estimators of the reliability were derived with the priors on the unknown parameters rather than the reliability itself. However, in practice it is much easier and more reasonable to give a prior on θ directly than to give a prior on μ. Furthermore, it is very hard to provide the 'equivalent' priors on different unknown quantities, even though it can be taken into consideration the relationship between μ and θ in the equation (2.2). Thus some Bayes estimators with the priors on the reliability will be deriven in the next section.

3. Some Proposed Bayes Estimators

In this section we consider Bayes estimation of reliability θ. Suppose, for a given(fixed) time t, the beta prior distribution with parameters α and β , $\Theta \sim \mathcal{B}^\theta(\alpha,\beta)$, $\alpha,\beta > 0$, for θ is used. $\sigma^2$is assumed to be known. Based upon a random sample $\underline{X} = (X_1, X_2, \cdots, X_n)$ from $\mathcal{LN}(\mu,\sigma^2)$, the likelihood function can be written in terms of θ as

$$L(\theta|\underline{X}) \propto \exp\left\{-\frac{n}{2}\Phi^{-1}(\theta)^2 + \frac{\Phi^{-1}(\theta)(\sum \ln X_i - n\ln t)}{\sigma}\right\}, \tag{3.1}$$

$$0<\theta<1\ ,0<\sigma^2,\ 0<X_i<\infty,\ i=1,2,\cdots,n.$$

Then the posterior density of θ given $\underline{X}=\underline{x}$ is

$$\Pi(\theta|\underline{x}) = \exp\left\{-\frac{n}{2}\Phi^{-1}(\theta)^2 + \frac{\Phi^{-1}(\theta)(\sum \ln x_i - n\ln t)}{\sigma}\right\}\theta^{\alpha-1}(1-\theta)^{\beta-1}/I_{\alpha,\beta}, \tag{3.2}$$

where

$$I_{\alpha,\beta} = \int_0^1 \exp\left\{-\frac{n\Phi^{-1}(\theta)^2}{2} + \frac{\Phi^{-1}(\theta)(\sum \ln x_i - n\ln t)}{\sigma}\right\}\theta^{\alpha-1}(1-\theta)^{\beta-1}\,d\theta. \tag{3.3}$$

Hence with the squared error loss, the following theorem can be obtained easily.

Theorem 1. Let $\Pi(\theta) = \theta^{\alpha-1}(1-\theta)^{\beta-1}/B(\alpha,\beta),\ \ 0<\theta<1$, be a prior density of θ. Then the Bayes estimator $\hat{\theta}_S$ of the reliability function is

$$\hat{\theta}_S = \frac{I_{\alpha+1,\beta}}{I_{\alpha,\beta}}, \tag{3.4}$$

where $I_{a,b}$ is given by the equation (3.3), and $B(\alpha,\beta)$ a the beta function with parameters α and β.

If $\alpha=\beta=1$ for a beta prior distribution, then θ follows a uniform distribution $\mathcal{U}(0,1)$. Thus the posterior distribution of θ given $\underline{X}=\underline{x}$ is, for $0<\theta<1$,

$$\Pi(\theta|\underline{x}) = \sqrt{\frac{n+1}{2\pi}}\frac{1}{\varphi(\Phi^{-1}(\theta))}\exp\left\{-\frac{n+1}{2\sigma^2}\left(\sigma\Phi^{-1}(\theta) - \frac{\sum \ln x_i - n\ln t}{n+1}\right)^2\right\} \tag{3.5}$$

,where $\varphi(\cdot)$ denotes the pdf of a standard normal distribution. Hence one can get the following corollary.

Corollary 1. Let $\Pi(\theta)=1,\ \ 0<\theta<1$, be a prior density of θ. Then the Bayes estimator $\tilde{\theta}_S$ of θ is

$$\tilde{\theta}_S = \Phi\left[\frac{\sum \ln x_i - n\ln t}{(n+1)\sigma}\sqrt{\frac{n+1}{n+2}}\right]. \tag{3.6}$$

proof.

Consider the one-to-one transformation $Y = \Phi^{-1}(\theta)$. Then $\tilde{\theta}_S$ is the posterior mean. That is,

$$\begin{aligned}\tilde{\theta}_S &= \int_{-\infty}^{\infty} \Phi(y)\sqrt{\frac{n+1}{2\pi}}\exp\left\{-\frac{n+1}{2}\left(y-\frac{\sum \ln x_i - n\ln t}{(n+1)\sigma}\right)^2\right\}dy \\ &= \int_{-\infty}^{\infty} \Phi\left[\frac{\sum \ln x_i - n\ln t}{(n+1)\sigma} - \frac{z}{\sqrt{(n+1)}}\right]\varphi(z)\,dz \\ &= \Phi\left[\frac{\sum \ln x_i - n\ln t}{(n+1)\sigma}\sqrt{\frac{n+1}{n+2}}\right],\end{aligned}$$

by 10,010.8 in Owen(1980).

Notice that the Bayes estimator $\tilde{\theta}_S$ is approximately equal to the MLE

$$\Phi\left(\frac{\sum \ln x_i - n\ln t}{n\sigma}\right) \tag{3.7}$$

for large sample. Also as $\xi^2 \to \infty$, the Bayes estimator (2.3) is approximately equal to the Bayes estimator $\tilde{\theta}_s$ and MLE for large sample size.

Now, we consider the loss function suggested by Harris(1976) for the case k=2, which is

$$L(\theta,\delta) = |\frac{1}{1-\delta} - \frac{1}{1-\theta}|^2. \tag{3.8}$$

The reasons of proposing such a loss function were as follows:

"If the system reliability is 0.99, on the average it should fail one time in 100, whereas if the system reliability is 0.999, it should fail one time in 1000 and hence is ten times as good. Thus the loss function should depend on how well one estimates $(1-\delta)^{-1}$."

Under the Harris loss the Bayes estimator $\hat{\theta}_H$ of the reliability can be deriven from the following relation

$$\frac{1}{1-\delta} = E^{\theta|\underline{x}}\left[\frac{1}{1-\theta}\right] \equiv \gamma(\underline{x}). \tag{3.9}$$

Thus

$$\hat{\theta}_H = 1 - \frac{1}{\gamma(\underline{x})}, \tag{3.10}$$

where

$$\gamma(\underline{x}) = E^{\theta|\underline{x}}\left[\frac{1}{1-\theta}\right] = \frac{I_{\alpha,\beta-1}}{I_{\alpha,\beta}}. \tag{3.11}$$

Then the following theorem and its corollary hold.

Theorem 2. Under the Harris loss function, the Bayes estimator $\hat{\theta}_H$ of the reliability function with respect to the beta prior, $\mathcal{B}^\theta(\alpha,\beta)$, is given by

$$\hat{\theta}_H = 1 - \frac{I_{\alpha,\beta}}{I_{\alpha,\beta-1}}, \tag{3.12}$$

where $I_{\alpha,\beta}$ is given by the equation (3.3).

Corollary 2. Under the Harris loss function, the Bayes estimator $\tilde{\theta}_H$ of θ with respect to the uniform prior over $\mathcal{U}(0,1)$ is

$$\tilde{\theta}_H = 1 - \frac{1}{\int_{-\infty}^{\infty} \varphi(z)\big(1-\Phi(h(\underline{x},t,n))\big)^{-1}\,dz}, \tag{3.13}$$

where

$$h(\underline{x},t,n) = \frac{(\sum \ln x_i - n\ln t)}{(n+1)\sigma} + \frac{z}{\sqrt{n+1}}. \tag{3.14}$$

Next, we consider a Bayes estimator for the reliability θ when the paramter μ is known. Without loss of generality, it can be assumed that $\mu = 0$. For a given time t, if a prior for θ is a beta distribution with parameters α and β, then the posterior density of θ given $\underline{x}$ is

$$\Pi(\theta|\underline{x}) = \Phi^{-1}(\theta)^n \exp\left\{-\frac{\Phi^{-1}(\theta)^2\sum(\ln x_i)^2}{2(\ln t)^2}\right\}\theta^{\alpha-1}(1-\theta)^{\beta-1}/J_{\alpha,\beta}, 0<\theta<1, \tag{3.15}$$

where

$$J_{\alpha,\beta} = \int_0^1 \Phi^{-1}(\theta)^n \exp\left\{-\frac{\Phi^{-1}(\theta)^2\sum(\ln x_i)^2}{2(\ln t)^2}\right\}\theta^{\alpha-1}(1-\theta)^{\beta-1}\,d\theta \tag{3.16}$$

Hence the following theorem can be obtained easily.

Theorem 3. Under the squared error loss and a beta prior with parameters α and β, the Bayes estimator $\hat{\theta}_S^*$ is

$$\hat{\theta}_S^* = E^{\theta|\underline{x}}(\theta) = \frac{J_{\alpha+1,\beta}}{J_{\alpha,\beta}}, \tag{3.17}$$

where $J_{\alpha,\beta}$ is given by the equation (3.16).

Also using the lemma by Padgett and Wei(1977), one can get the following corollary.

Corollary 3. Under the squared error loss and a uniform prior $\mathcal{U}(0,1)$ for θ, the Bayes estimator $\hat{\theta}_S^{**}$ is

$$\hat{\theta}_S^{**} = P\left[T_{n+1} < \frac{-\ln t}{\sqrt{\frac{\sum(\ln x_i)^2 + (\ln t)^2}{n+1}}}\right], \tag{3.18}$$

where T_{n+1} is a random variable having Student's t distribution with n+1 degrees of freedom.

Remark. As $n \to \infty$, $(\sum(\ln x_i)^2 + (\ln t)^2)/(n+1) \to \sigma^2$, a.e. and thus T_{n+1} follows a standard normal distribution as a limiting distribution. Therefore, the estimator $\hat{\theta}_S^{**}$ is a strongly consistent estimator of θ.

If the Harris loss is used, the following theorem and its corollary can be obtained similarly.

Theorem 4. Under the Harris loss, the Bayes estimator $\hat{\theta}_H^*$ with respect to a beta prior with parameters α and β is given by

$$\hat{\theta}_H^* = 1 - \frac{J_{\alpha,\beta}}{J_{\alpha,\beta-1}}, \tag{3.19}$$

where $J_{\alpha,\beta}$ is given by the equation (3.16).

Corollary 4. Under the Harris loss, the Bayes estimator $\hat{\theta}_H^{**}$ with respect to a uniform prior $\mathcal{U}(0,1)$ is

$$\theta_H^{**} = 1 - \frac{1}{\int_0^\infty g(z)(\Phi(\ln t\sqrt{z}))^{-1}\,dz}, \tag{3.20}$$

where $g(\cdot)$ denotes the pdf of a gamma distribution with density

$$\kappa^{\psi} z^{\psi-1} e^{-\kappa z}/\Gamma(\psi), \quad z > 0.$$

Remark. Even though the parameters are unknown, it is possible to have some information about the relation between mean and variance, such as the coefficient of variation(CV). For the lognormal model, the square of CV is $e^{\sigma^2} - 1$. Thus for known CV, the Bayes estimators can be obtained by replacing σ^2 in the equations (3.4), (3.6), (3.12), and (3.13) with $\ln(CV^2 + 1)$.

4. Monte Carlo Simulation Study

To evaluate the performances of the proposed Bayes estimators for known σ^2 and the given mission time t, some Monte Carlo simulations were carried out. The International Mathematical and Statistical Libraries Subroutines MDNOR, MDNRIS, and GGNLG were used and the Gauss-Legendre quadratures based on 50 points were used for the numerical integration and simulations were replicated 1000 times. We consider both the squared-error loss and the Harris loss with several prior distributions for θ and μ. The estimated mean squared errors(MSE's) and biases of $\hat{\theta}_S$,$\tilde{\theta}_S$,$\hat{\theta}_H$,$\tilde{\theta}_H$, and $\hat{\theta}_B$ were computed and tabulated in Tables 1 and 2 (biases are given in the parenthesis).

From Tables 1 and 2, one can observe the followings:

1). The Bayes estimators of θ with respect to the prior distribution for θ perform better than the Bayes estimator with respect to the prior distribution for μ in terms of MSE (bias) , provided that the prior for θ spreads near the true value of θ regardless of the type of a loss.

2). The estimated MSE's (biases) decreases as n increases for both the squared error loss and the Harris loss.

3). The estimated MSE's (biases) become close to each other as the true value of θ approaches to 0 or 1 for both losses.

4). The performances of the proposed estimators are relatively sensitive to the prior. That is, the MSE's with a proper prior are smaller than those with a wrong prior.

5). For a given prior distribution, the estimator under the Harris loss performs better than the estimator under the squared error loss as θ becomes larger.

References

1. Adams, J.D.(1962), "Failure Time Distribution Estimation", *Semiconductor Reliability*, 2, 41-52.
2. Evans, R.A.(1969), "Prior Knowledge, Engineers versus Statisticians", IEEE *Trans. Reliability*, R-18, 143.
3. Harris, B.(1976), "A Survey of Statistical Methods in Systems Reliability using Bernoulli Sampling of Components", *Proc. of the Conference on the Theory and Applications of Reliability with Emphasis on Bayesian and Nonparametric Methods*, New York ; Academic.
4. Howard, B.T., and Dodson, G.A.(1961), "High Stress Aging to Failure of Semiconductor Devices", *Proc. Seventh National Sympos. Reliability and Quality Control*, 201-207.
5. Mann, N.R., Schafer R.E., and Singpurwalla, N.D.(1974), *Methods for Statistical Analysis of Reliability and Life Data*, New York ; Wiley.
6. Owen, D.B.(1980), "A Table of Normal Integrals", *Commun. Statist.-Simula. Computa.*, B9(4), 389-419.
7. Padgett, W.J., and Johnson, M.P.(1983), " Some Bayesian Lower Bounds on Reliability in the Lognormal Distribution", *Canadian J. Statist.*, 11, 137-147.
8. Padgett, W.J., and Tsokos, C.P.(1977), "Bayes Estimation of Reliability for the Lognormal Failure Model", *Proc. of the Conference on the Theory and Applications of Reliability with Emphasis on Bayesian and Nonparametric Methods*, New York ; Academic.
9. Padgett, W.J., and Wei, L.J.(1977), "Bayes Estimation of Reliability for the Two Parameter Lognormal Distribution", *Commun. Statist.-Theor. Meth.*, A6, 443-447.
10. Padgett, W.J., and Wei, L.J.(1978), "Bayesian Lower Bounds on Reliability for the Lognormal Model", IEEE *Trans. Reliability*, R-27, 161-165.
11. Ratnaparkhi, M.V., and Park, W.J.(1986), "Lognormal Distribution- Model for Fatigue Life and Residual Strength of Composite Materials", IEEE *Trans. Reliability*, R-35, 312-315.

Table 1. Estimates of MSE's and Biases under the Squared-Error Loss

				Prior for θ						Prior for μ		
(μ, σ^2)	n	t	θ	$B(1,9)$	$B(3,7)$	$B(1,1)$	$B(5,5)$	$B(7,3)$	$B(9,1)$	$\mathcal{N}(0,1)$	$\mathcal{N}(1,0.649)$	$\mathcal{N}(1,0.105)$
(0,.5)	10	0.5	.8365	.07242	.03951	.00778	.01777	.00601	.00278	.00680	.00528	.01150
				(-.25896)	(-.18623)	(-.04233)	(-.11653)	(-.05044)	(.01108)	(-.02198)	(.00370)	(.10006)
		0.7	.6930	.05739	.02831	.01056	.01100	.00481	.00886	.01141	.01050	.03285
				(-.22707)	(-.15055)	(-.02948)	(-.07546)	(-.00223)	(.06856)	(-.01577)	(.01930)	(.16842)
		1.0	.5000	.02877	.01156	.01153	.00577	.01150	.02864	.01371	.01515	.06374
				(-.15281)	(-.07671)	(-.00029)	(-.00020)	(.07630)	(.15242)	(-.00032)	(.03938)	(.23255)
		1.2	.3983	.01666	.00654	.01128	.00730	.01949	.04339	.01307	.01573	.07496
				(-.10868)	(-.03469)	(.01588)	(.04071)	(.11707)	(.19399)	(.00831)	(.04728)	(.25059)
		1.5	.2832	.00740	.00475	.01024	.01211	.03048	.06059	.01081	.01426	.07787
				(-.05874)	(.01091)	(.03192)	(.08332)	(.15784)	(.23404)	(.01674)	(.05206)	(.25290)
		2.0	.1635	.00279	.00597	.00775	.01769	.03937	.07222	.00679	.00986	.06324
				(-.01136)	(.05013)	(.04192)	(.11618)	(.18586)	(.25857)	(.02156)	(.04909)	(.22417)
	30	0.5	.8365	.01338	.00793	.00225	.00424	.00215	.00150	.00211	.00193	.00315
				(-.10522)	(-.07628)	(-.01443)	(-.04829)	(-.02133)	(.00451)	(-.00683)	(.00197)	(.04291)
		0.7	.6930	.01311	.00748	.00387	.00408	.00283	.00362	.00398	.00389	.00774
				(-.09921)	(-.06584)	(-.00973)	(-.03296)	(-.00062)	(.03111)	(-.00431)	(.00804)	(.06830)
		1.0	.5000	.00811	.00463	.00472	.00353	.00480	.00845	.00503	.00526	.01247
				(-.06803)	(-.03341)	(.00145)	(.00125)	(.03591)	(.07052)	(.00150)	(.01553)	(.08752)
		1.2	.3983	.00543	.00348	.00451	.00379	.00642	.01141	.00475	.00514	.01317
				(-.04758)	(-.01390)	(.00756)	(.02010)	(.05438)	(.08888)	(.00460)	(.01827)	(.09018)
		1.5	.2832	.00308	.00274	.00373	.00436	.00808	.01400	.00378	.00425	.01183
				(-.02422)	(.00691)	(.01335)	(.03870)	(.07111)	(.10407)	(.00746)	(.01960)	(.08558)
		2.0	.1635	.00150	.00223	.00233	.00443	.00824	.01383	.00216	.00254	.00778
				(-.00290)	(.02306)	(.01632)	(.05013)	(.07821)	(.10724)	(.00873)	(.01782)	(.06926)
(0,.7)	10	0.5	.8365	.06944	.03690	.00841	.01583	.00520	.00370	.00789	.00643	.02064
				(-.25320)	(-.17884)	(-.03925)	(-.10702)	(-.03826)	(.02661)	(-.01986)	(.01415)	(.13822)
		0.7	.6930	.05275	.02516	.01031	.00937	.00484	.01078	.01142	.01109	.04680
				(-.21721)	(-.14040)	(-.02483)	(-.06477)	(.00930)	(.08124)	(-.01269)	(.03012)	(.20709)
		1.0	.5000	.02840	.01123	.01099	.00550	.01132	.02857	.01309	.01519	.08109
				(-.15241)	(-.07625)	(.00042)	(.00030)	(.07685)	(.15300)	(.00045)	(.04754)	(.27048)
		1.2	.3983	.01798	.00681	.01086	.00665	.01796	.04098	.01268	.01598	.09481
				(-.11512)	(-.04066)	(.01406)	(.03506)	(.11156)	(.18849)	(.00758)	(.05419)	(.29093)
		1.5	.2832	.00917	.00464	.01021	.01042	.02739	.05615	.01110	.01532	.10247
				(-.07165)	(-.00052)	(.02843)	(.07298)	(.14825)	(.22489)	(.01497)	(.05857)	(.30013)
		2.0	.1635	.00369	.00525	.00852	.01596	.03711	.06976	.00800	.01217	.09441
				(-.02616)	(.03876)	(.03993)	(.10756)	(.17942)	(.25380)	(.02057)	(.05721)	(.28452)
	30	0.5	.8365	.01412	.00831	.00283	.00445	.00239	.00198	.00273	.00253	.00524
				(-.10682)	(-.07617)	(-.01442)	(-.04632)	(-.01736)	(.01065)	(-.00712)	(.00475)	(.05980)
		0.7	.6930	.01277	.00727	.00418	.00406	.00307	.00419	.00434	.00428	.01038
				(-.09651)	(-.06269)	(-.00915)	(-.02927)	(.00369)	(.03614)	(-.00442)	(.01072)	(.08434)
		1.0	.5000	.00832	.00479	.00485	.00363	.00485	.00844	.00517	.00545	.01526
				(-.06873)	(-.03412)	(.00060)	(.00053)	(.03518)	(.06978)	(.00061)	(.01721)	(.10212)
		1.2	.3983	.00599	.00376	.00472	.00381	.00620	.01096	.00499	.00544	.01623
				(-.05141)	(-.01753)	(.00587)	(.01662)	(.05101)	(.08559)	(.00332)	(.01961)	(.10514)
		1.5	.2832	.00377	.00300	.00414	.00428	.00774	.01345	.00426	.00482	.01547
				(-.03091)	(.00105)	(.01131)	(.03358)	(.06664)	(.10016)	(.00608)	(.02108)	(.10230)
		2.0	.1635	.00205	.00250	.00298	.00461	.00853	.01439	.00288	.00341	.01188
				(-.00986)	(.01819)	(.01534)	(.04719)	(.07707)	(.10776)	(.00806)	(.02027)	(.08915)

Table 2. Estimates of MSE's and Biases under the Harris Loss

(μ, σ^2)	n	t	θ	Prior for θ $B(1,9)$	$B(3,7)$	$B(1,1)$	$B(5,5)$	$B(7,3)$	$B(9,1)$
(0,.5)	10	0.5	.8365	.06181	.03141	.00563	.01261	.00398	.00371
				(-.23709)	(-.16263)	(-.00918)	(-.09172)	(-.02515)	(.03582)
		0.7	.6930	.04973	.02273	.01004	.00825	.00547	.01322
				(-.20849)	(-.12935)	(.00508)	(-.05194)	(.02322)	(.09533)
		1.0	.5000	.02523	.00974	.01358	.00660	.01588	.03734
				(-.13902)	(-.05985)	(.02846)	(.01968)	(.09909)	(.17791)
		1.2	.3983	.01481	.00624	.01402	.00951	.02520	.05355
				(-.09748)	(-.02041)	(.03971)	(.05814)	(.13766)	(.21770)
		1.5	.2832	.00688	.00556	.01295	.01517	.03681	.07127
				(-.05055)	(.02202)	(.04925)	(.09753)	(.17530)	(.25486)
		2.0	.1635	.00294	.00709	.00950	.02055	.04492	.08155
				(-.00643)	(.05751)	(.05182)	(.12633)	(.19906)	(.27508)
	30	0.5	.8365	.01123	.00637	.00198	.00327	.00176	.00165
				(-.09445)	(-.06545)	(-.00252)	(-.03747)	(-.01059)	(.01507)
		0.7	.6930	.01129	.00626	.00382	.00353	.00297	.00447
				(-.08913)	(-.05535)	(.00297)	(-.02210)	(.01055)	(.04255)
		1.0	.5000	.00726	.00428	.00504	.00375	.00568	.01007
				(-.06041)	(-.02518)	(.01172)	(.01009)	(.04534)	(.08053)
		1.2	.3983	.00503	.00347	.00491	.00427	.00748	.01315
				(-.04155)	(-.00724)	(.01576)	(.02740)	(.06232)	(.09747)
		1.5	.2832	.00301	.00294	.00409	.00493	.00909	.01555
				(-.02010)	(.01161)	(.01892)	(.04402)	(.07706)	(.11067)
		2.0	.1635	.00155	.00243	.00252	.00482	.00890	.01483
				(-.00076)	(.02564)	(.01911)	(.05319)	(.08180)	(.11137)
(0,.7)	10	0.5	.8365	.05953	.02937	.00664	.01124	.00385	.00556
				(-.23216)	(-.15575)	(-.00484)	(-.08233)	(-.01259)	(.05236)
		0.7	.6930	.04570	.02015	.01023	.00718	.00611	.01588
				(-.19929)	(-.11975)	(.00939)	(-.04164)	(.03454)	(.10807)
		1.0	.5000	.02484	.00940	.01302	.00633	.01571	.03729
				(-.13860)	(-.05936)	(.02925)	(.02022)	(.09967)	(.17853)
		1.2	.3983	.01589	.00630	.01350	.00868	.02352	.05099
				(-.10351)	(-.02595)	(.03876)	(.05291)	(.13255)	(.21255)
		1.5	.2832	.00834	.00521	.01297	.01333	.03366	.06685
				(-.06263)	(.01149)	(.04760)	(.08813)	(.16665)	(.24661)
		2.0	.1635	.00368	.00635	.01065	.01902	.04312	.07982
				(-.02011)	(.04747)	(.05238)	(.11920)	(.19422)	(.27198)
	30	0.5	.8365	.01195	.00677	.00257	.00353	.00210	.00230
				(-.09607)	(-.06524)	(-.00193)	(-.03527)	(-.00625)	(.02173)
		0.7	.6930	.01106	.00616	.00417	.00361	.00331	.00516
				(-.08673)	(-.05245)	(.00336)	(-.01861)	(.01472)	(.04749)
		1.0	.5000	.00746	.00442	.00516	.00384	.00572	.01004
				(-.06113)	(-.02591)	(.01084)	(.00935)	(.04459)	(.07978)
		1.2	.3983	.00552	.00371	.00511	.00426	.00724	.01268
				(-.04515)	(-.01064)	(.01438)	(.02416)	(.05919)	(.09441)
		1.5	.2832	.00363	.00316	.00452	.00484	.00878	.01508
				(-.02629)	(.00628)	(.01757)	(.03944)	(.07314)	(.10732)
		2.0	.1635	.00208	.00272	.00323	.00509	.00933	.01561
				(-.00707)	(.02148)	(.01904)	(.05102)	(.08147)	(.11275)